Long period return level estimates


Photo: F. Zwiers

Francis Zwiers and M.A. Ben Alaya, PCIC, University of Victoria
Xuebin Zhang, Climate Research Division, Environment and Climate Change Canada
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Outline

- Introduction
- Problem definition
- Possible solution
- Conclusions

The elegant theoretical basis

Photo: F. Zwiers

Introduction

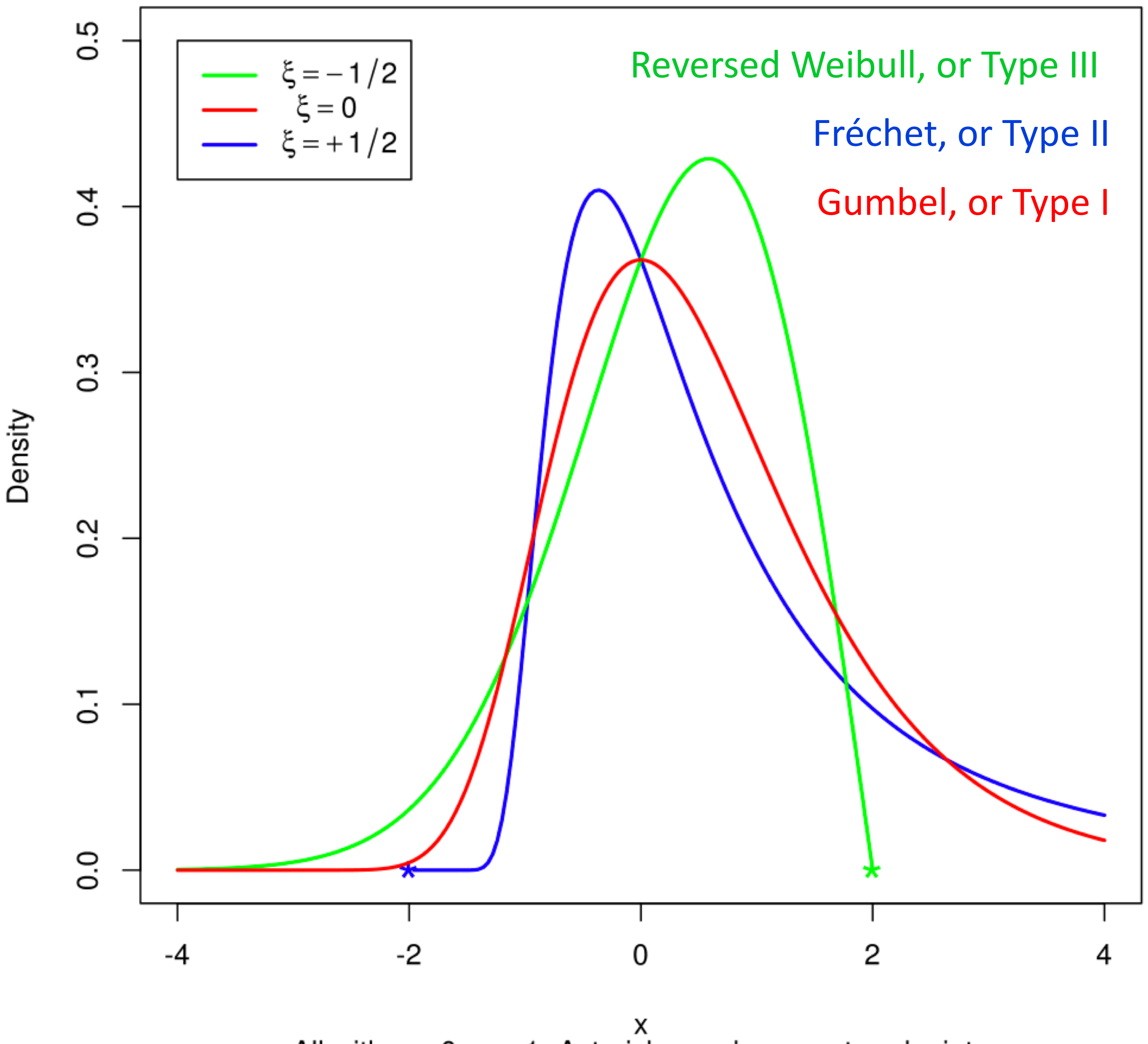
- Models for the main body of a probability distribution are not guaranteed to represent the upper tail well → rely on extreme value theory
- Two general approaches
  - Block maximum
    - Fixed length blocks (typically a year)
    - Analyze a time series of block maxima
    - Leads to the Generalized Extreme Value (GEV) distribution
  - Peaks over threshold
    - Set a high threshold
    - Analyze exceedances above the threshold, usually after de-clustering
    - Leads to the Generalized Pareto distribution (GPD)

Block maximum approach

- Approach most widely used in engineering design problems
- Natural block length is a year ➔ annual maxima
- Seeks to estimate a point in the upper tail of the distribution (e.g., an n-year ”return level”)
- Most work uses the Generalized Extreme Value (GEV) distribution

$$F(Y = y|\mu,\sigma,\xi) = \begin{cases} \exp\left\{-\left[1+\frac{\xi(y-\mu)}{\sigma}\right]^{-1/\xi}\right\}, \xi < 0, y < \mu - \sigma/\xi & \text{Reversed Weibull, or Type III} \\ \exp\left\{-\left[1+\frac{\xi(y-\mu)}{\sigma}\right]^{-1/\xi}\right\}, \xi > 0, y > \mu - \sigma/\xi & \text{Fréchet, or Type II} \\ \exp\left\{-\exp\left[-\frac{y-\mu}{\sigma}\right]\right\}, \xi = 0 & \text{Gumbel, or Type I} \end{cases}$$

Generalized extreme value densities



All with $\mu = 0$, $\sigma = 1$. Asterisks mark support-endpoints

R D Gill - Created by R D Gill, 4 January 2013, using R script

Block maximum approach

- Basis for the GEV is the ”Extremal Types Theorem”

Let $M_n = \max\{X_1, X_2, \dots, X_n\}$ where X_i are iid random variables. If for some constants $a_n > 0, b_n$,

$$P\{a_n(M_n - b_n) \leq x\} \overset{w}{\to} G(x)$$

for some nondegenerate G, then G is one of the three extreme value types that comprise the GEV distribution

- This theorem has been generalized to various types of stationary processes

Real world applications

- Note that this is a *limit* theorem, like the Central Limit Theorem
- Working assumption is that 1-year blocks are large enough so that convergence to the GEV has more or less occurred
- But … observed processes are generally not iid or even stationary
- There can be strong dependence and a strong annual cycle (e.g., think of stream flow in snow dominated catchments; rainfall in monsoon regions; temperature in midlatitude continental regions)
- There may also be “surprises” in the upper tail (e.g., think of tropical cyclones or atmospheric rivers)
- Thus the effective block length can be small, raising the question of whether an approximation proposed in a limit theorem can be used

Real world applications

- Other distributions such as Log-Pearson Type III and Log-Normal are also sometimes used based on empirical assessments of the quality of the fit to the available sample of block maxima

- For example, the Log-Pearson Type III is used for flood frequency analysis in the United States (e.g., USGS Bulletin 17c)


Stability?

Can practitioners safely extrapolate fitted EV distributions?

- Most engineering analyses use the block maximum approach
  - Very often assume the GEV distribution
  - Increasingly require ambitious extrapolation deep into the upper tail
- Example
  - In Canada, wind load estimates are based on 50-year return levels for annual maximum wind pressure, which are multiplied by a fixed ”load factor”
  - Higher importance buildings use larger load factors
  - The use of fixed “load factors” leads to differences in building reliability
  - There is therefore a move to adopt ”uniform risk” design procedures that eliminate fixed load factors but require return level estimates for much longer periods (up to 1000’s of years for “post disaster” buildings)

Can practitioners safely extrapolate fitted EV distributions?

- Implicitly assume that the sampling of extremes results in convergence, and thus a stable upper tail
  - Convergence to the GEV (and thus max-stable conditions) is only occasionally discussed in the practitioner literature
  - Not testable with available observational records
- Analysis of the CanRCM4 large ensemble of regional climate simulations suggests we should think more deeply

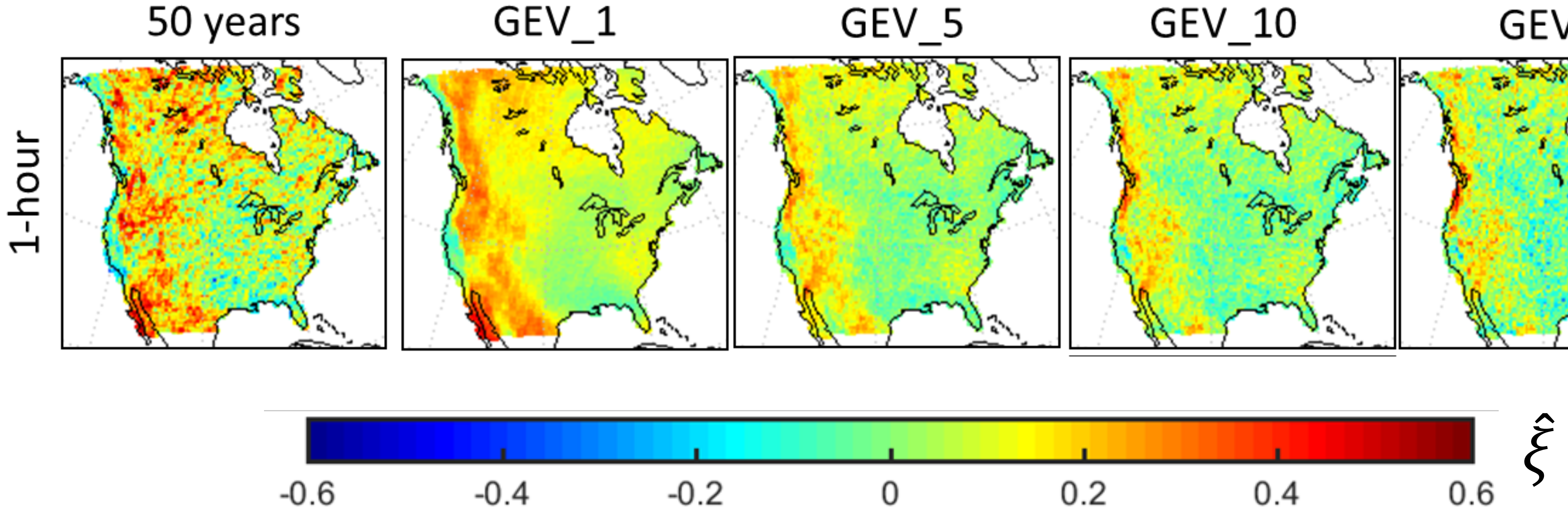
CanRCM4 large ensemble

- 50-members, 50 km resolution, driven by the CanESM2 large ensemble
- historical + RCP8.5 forcing
- hourly precipitation archived for 35-members
- considering 1951-2000 only, we have 35x50=1750 annual maxima

GEV fitting method

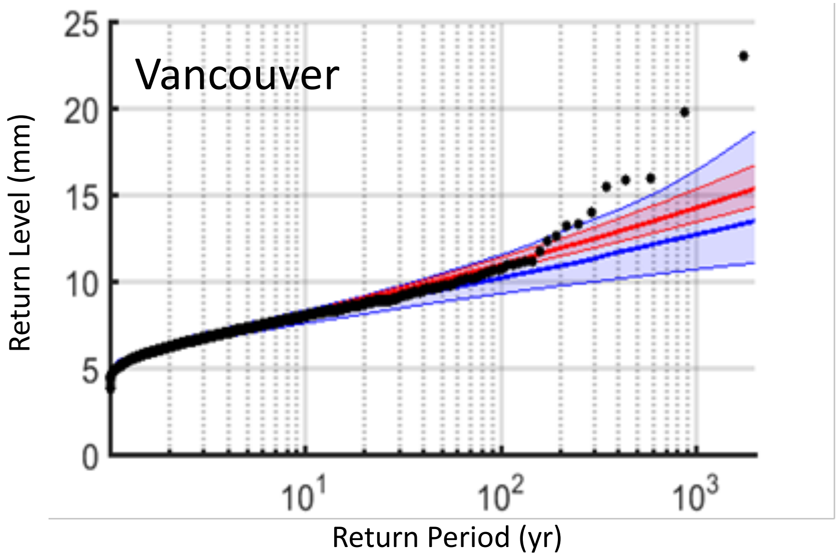
- assume stationarity over 1951-2000
- fit via maximum likelihood
- results are similar if using probability weighted moments

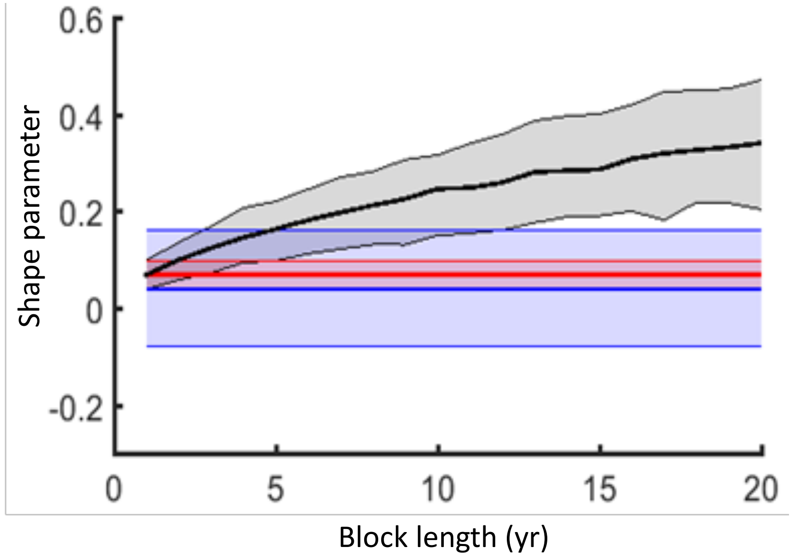
Shape parameters of extreme 1-hour precipitation



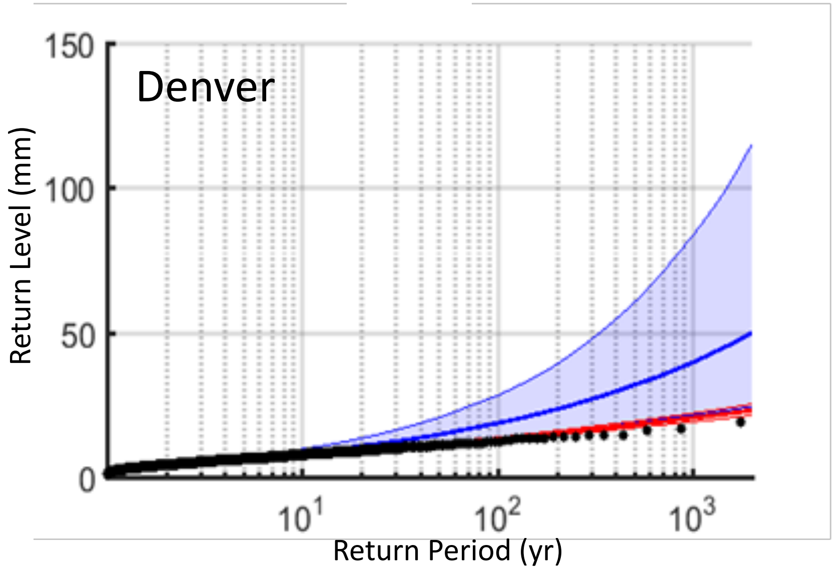
GEV fits to block maxima at 2 locations

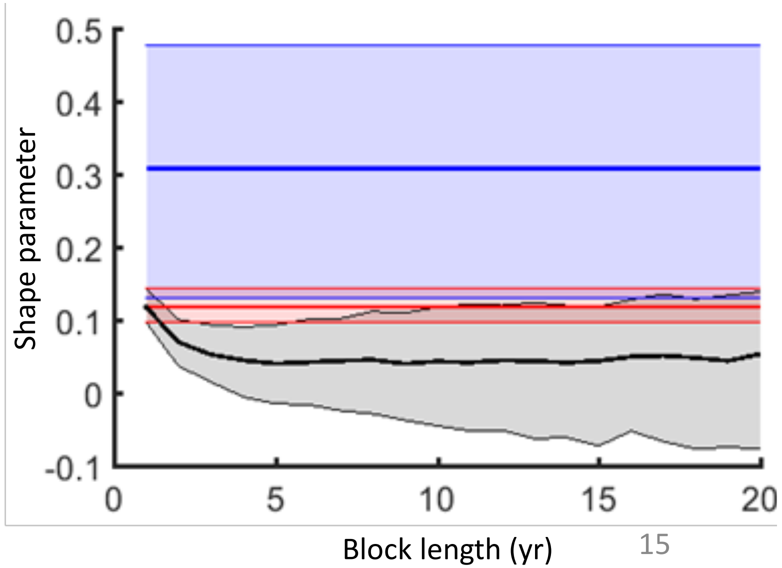
Extreme quantiles based on 1750-years of CanRCM4 simulated 1-hour precipitation accumulations for 1951-2000





- ▪▪▪▪ Empirical distribution from 1750 annual maxima
- GEV from 50 annual maxima (blue)
- GEV from 1750 annual maxima (red)
- GEV from 1 to 20 year block maxima (black)





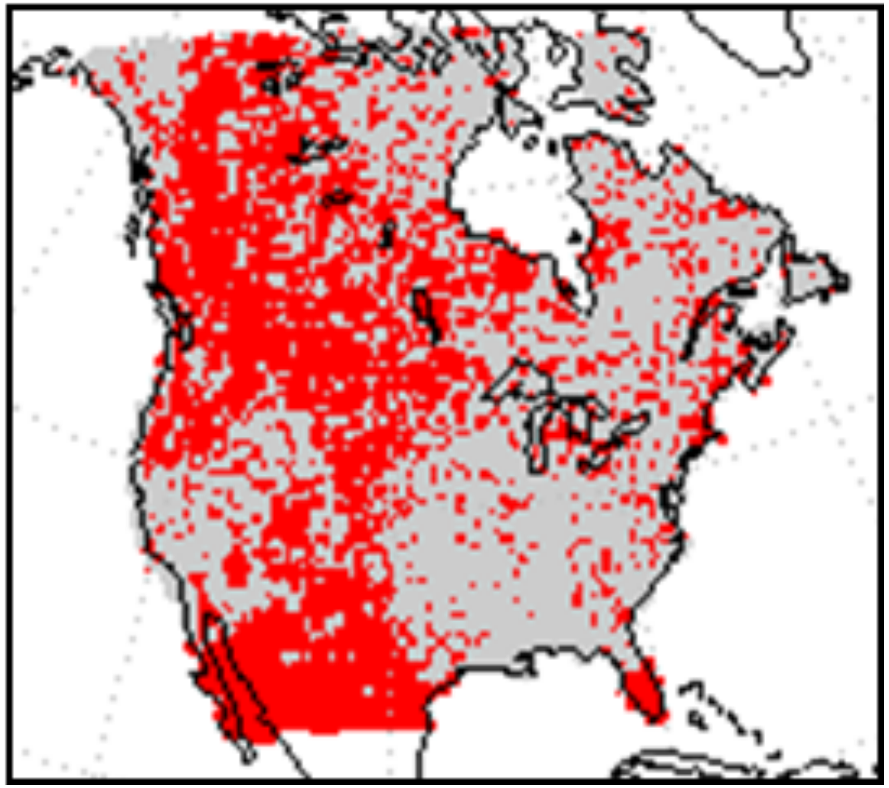
Is there something unusual about the 50-year sample?
(red ➔ rejection of χ^2 test at 5% significance level)

50-year sample vs remaining 1700 years

GEV goodness of fit to annual maxima
(red ➔ rejection of χ^2 test at 5% significance level)

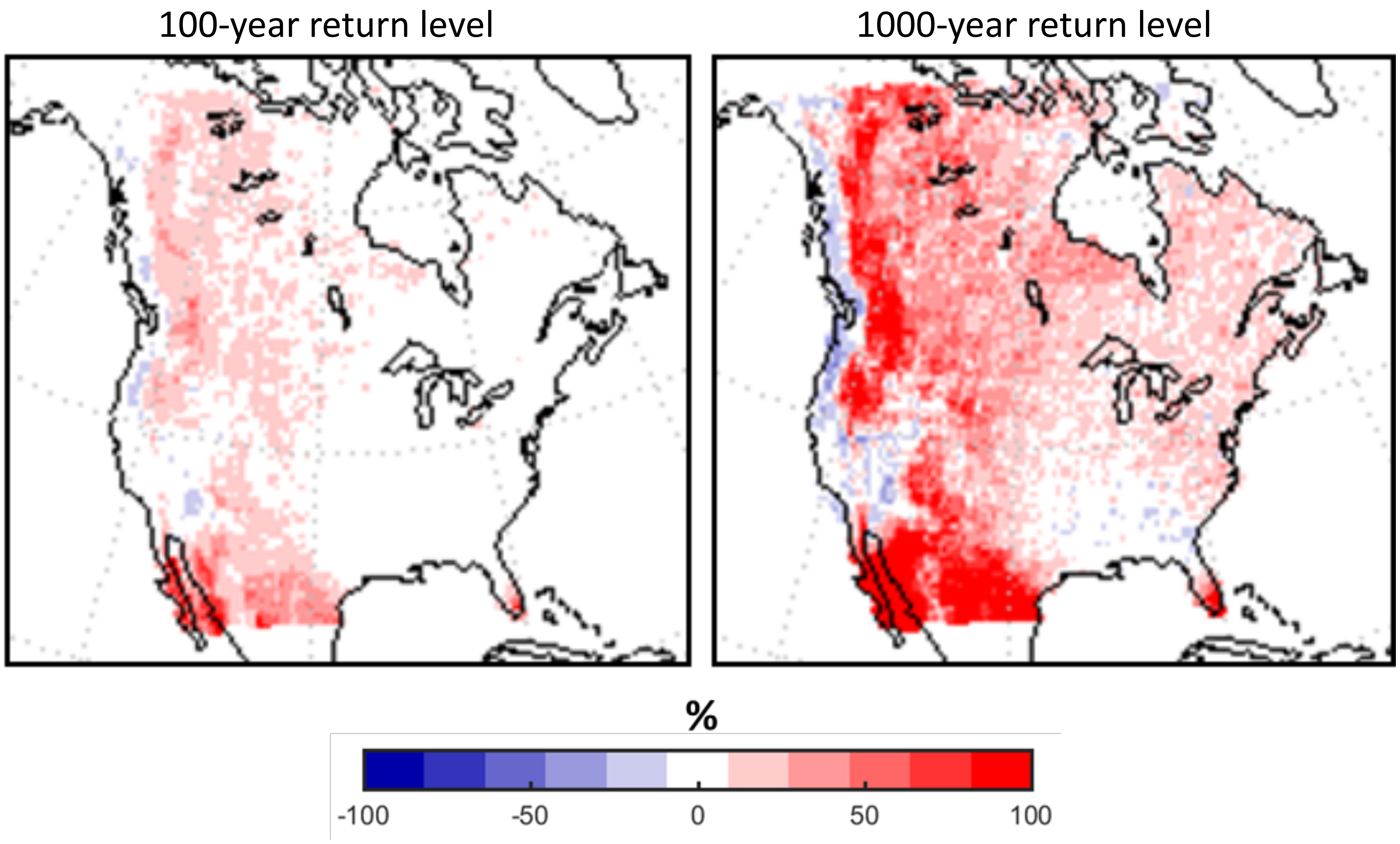
50-year sample

1750-year sample



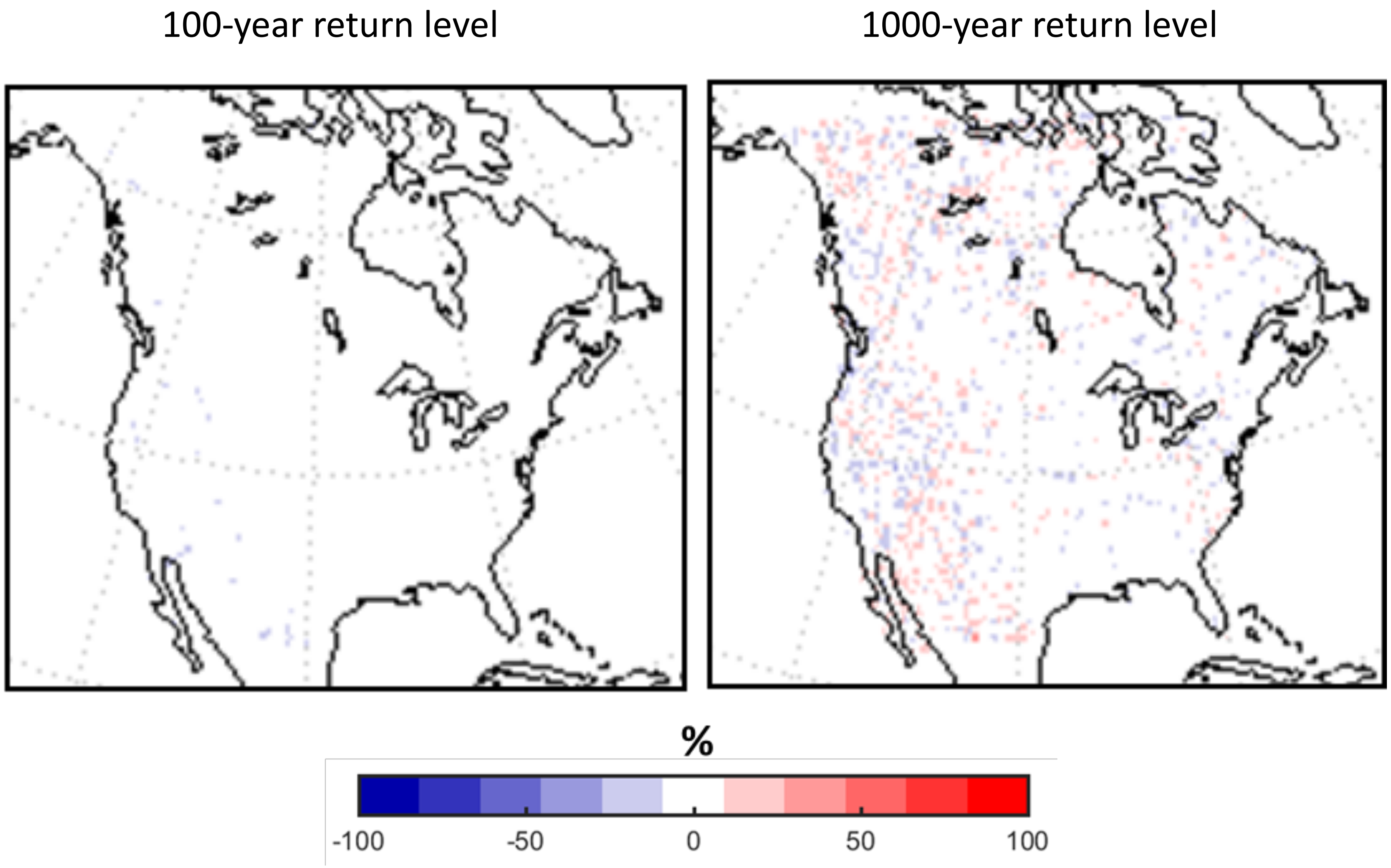
Relative bias of extreme quantile estimates

Relative bias in extreme quantiles of CanRCM4 simulated 1-hour precipitation accumulations for 1951-2000 based on fitting a GEV distribution to 1750 annual extremes for 1951-2000

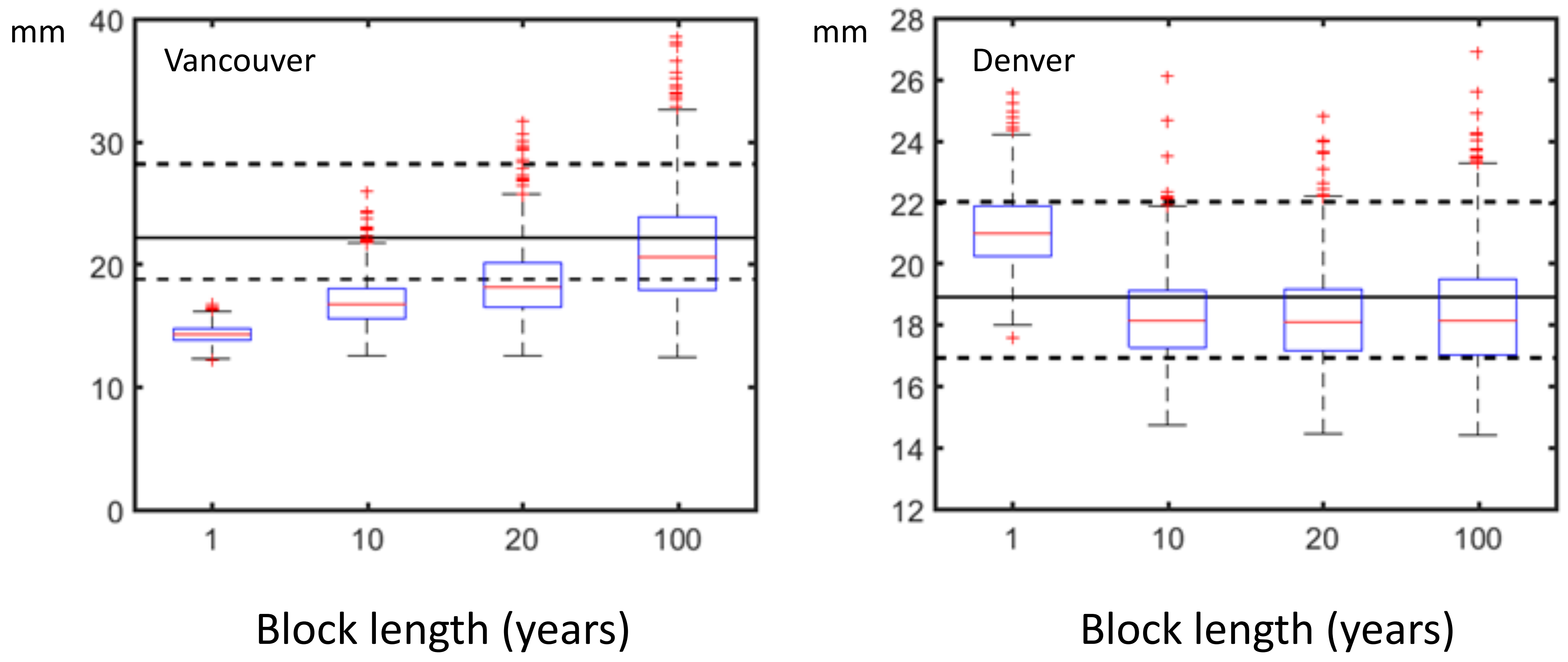


Relative bias of extreme quantile estimates

Relative bias in extreme quantiles of CanRCM4 simulated 1-hour precipitation accumulations for 1951-2000 based on fitting a GEV distribution to <u>175 decadal extremes</u> for 1951-2000

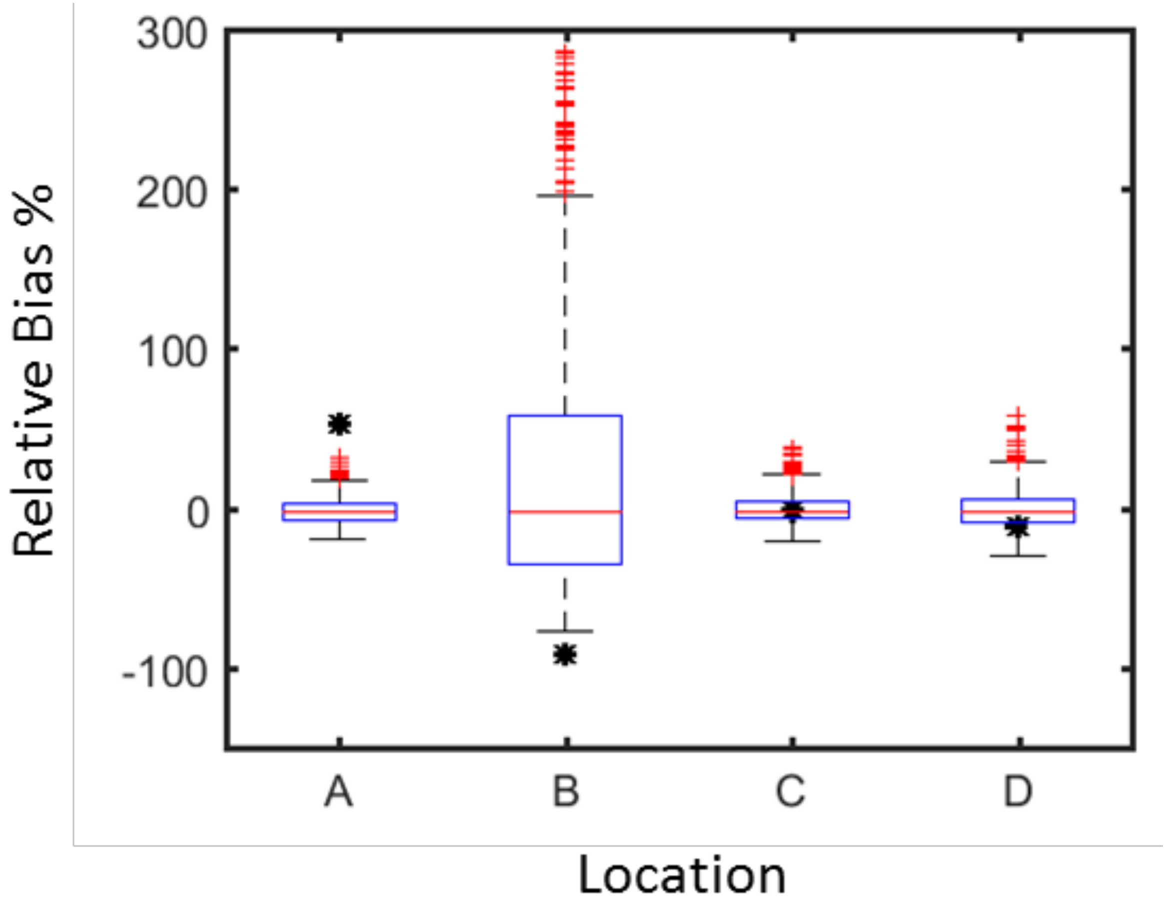


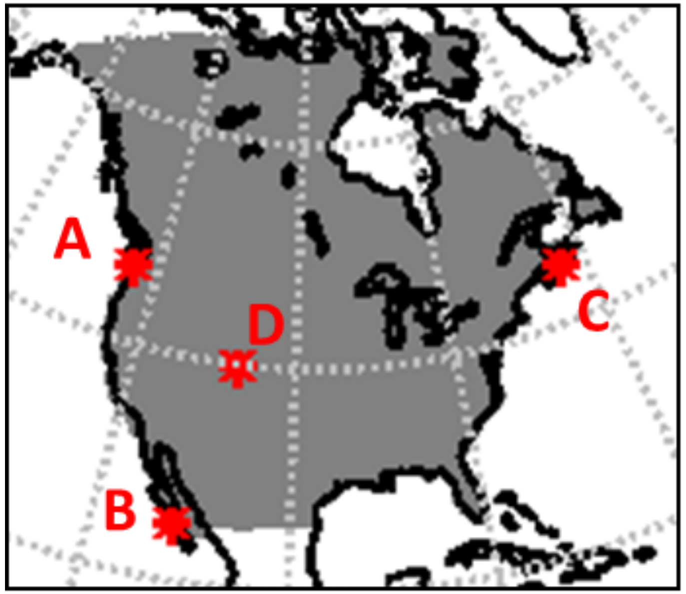
1000-year return level estimates vs block length (using 1000 bootstrap samples)



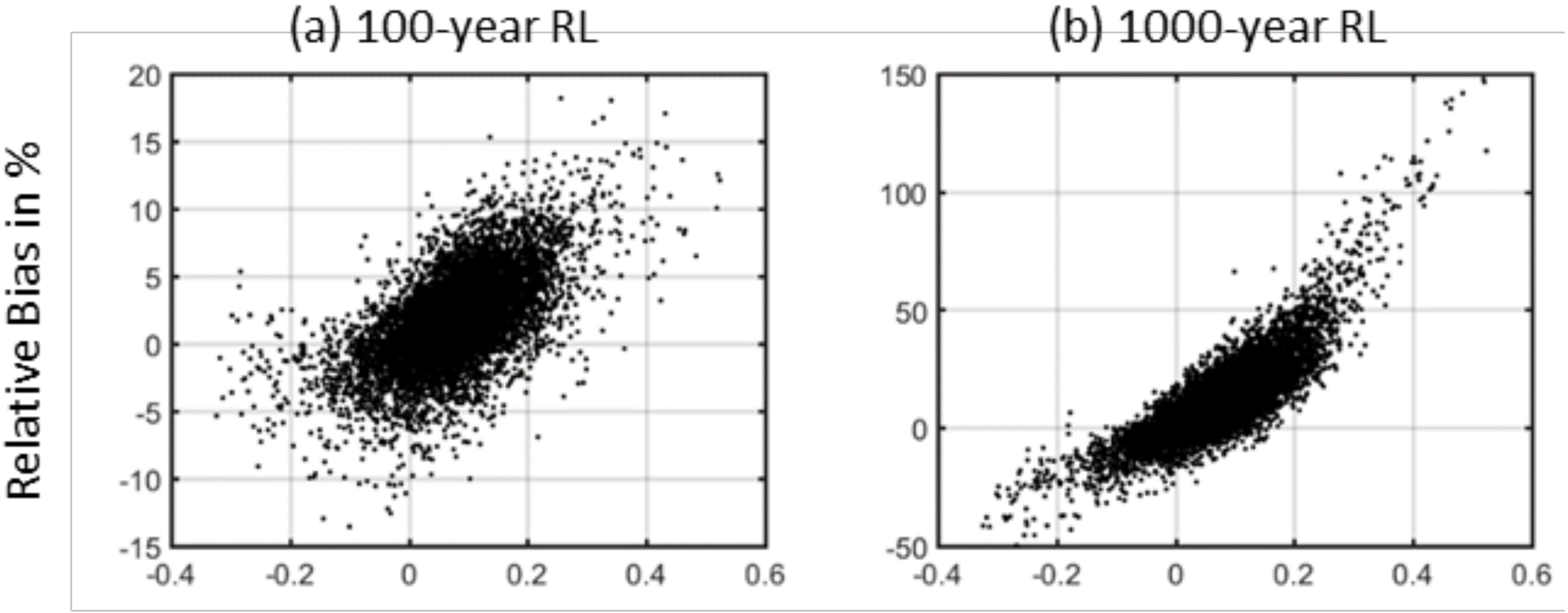
Relative bias in 1000-year return level estimates from annual maxima

(✷empirical vs fitted, and based on samples from the fitted distribution)





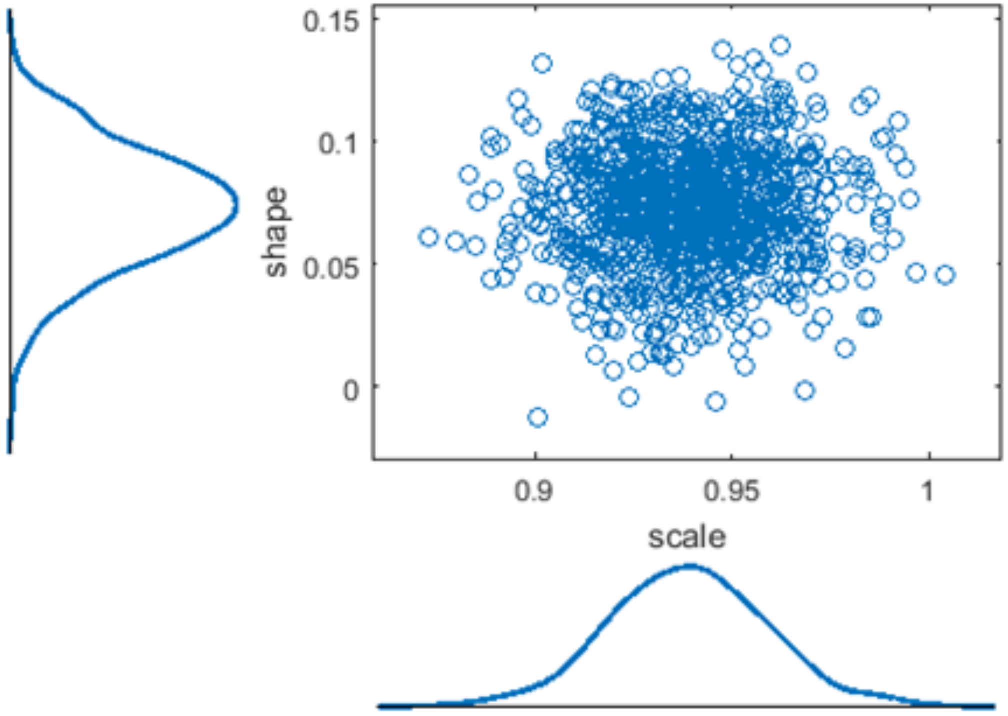
Relative bias in return levels



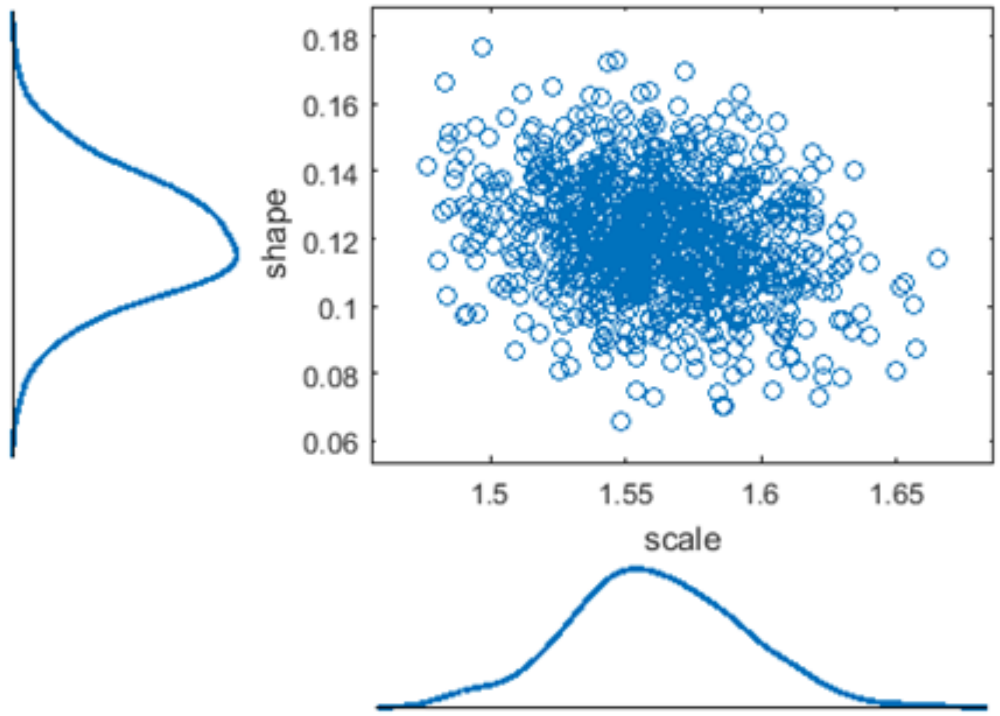
$$D = \xi_{1\,year} - \xi_{10\,year}$$

Joint distribution of estimated scale and shape parameters

Vancouver

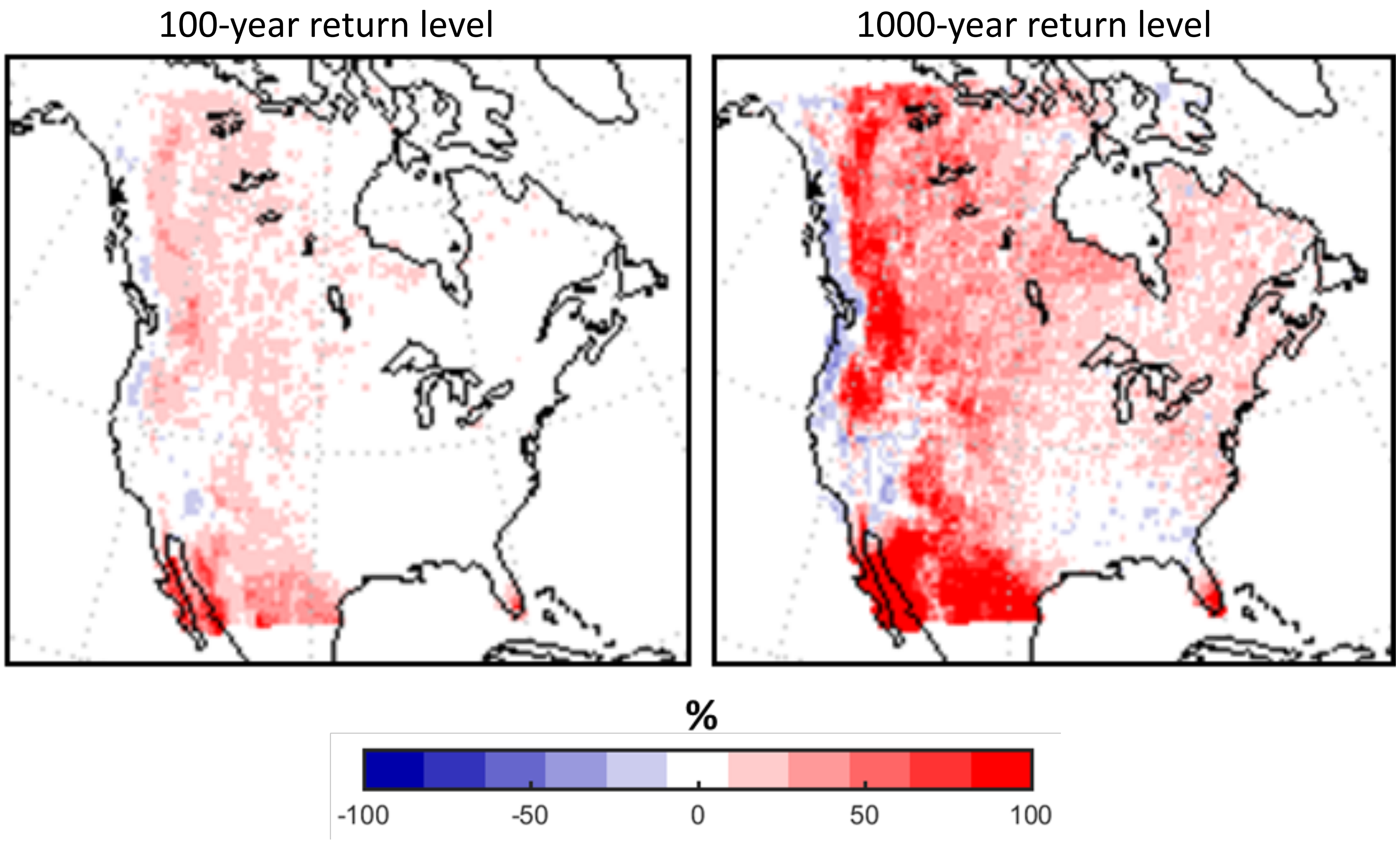


Denver



Relative bias of extreme quantile estimates

Relative bias in extreme quantiles of CanRCM4 simulated 1-hour precipitation accumulations for 1951-2000 based on fitting a GEV distribution to 1750 annual extremes for 1951-2000

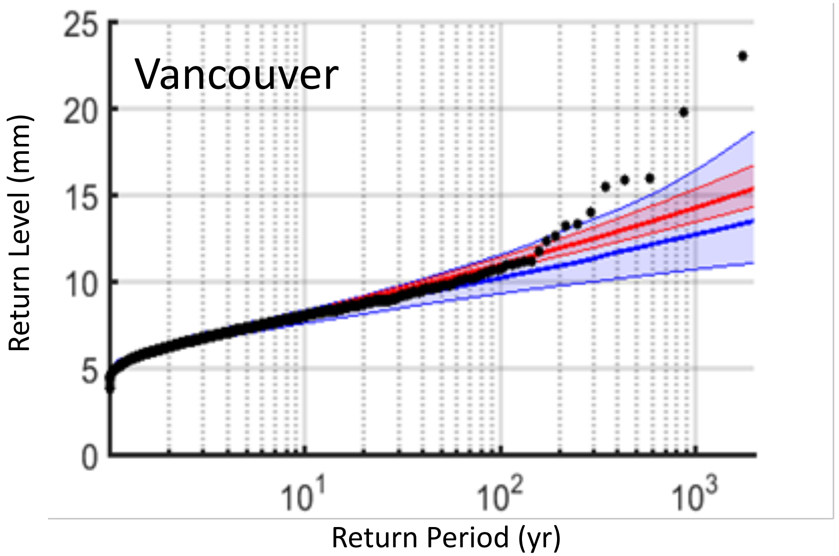


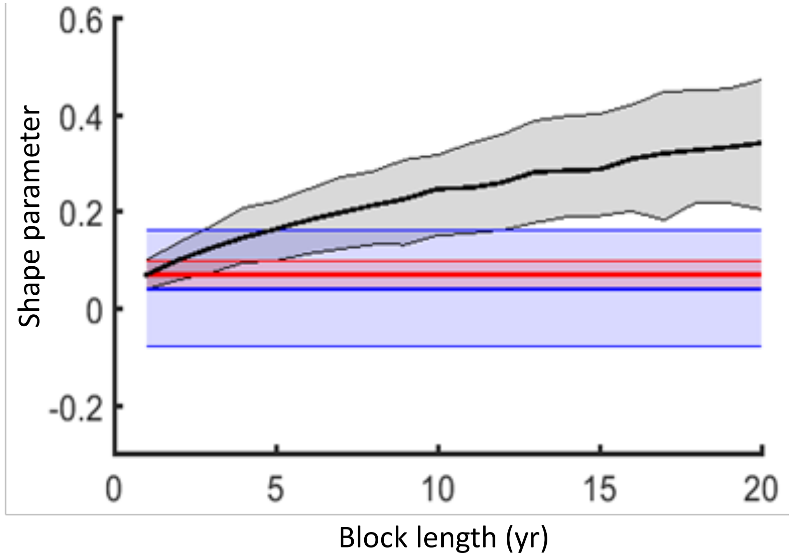
Discussion

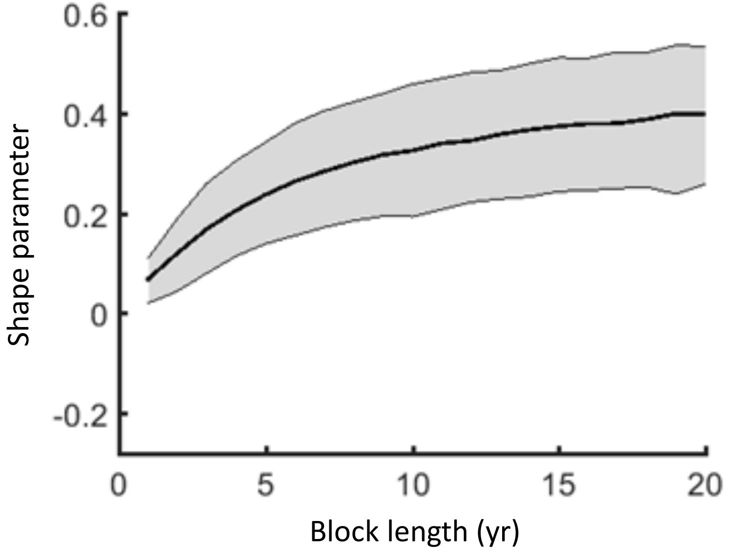
- CanRCM4 is not the real world, but …
- Extremes come from a mixture of processes, with the process producing the most intense extremes dominating the far upper tail
- Note that this is well recognized in the practitioner literature (e.g., it is explicitly discussed in the USGS Bulletin 17c), but practitioners usually deal with this at individual locations in an ad-hoc fashion
- The reliability of these ad-hoc approaches (e.g., based on storm classification) is unknown
- Fitting a mixture of two GEVs to the available sample of 1750 annual maxima at our test locations replicates the variation of the shape parameter with block length

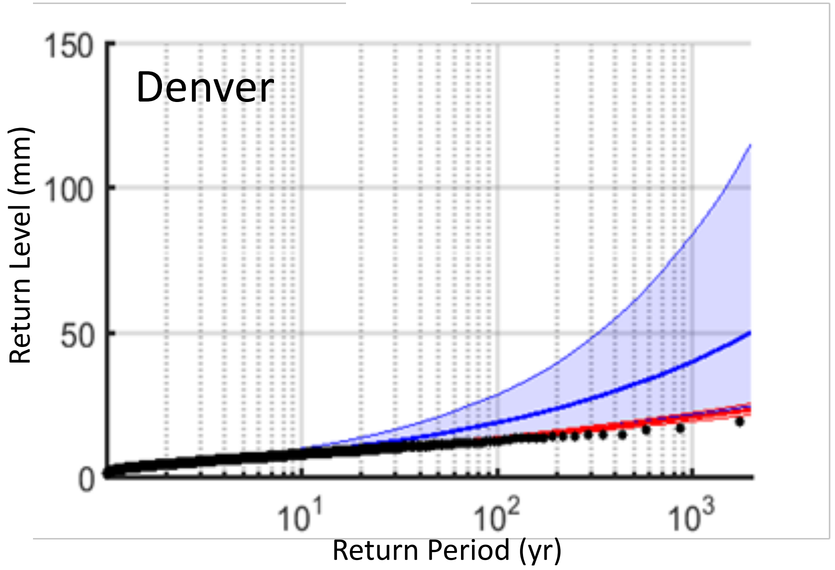
GEV fits to block maxima at 2 locations

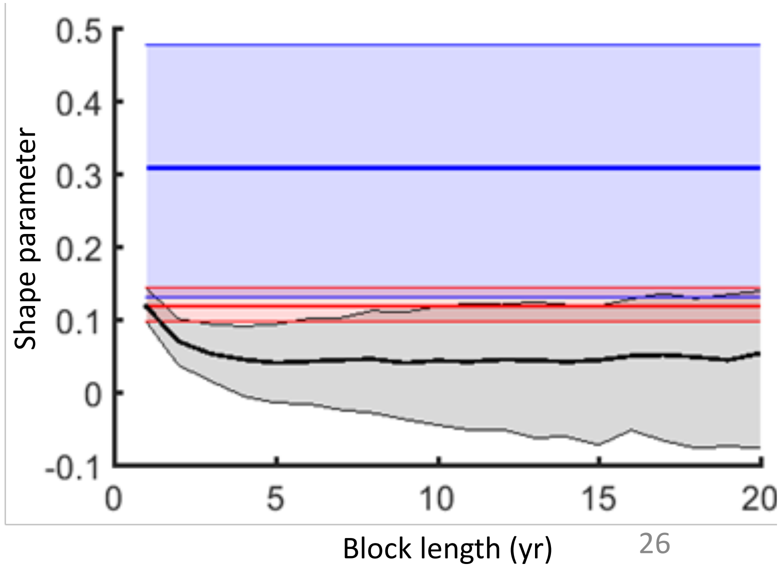
GEV fit to samples drawn from a mixture of 2 GEVs fitted to the sample of 1750 annual maxima

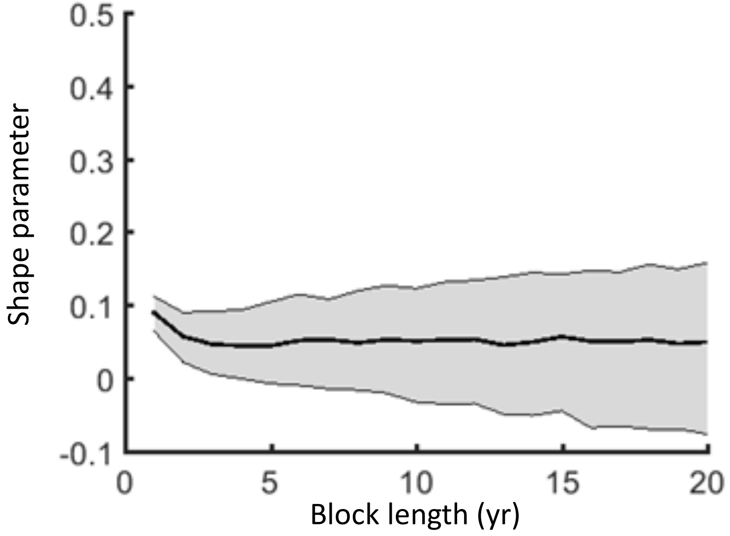












Discussion

- We should worry about tail stability and where we sample
- Sampling the annual maximum may leave us ignorant (in relative terms) about surprises deeper in the upper tail
- Using a peaks-over-threshold approach does not solve the problem
- Extrapolation into the deep tail requires information from somewhere
- It is either constructed from basic postulates, assumed, or perhaps can be objectively derived from further information about the underlying physics


A possible approach

Can physical considerations help?

- We obviously cannot directly assess stability with a typical 50-year sample of annual maxima …
- Perhaps a multivariate approach can help, whereby we decompose extreme precipitation into more than one component representing different aspects of the underlying physical processes
- One possible decomposition is PCP=PW×PE, where
  - PW is the precipitable water in the atmospheric column
  - PE is the precipitation efficiency (the fraction of PW that is precipitated during the event)
- PW is generally bounded, whereas PE can be heavy tailed, with PE >> 1 possible.

Proposal …

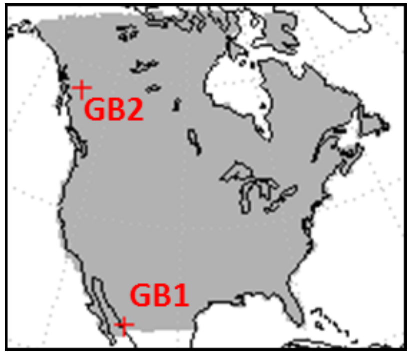
- Model the joint behaviour of extreme PW and PE and then use Monte Carlo methods to estimate the marginal distribution of PWxPE
- Options
  - Heffernan and Tawn (2004) conditional dependence model
  - Ben Alaya et al (2018) extreme value copula based model

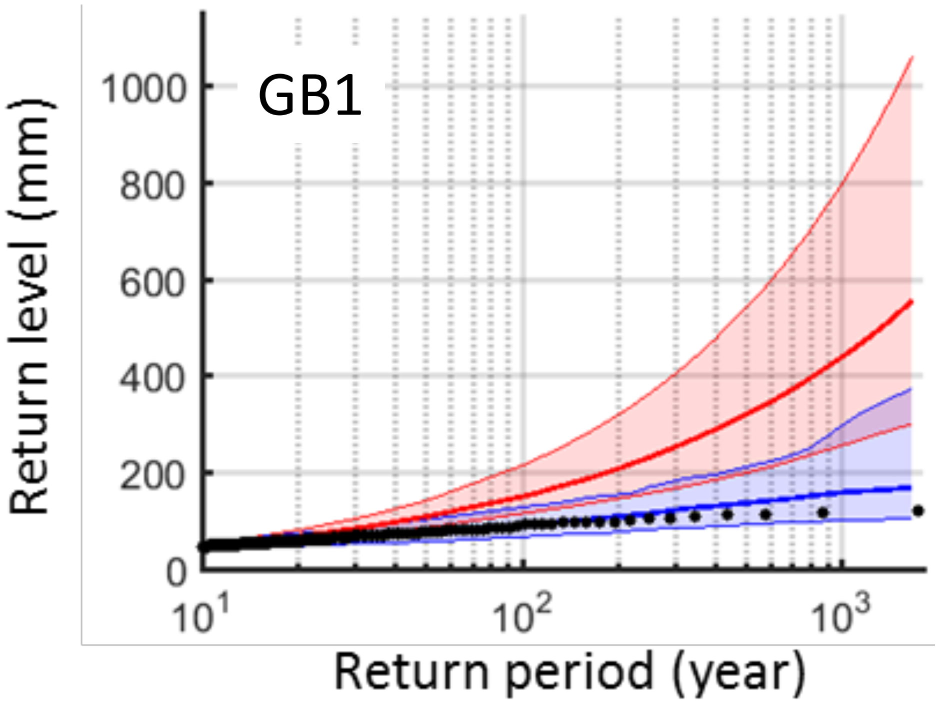
Approach

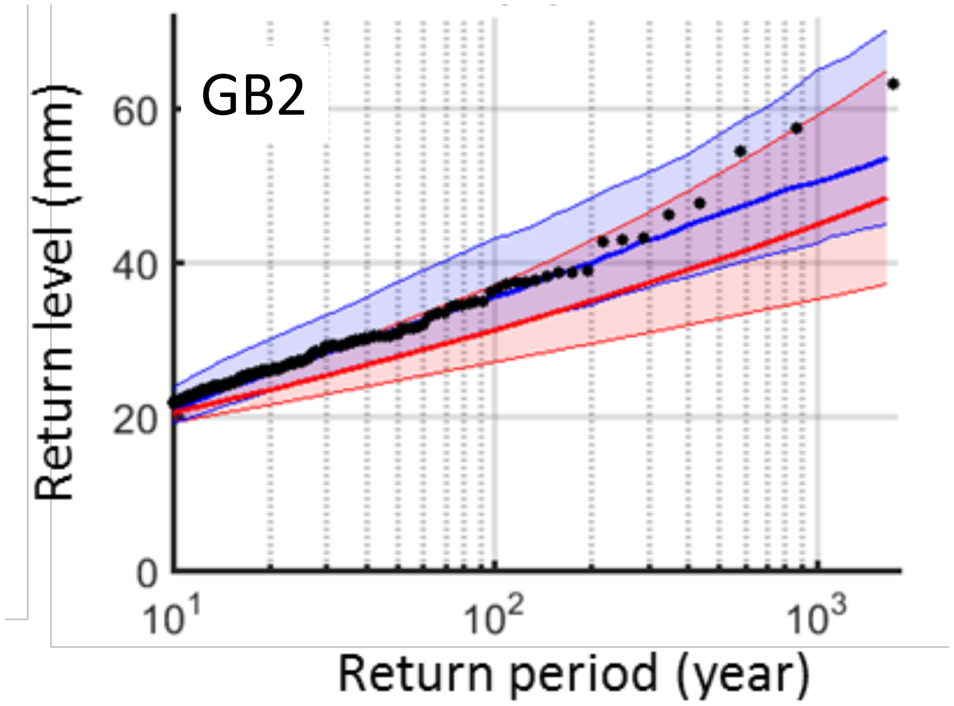
1. Fit semi-empirical marginal distributions to PW and PE
   - Empirical, plus Generalized Pareto in the upper tail
2. Transform the full marginal PW and PE to marginal Laplace distributions
3. Build a dependence model for extreme values of the transformed variables (PW, PE) by describing the conditional distributions of extreme $PW|PE$ and $PE|PW$
4. Repeatedly sample the joint extreme (PW, PE) distribution, transform back to (PW, PE), and multiply to obtain a Monte Carlo estimate of the distribution of extreme PCP=PW*PE

Some results

Estimated return levels for extreme 6-hour precipitation at two locations





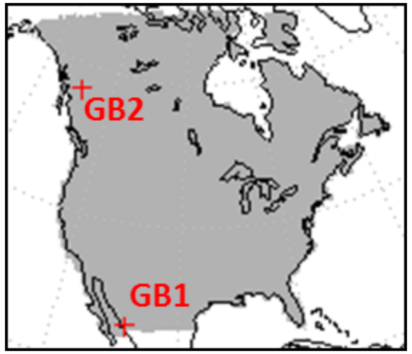


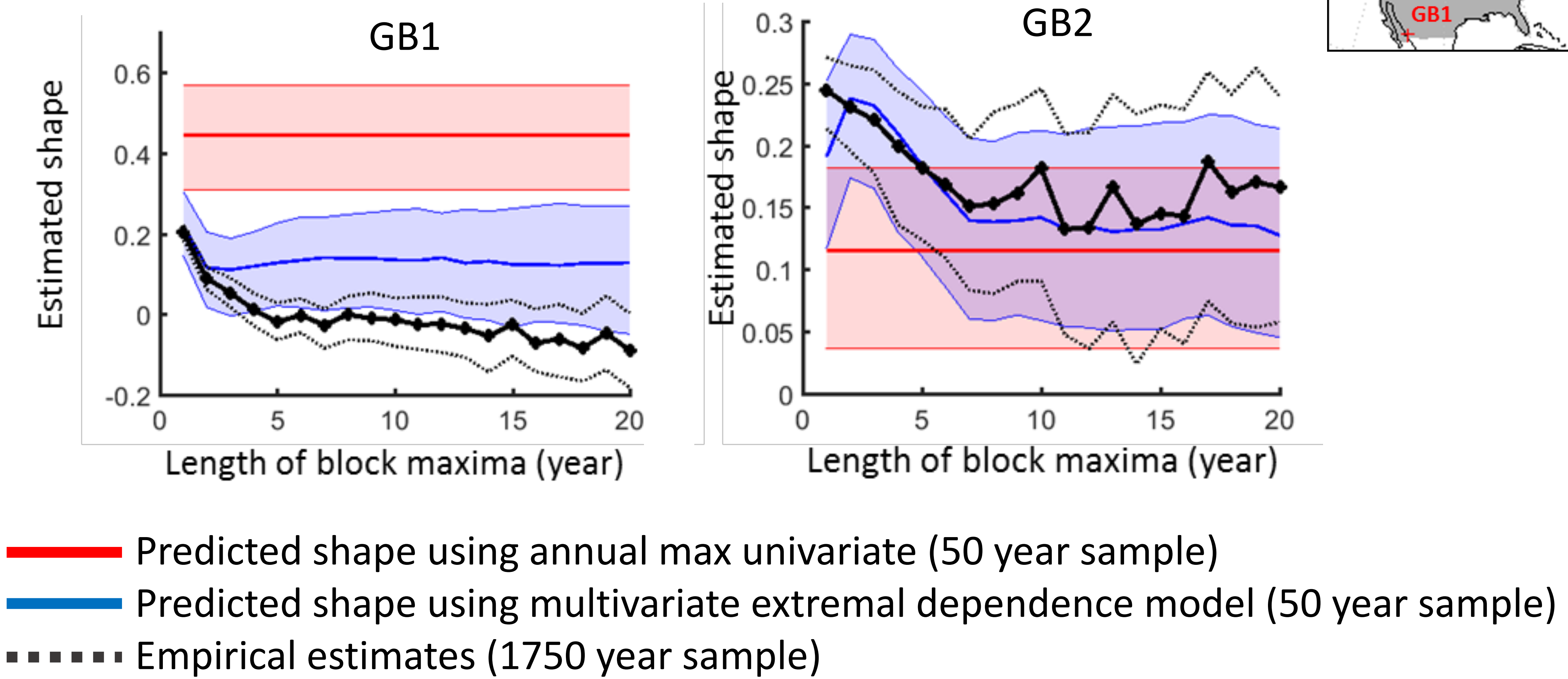
Annual max univariate (50 year sample)

Multivariate extremal dependence model (50 year sample)

Empirical quantile estimates (1750 year sample)

Estimated shape parameters for extreme 6-hour precipitation at two locations



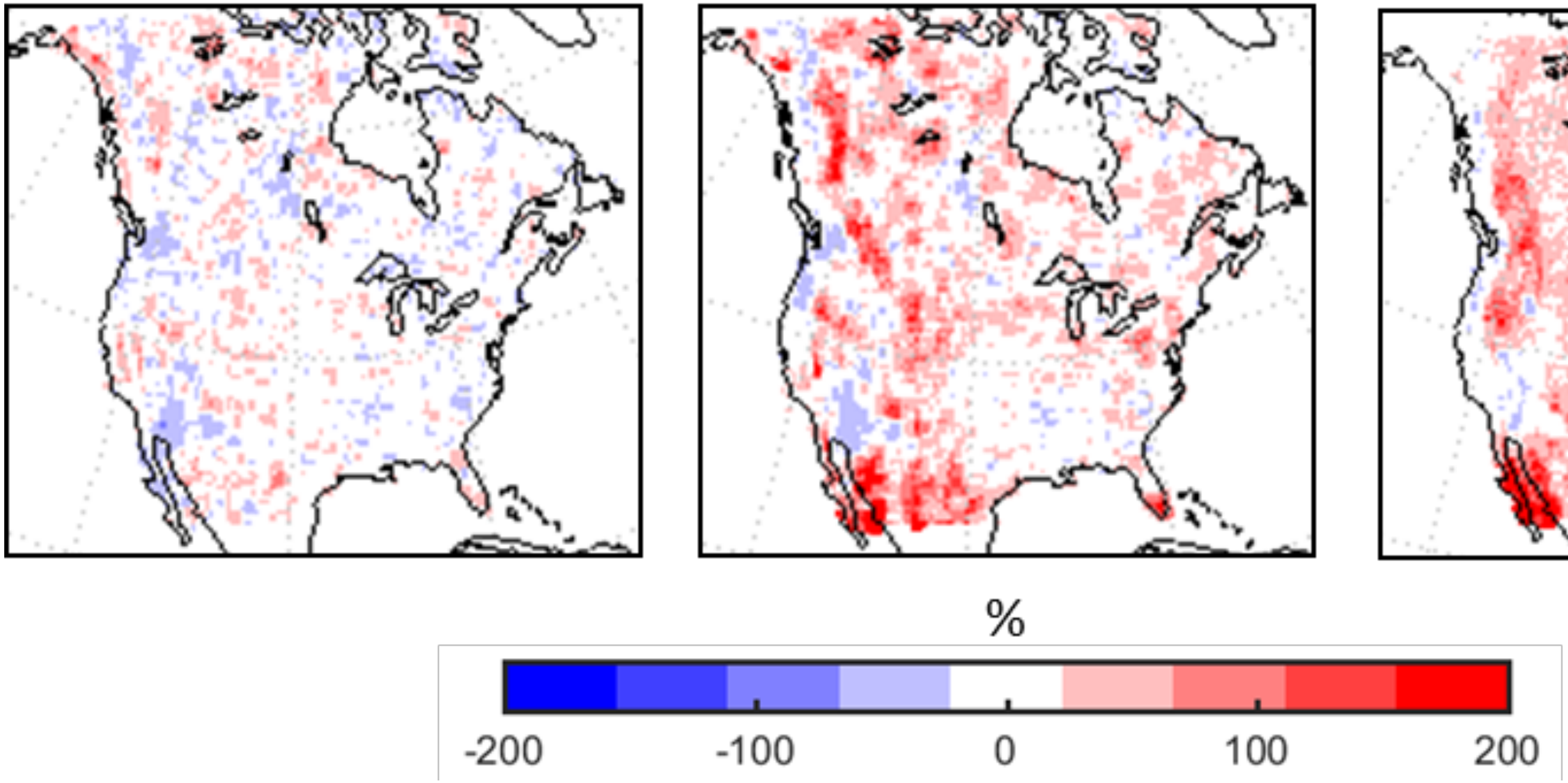


Relative bias in 1000-year return level estimates

Compound Approach
(50-year sample of precip components

Univariate Approach
(50-year sample of annual maxima)

Univariate Approach
(1750-year sample of annual maxima)

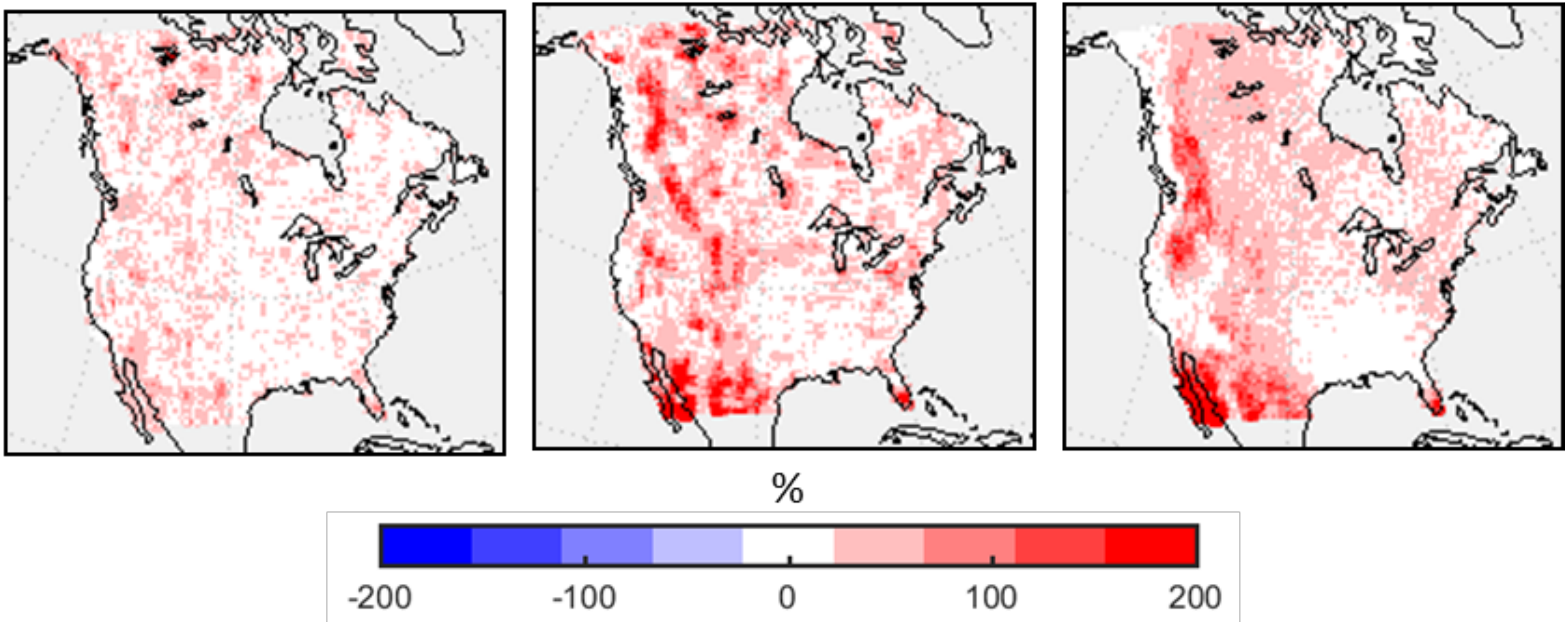


Relative RMSE in 1000-year return level estimates

Compound Approach
(50-year sample of precip components

Univariate Approach
(50-year sample of annual maxima)

Univariate Approach
(1750-year sample of annual maxima)

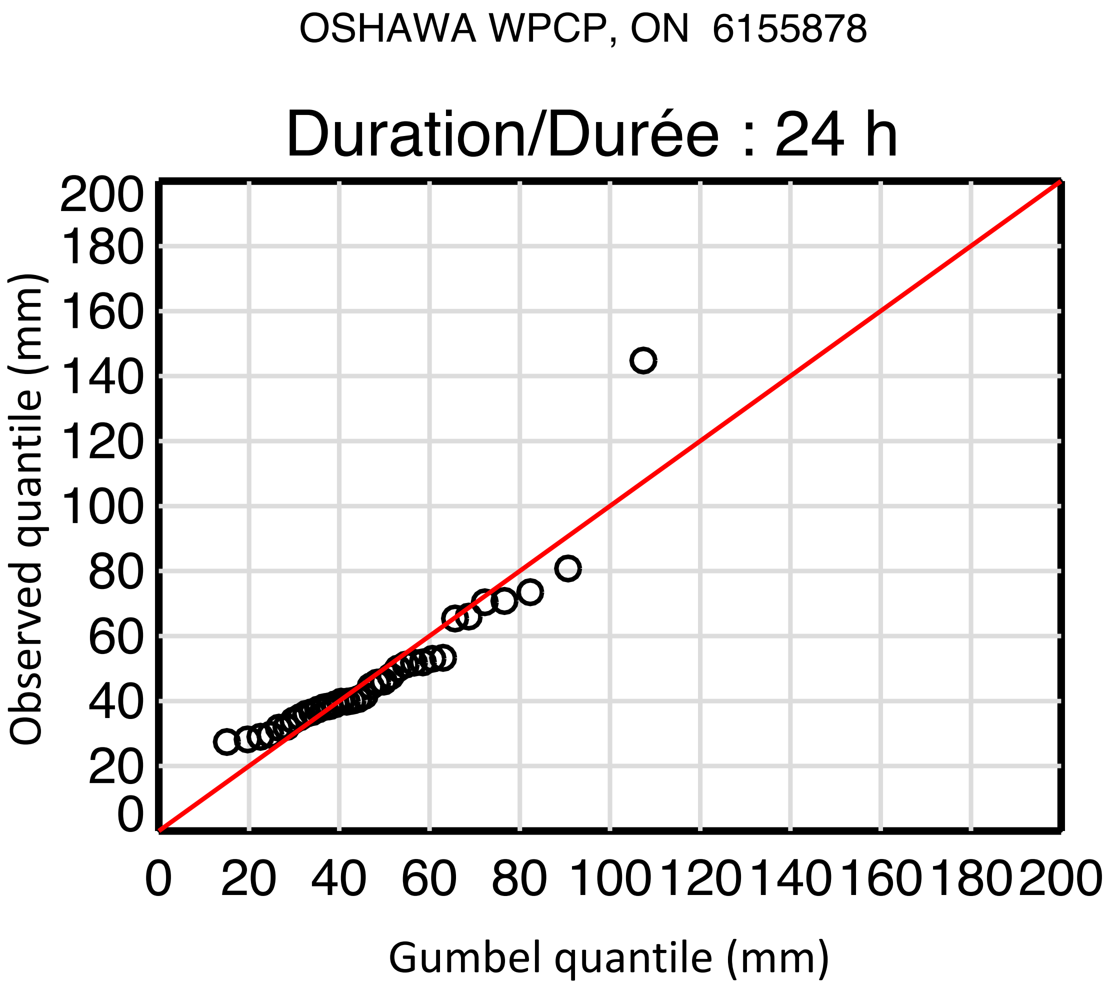


Conclusions

Photo: F. Zwiers

Conclusions

- Traditional univariate analysis assumes a stable upper tail
- The underlying process generating extremes, may however, be very complex, implying that stability will only be attained once blocks are large enough to consistently sample extremes from the physical process responsible for the largest events
- Large multi-year blocks are infeasible with short historical records



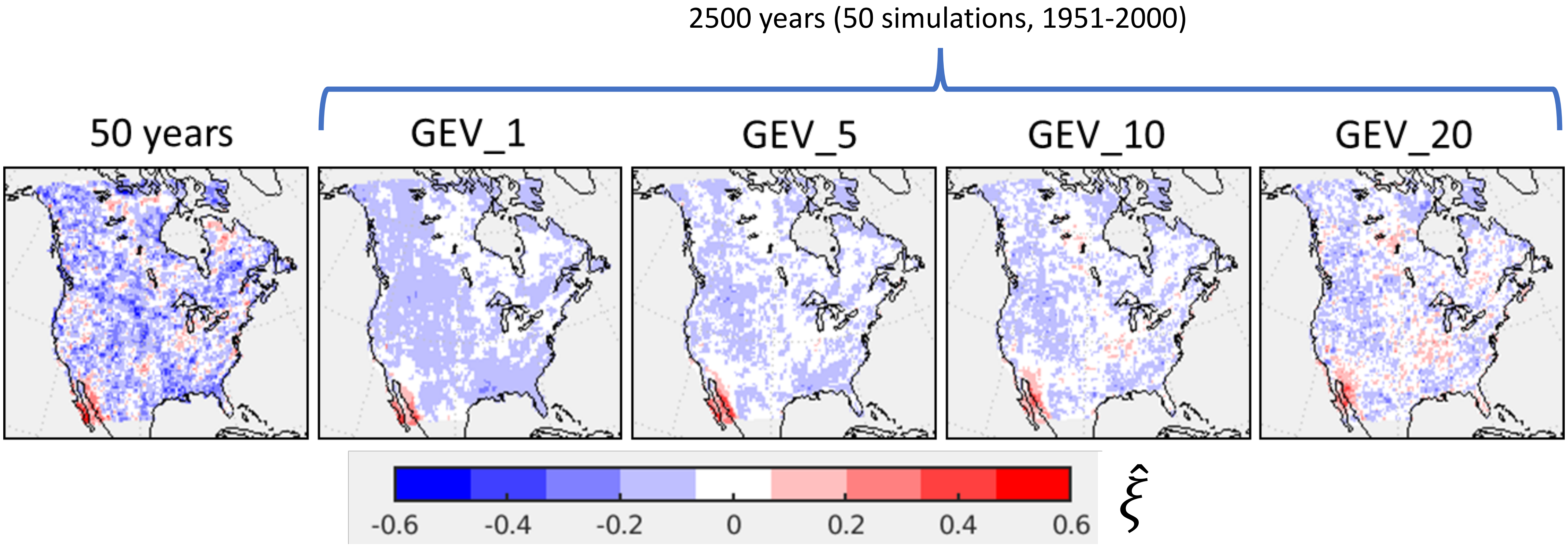
Conclusions

- It is therefore necessary to better use information available in the historical record
- One option is to extract information from the constituent variables that produce univariate extremes
- We illustrated this approach by decomposing precipitation as the product of precipitable water and precipitation efficiency
- The ”compound events” extremal dependence model appears to be able to capture fluctuations in tail shape that result from physical relationships between the component variables.
- Bias is, consequently, considerably reduced, even when using a modestly short 50-year sample.
- Note that additional information that allows this to happen comes from PW

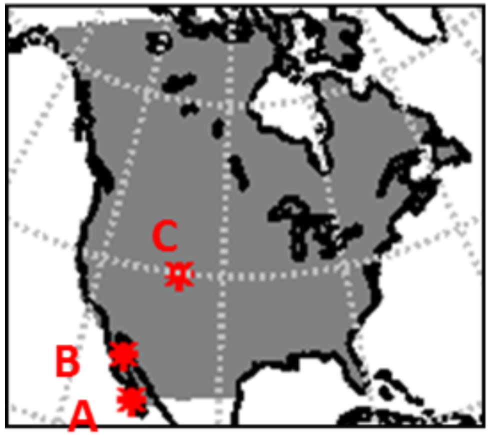
Problem is not limited to extreme precipitation

- We see similar issues with extreme wind speed
  - Fitting GEV distributions to annual maxima of model simulated "instantaneous" wind speed tends to find bounded distributions
  - Leads to negative bias in long-return period extreme wind speed estimates
  - Engineers use extreme wind pressures → substantially under-estimated extreme wind loads

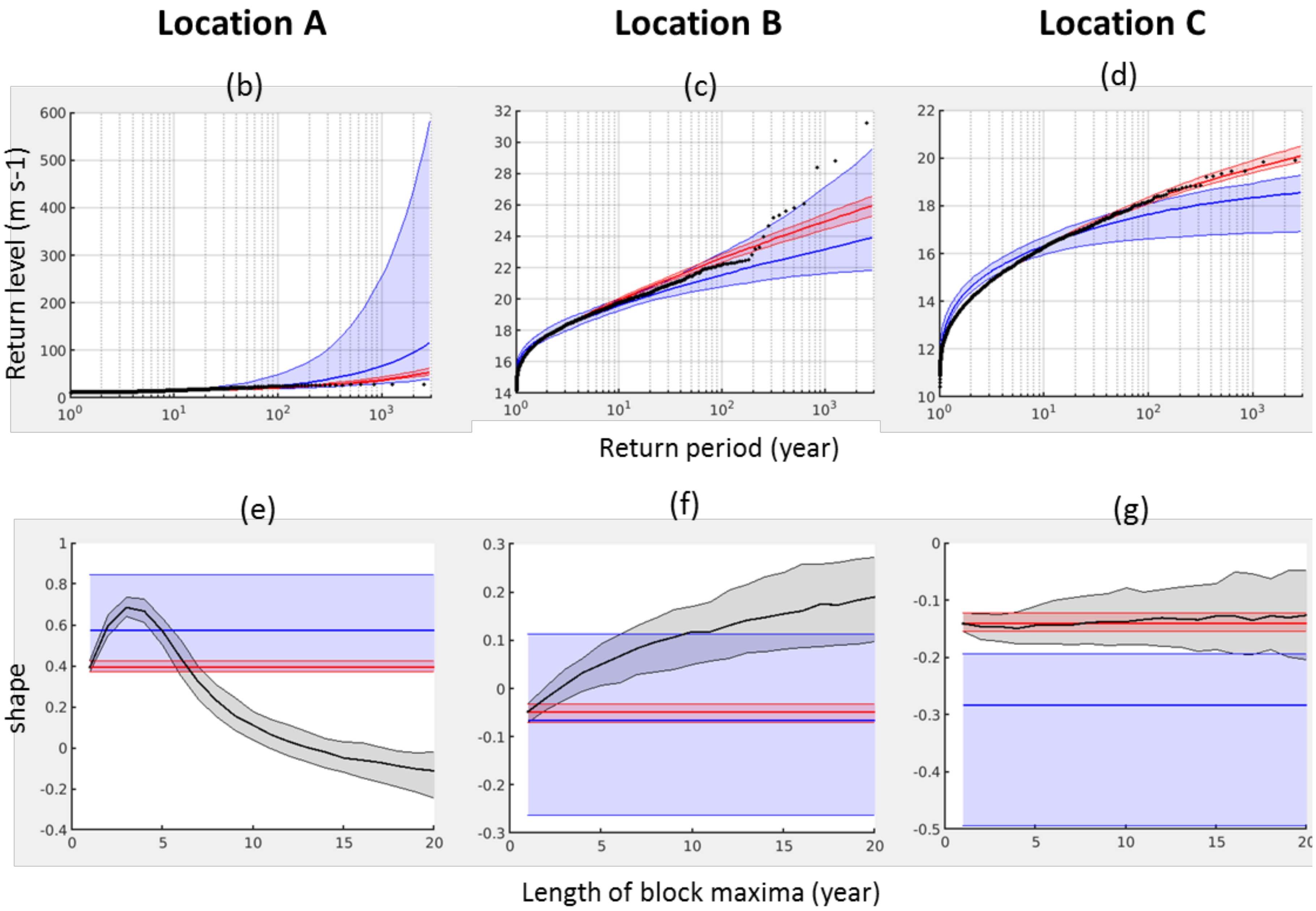
Shape parameters of extreme instantaneous windspeed



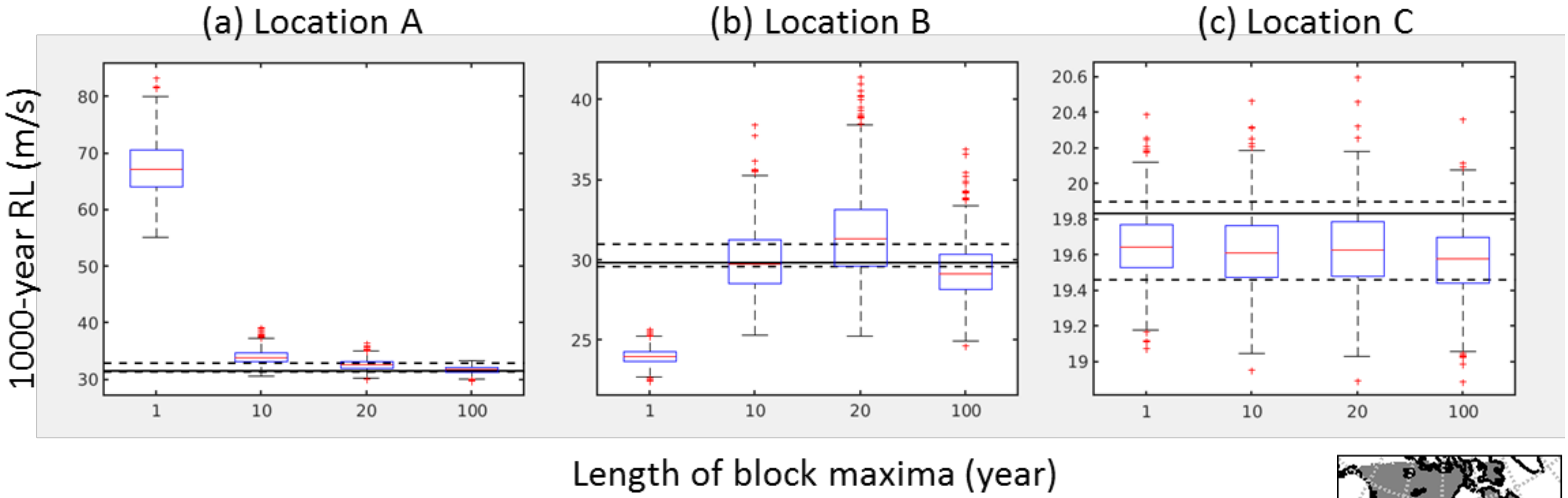
GEV fit to annual max wind speed based on 50 and 2500 values

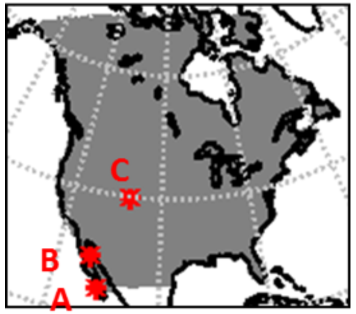


Estimated shape parameter vs block length

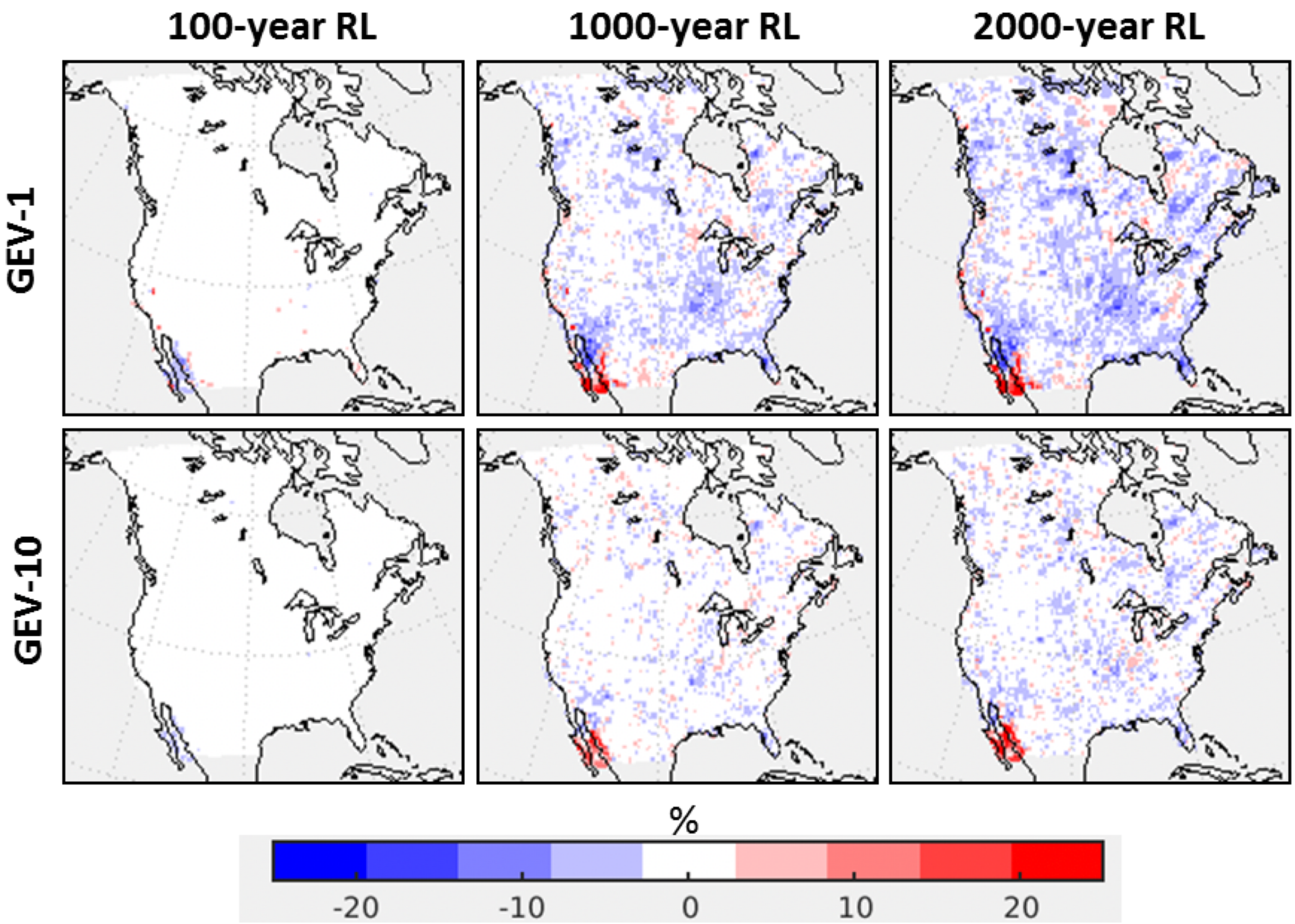


Bias vs block length for estimated 1000-year extreme windspeed based on GEV fits to block maxima





Bias of extreme windspeed return level estimates based on GEV fit to annual maxima and decadal maxima




Questions?
https://www.pacificclimate.org/
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