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CHANGES IN EXTREME PRECIPITATION

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Canada

Overview

- Observed changes at the global and regional scales
- Understanding the causes
- Projected future changes
- Linking to applications
- Focus on the concepts of methods rather than detailed/rigorous math/stats treatment

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PART I

WHAT AND HOW DO WE KNOW

Canada

Some basics: the Clausius-Clapeyron relation

- Saturation vapor pressure is a quasi-exponentially increasing function of temperature

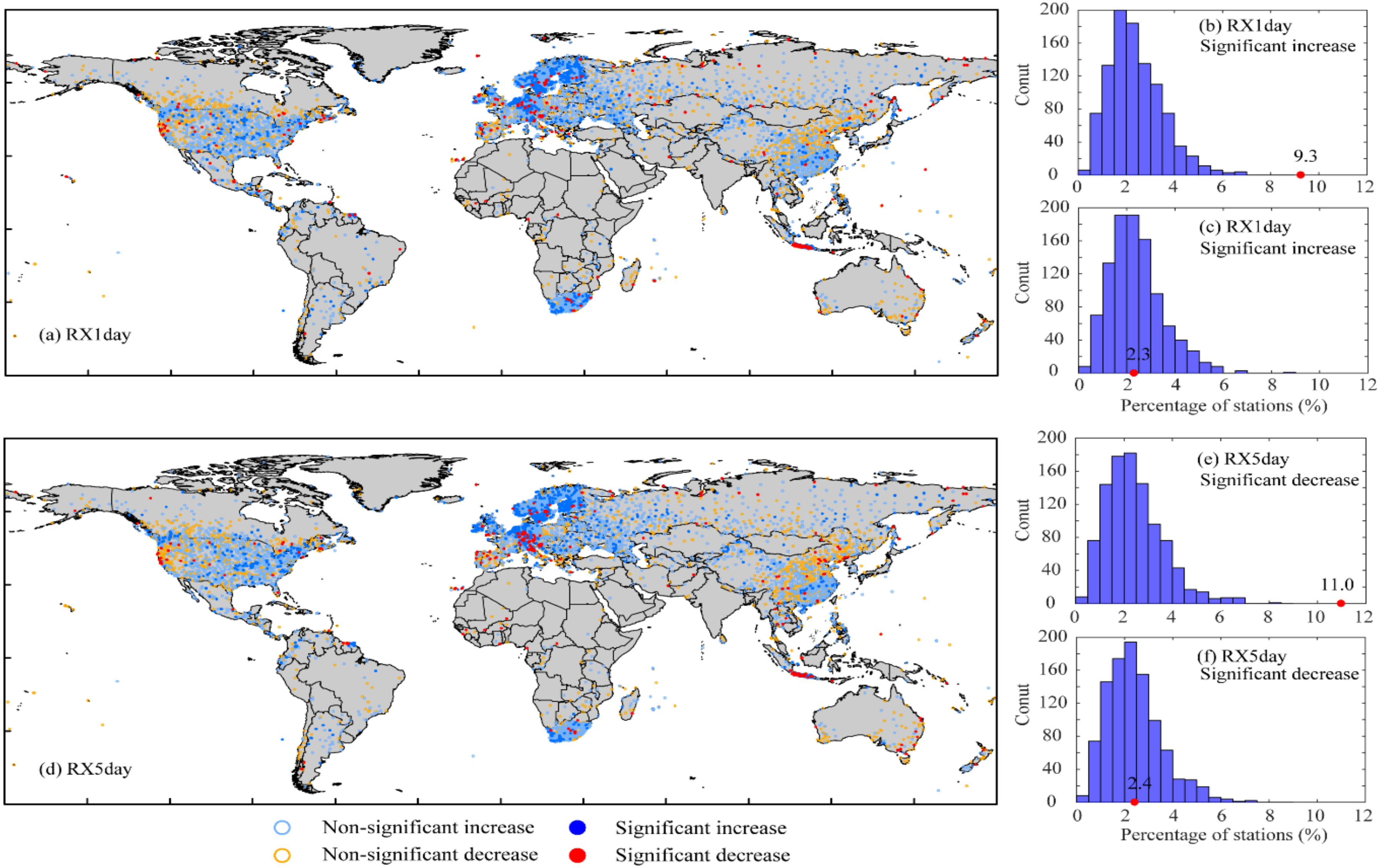
$$e_s(T) = 6.1094 \exp\left(\frac{17.625T}{T + 243.04}\right)$$

- The gross features of the general circulation will stay the same under climate change
- As a first approximation, the distribution of relative humidity will stay the same and specific humidity will increase with temperature at about 6-7%/K

Expected changes in hydrological cycle

- Water vapor tends to increase at the Clausius-Clapeyron rate about 6-7%/K
- P-E balances the horizontal advection of water vapor
- Global precipitation is affected by energy balance, i.e., latent heat needs to be balanced by long wave radiation cooling Increase that increases with temperature at about half of C-C rate. As a consequence, global precipitation increases with temperature at a rate much smaller than 6-7%/K
- Extreme precipitation is more affected by the availability of atmospheric moisture and generally increases at the C-C rate, depending on space/time scale
- Changes in hydrological cycle differ regionally

There are likely more land regions where the number of heavy precipitation events has increased than where it



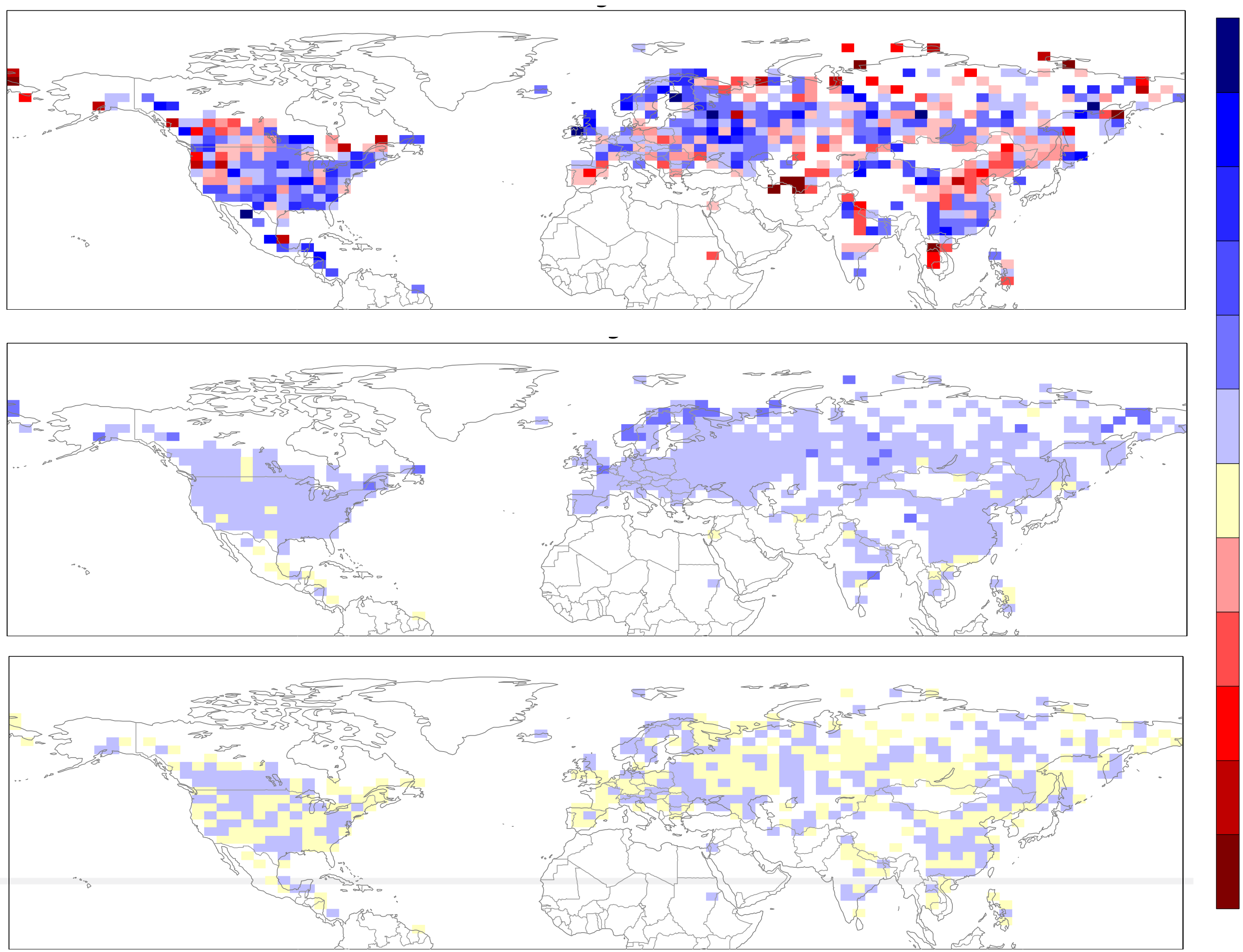
Sun et al. (2019) in preparation, see also Westra et al. (2013)

Anthropogenic influences have contributed to intensification of heavy precipitation over land regions where data are sufficient (IPCC AR5 SPM)

Observed

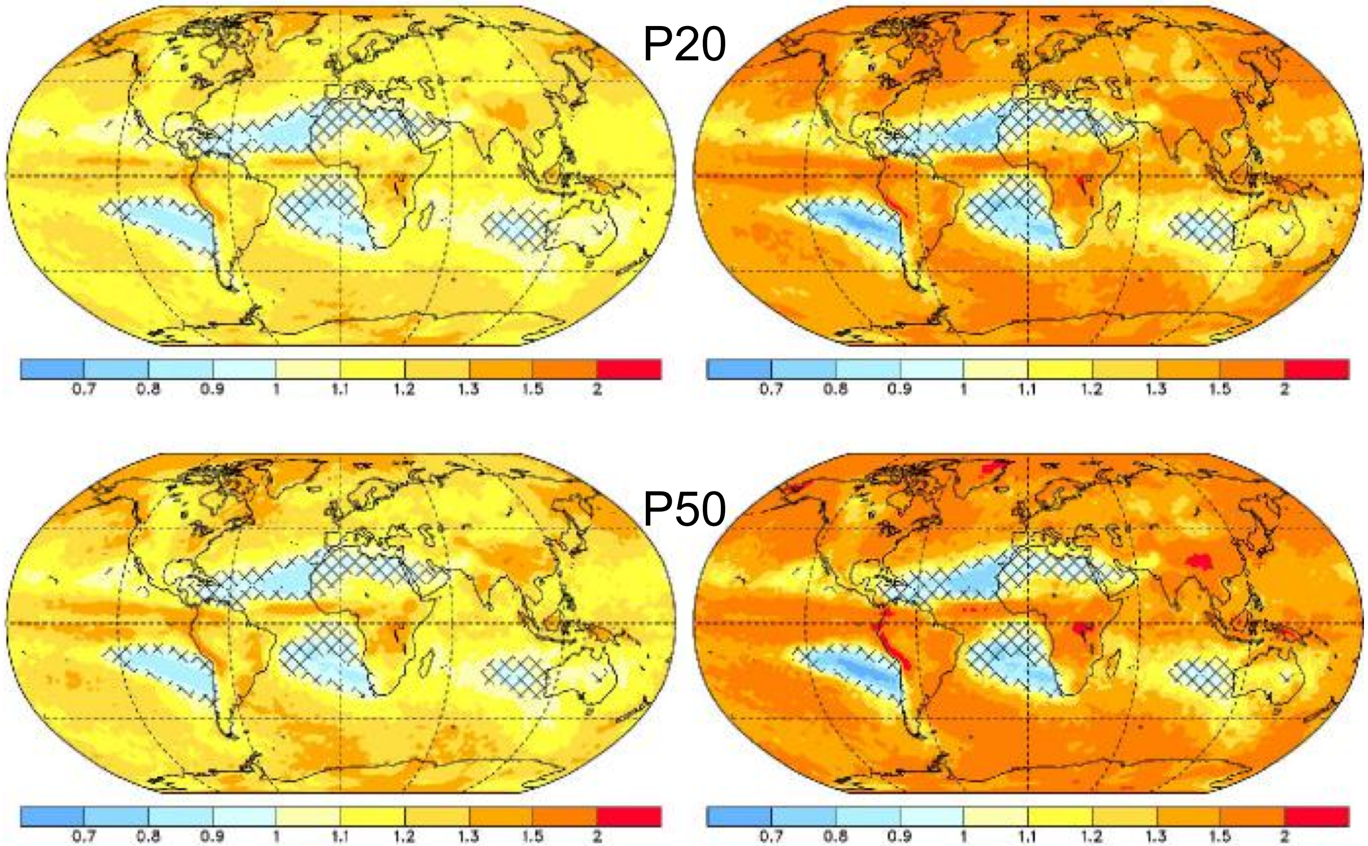
Models
(ALL forcings)

Models
(NAT forcings)



Zhang et al., [2013](#) (see also Min et al [2011](#))

Extreme precipitation events over most of the mid-latitude land masses and over wet tropical regions will very likely become more intense and more frequent (IPCC AR5 SPM)



Multi-model median risk ratio for 20-yr (P20) and 50-yr (P50) daily precipitation events in current climate (global warming at 1.0°C) at 1.5°C (left) and 2°C (right) global warming.

Kharin et al. 2019

Mann-Kendall test for trend

Mann-Kendall test

The Mann–Kendall test (Mann 1945; Kendall 1955) is a nonparametric test for randomness against trend. According to Mann the null hypothesis of randomness H_0 states that the data $(Y_1, Y_2, \ldots, Y_n)$ are a sample of n independent and identically distributed random variables. The test statistic S is defined as

$$S = \sum_{k=1}^{n-1} \sum_{j=k+1}^{n} \operatorname{sgn}(Y_j - Y_k), \tag{A1}$$

where

$$\operatorname{sgn}(x) = \begin{cases} 1 & \text{if } x > 0, \\ 0 & \text{if } x = 0, \\ -1 & \text{if } x < 0. \end{cases} \tag{A2}$$

The distribution of S under H_0 is symmetrical and is normal in the limit as $n \to \infty$. Under H_0, the mean of S is zero and, in case of no ties (e.g., no multiple values for the same sampling time), the variance of S is given by

$$V_S^2 = n\,(n - 1)(2n + 5)/18. \tag{A3}$$

A two-sided test for trend is then performed by comparing the following Z statistic:

$$Z = \begin{cases} (S - 1)/V_S & \text{if } S > 0 \\ 0 & \text{if } S = 0 \\ (S + 1)/V_S & \text{if } S < 0, \end{cases} \tag{A4}$$

with the critical value $Z_{\alpha/2}$ where $F_N(Z_{\alpha/2}) = \alpha/2$, F_N being the standard normal cumulative distribution function and α being the significance level for the test (Hirsch et al. 1982). The H_0 should be accepted if $|Z| \leq Z_{\alpha/2}$. A positive value of Z indicates an increasing trend, and a negative one a decreasing trend.

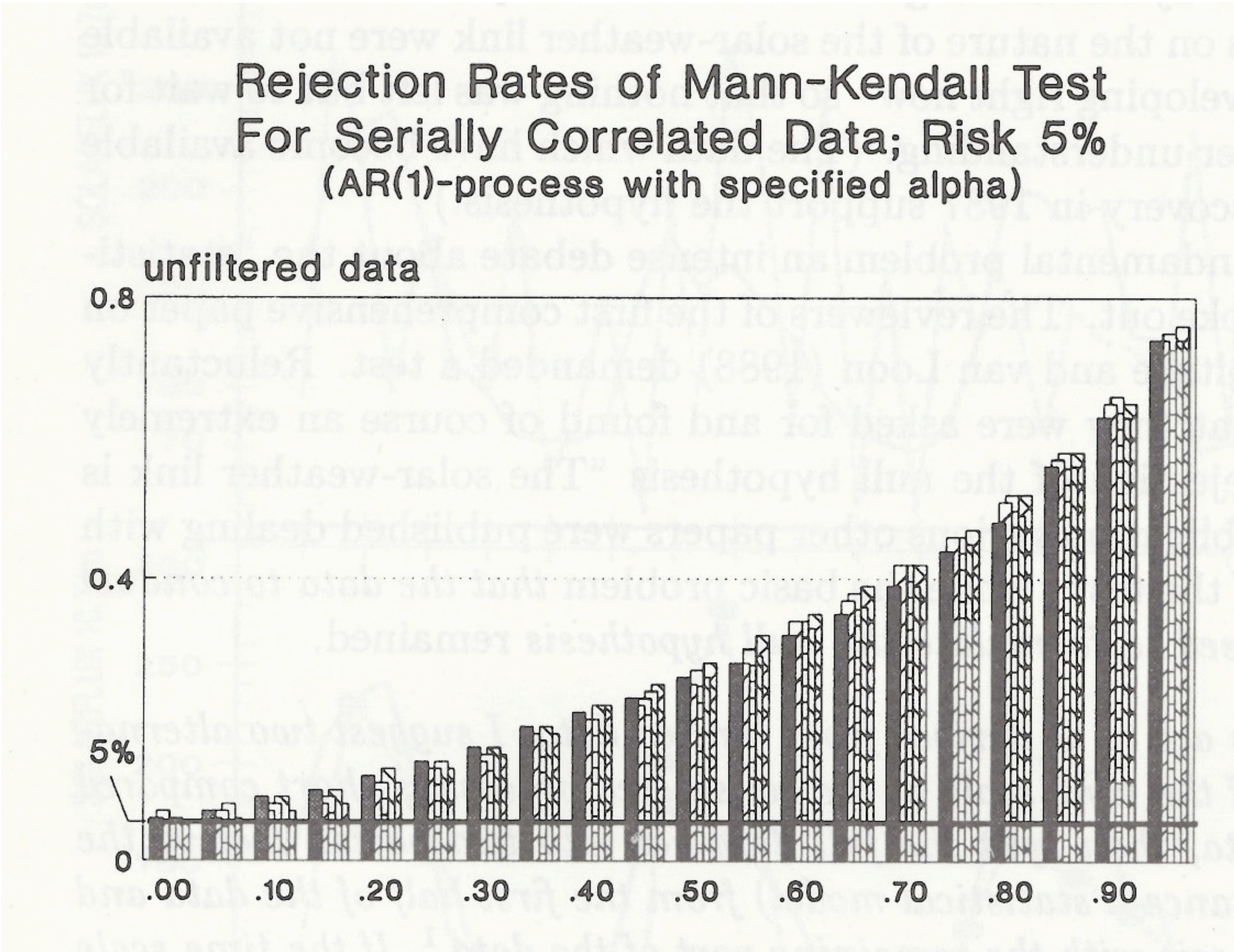
Nonparametric test, distribution free

Wang and Swail 2001, J. Climate

Assumption about the residuals: i.i.d.

- A sequence or other collection of random variables is **independent and identically distributed (i.i.d.)** if each has the same probability distribution as the others and all are mutually independent.
- *i.i.d.* is very common in statistics: observations in a sample are USUALLY assumed to be (more-or-less) i.i.d. for the purposes of statistical inference.
- The requirement that observations be i.i.d. tends to simplify the underlying mathematics of many statistical methods. However, in practical applications this is most often not realistic.
- We need to pay particular attention on this issue in almost all hypothesis tests

Serial correlation

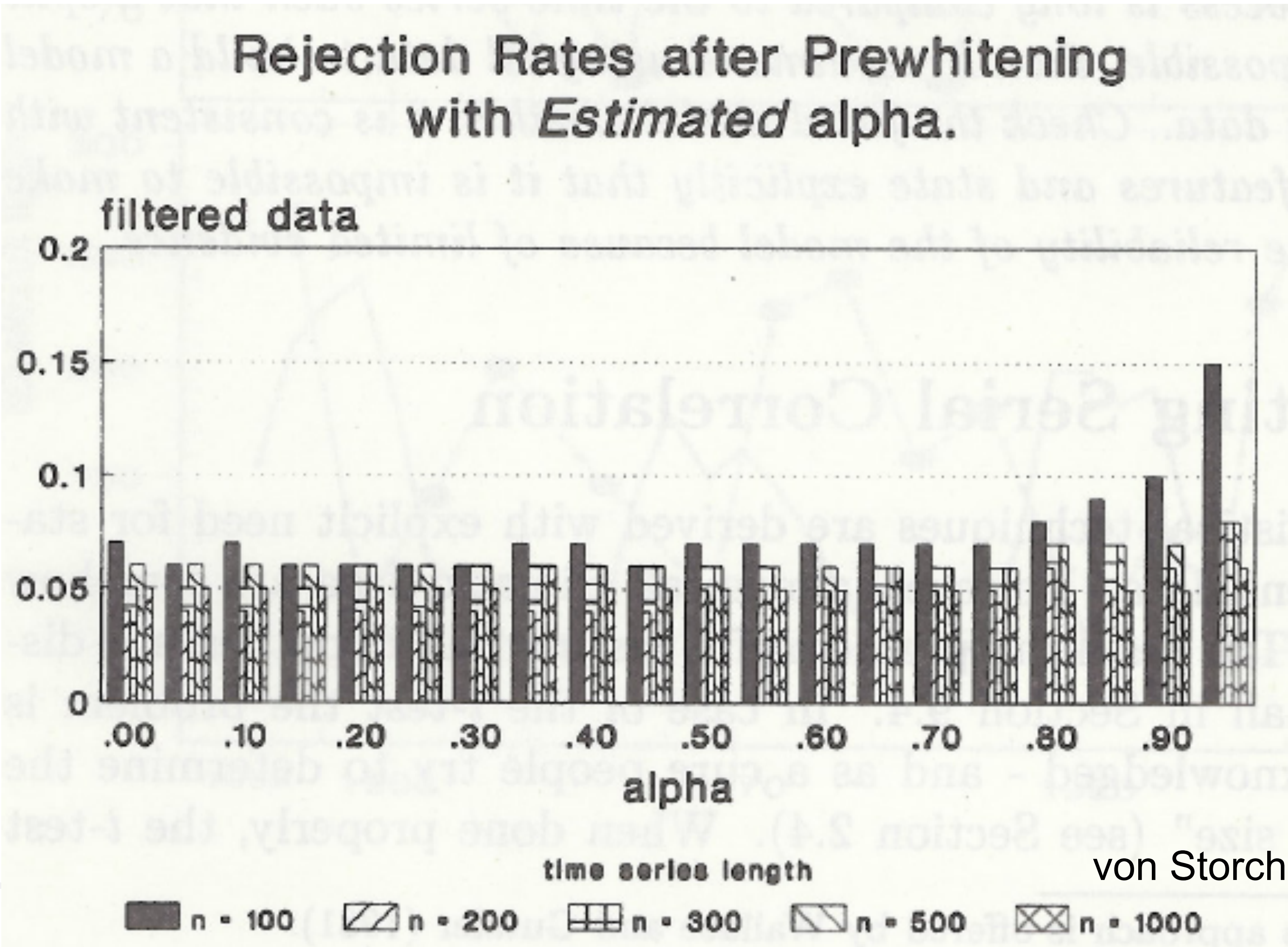


Von Storch and Navarra 1995

Treating serial correlation

- Pre-whitening: removal of serial correlation
- Estimate the proper number of degree of freedom
- Estimate uncertainty empirically using Block-bootstrap
- Generalized linear regression to explicitly consider autocorrelation.

Prewhitening: y(i+1)-alpha*y(i)



von Storch 1995

Effective sample size

- Effective time τ between independent samples can be estimated for autoregressive process
- Effective sample size $n = N\Delta t/\tau$
- Use n in place of N to compute test statistic/critical value

Block bootstrap

- Produce many series that do not have the property (e.g. trend) to be tested by resampling the original series
- Keep the serial correlation in the resampled data by resampling the data block by block
- Compute the statistics in the resampled data to come up with the critical values of the test statistic

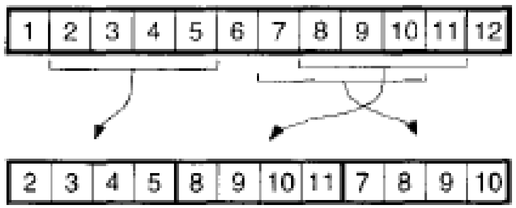


FIGURE 5.10 Schematic illustration of the moving-block bootstrap. Beginning with a time series of length $n = 12$ (above), $b = 3$ blocks of length $L = 4$ are drawn with replacement. The resulting time series (below) is one of $(n - L + 1)^b = 729$ equally likely bootstrap samples. From Wilks (1997b).

Trend estimation

Linear trends

- Simple, frequent and widespread use
- Strength and weakness well known

$$Y_i = a + bt_i + \varepsilon_i$$

LINEAR TREND: LEAST SQUARE FIT

- Least square estimates:

$$\hat{b} = \frac{S_{XY}}{S_{XX}}, \hat{a} = \bar{\mathbf{y}} - \hat{b}\bar{t}$$

- T-test for the statistical significance of the trend
- Test statistic:

$$\mathbf{t} = \frac{\hat{b}}{\hat{\sigma}_E / \sqrt{S_{XX}}} \sim \mathbf{t}\,(n-2),$$

$$S_{XX} = \sum_{i=1}^{n} (t_{\mathbf{i}} - \bar{t})^2, S_{YY} = \sum_{i=1}^{n} (\mathbf{y}_{\mathbf{i}} - \bar{\mathbf{y}})^2,$$

$$SS\varepsilon = S_{YY} - \hat{b}S_{XY}, \hat{\sigma}_E^2 = SS\varepsilon/(n-2)$$

- Gaussian assumption for the residual!

Linear trend: Sen's slope estimator

Without loss of generality we assume that $t_1 \le t_2 \le \cdots \le t_n$ are the sampling times which are not all equal. And let

$$N = \sum_{1 \le i < j \le n} \mathrm{sgn}(t_j - t_i), \qquad \text{(A5)}$$

where sgn(x) is as defined in (A2). Then, among all

values of $(t_j - t_i)$, $1 \le i \le j \le n$, only N values are nonzero. We now consider the set $\mathcal{R}$ of N distinct pairs (i, j) for which $t_j > t_i$, and define

$$X_{ij} = (Y_j - Y_i)/(t_j - t_i), \qquad (i, j) \in \mathcal{R}. \qquad \text{(A6)}$$

We then arrange the N values in (A6) in ascending order of magnitude and denote the kth smallest value by X_k ($k = 1, 2, \ldots, N$). Thus, the estimator of b based on Kendall's rank correlation is given by

$$\hat{b} = \begin{cases} X_{(N-1)/2+1} & \text{if } N \text{ is odd,} \\ (X_{N/2} + X_{N/2+1})/2 & \text{if } N \text{ is even.} \end{cases} \qquad \text{(A7)}$$

Let $N^* = Z_{\alpha/2} V_S$, and $M_1 = (N - N^*)/2$ and $M_2 = (N + N^*)/2$. Then (X_{M_1}, X_{M_2+1}) gives the $(1 - \alpha)$ confidence interval of estimator $\hat{b}$.

Wang and Swail 2001, J. Climate

Nonstationary extreme value models

GEV

$$G(y) = \begin{cases} \exp\{-\exp[\,1-(y-\mu)/\sigma]\} & ,\xi = 0, (EV-I) \\ \exp\{-[1-\xi(y-\mu)/\sigma]^{1/\xi}\}, & \xi > 0, y \leq \mu + \sigma/\xi (EV-II) \\ \exp\{-[1-\xi(y-\mu)/\sigma]^{1/\xi}\} & ,\xi < 0, y \geq \mu - \sigma/\xi (EV-III) \end{cases}$$

EV-I, Gumble distribution

EV-II, Frechet type I distribution

EV-III, Weibul type distribution

Non-stationary GEV

Co-variates in *GEV*

$$\mu = \alpha + \beta_i x_i$$

$$\log(\sigma) = \theta + \varphi_i x_i$$

Can be used for trend calculation

Estimation and testing

Maximum likelihood method

GEV

$$L(\mu,\sigma,\xi) = \prod_{i=1}^{m}\left(\exp\left\{-\left[1+\xi\left(\frac{y_i^{(r)}-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\}\times\prod_{k=1}^{r}\sigma^{-1}\left[1+\xi\left(\frac{y_i^{(k)}-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}-1}\right)$$

Nested models to determine predictors

Likelihood ratio test $$T = 2(l^1 - l^0) \sim \chi_q^2$$

Multiple testing and field significance

What if trends significant only in some places …

- Trends usually estimated for multiple locations, thus a multiple tests problem
- Is the trend significant globally?

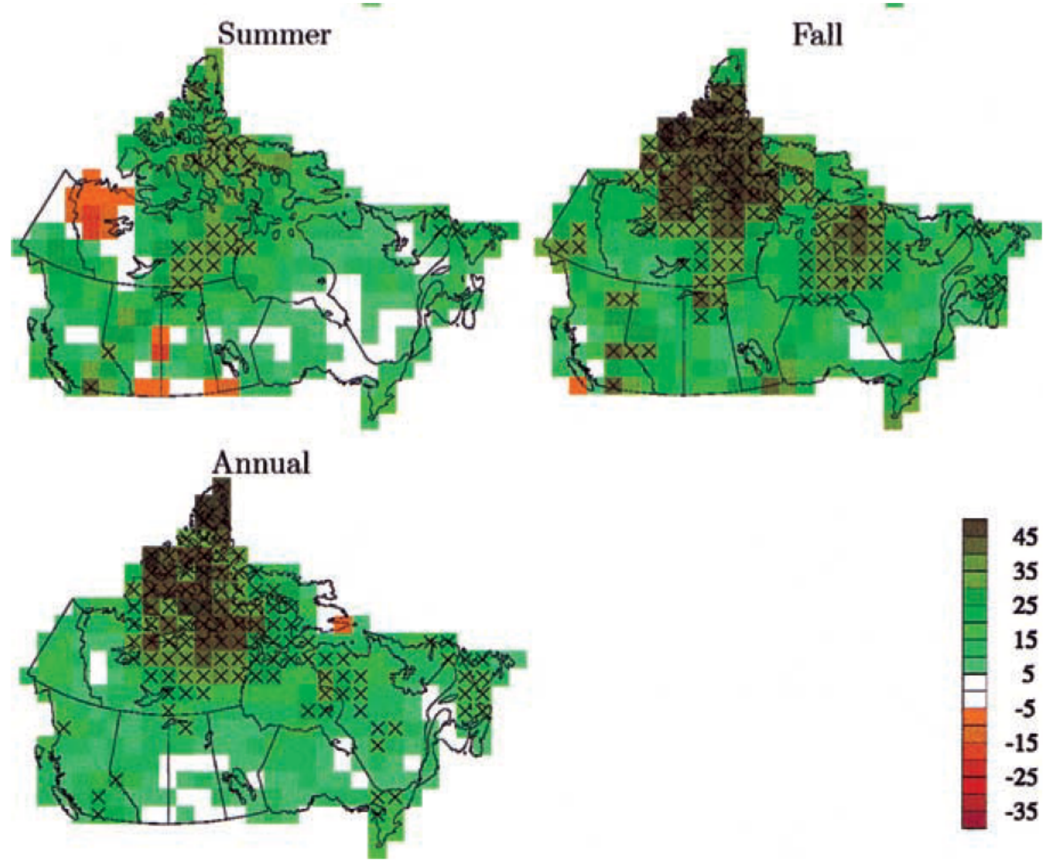


Fig. 13 Trends in precipitation totals from 1950–1998. Units are percent change over the 49-year period. Grid squares with trends statistically significant at 5% are marked by crosses.

Zhang et al. 2000

Concepts of type I and type II errors

- Type I error
  - Reject H_0 while it is true
  - Significance level
- Type II error
  - Failure to reject H_0 when it is false

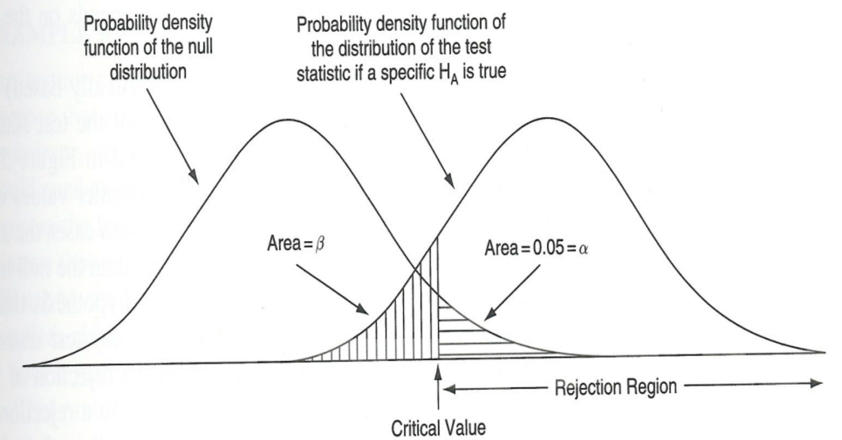


FIGURE 5.1 Illustration of the relationship of the rejection level, α, corresponding to the probability of a Type I error (horizontal hatching); and the probability of a Type II error, β (vertical hatching); for a test conducted at the 5% level. The horizontal axis represents possible values of the test statistic. Decreasing the probability of a Type I error necessarily increases the probability of a Type II error, and vice versa.

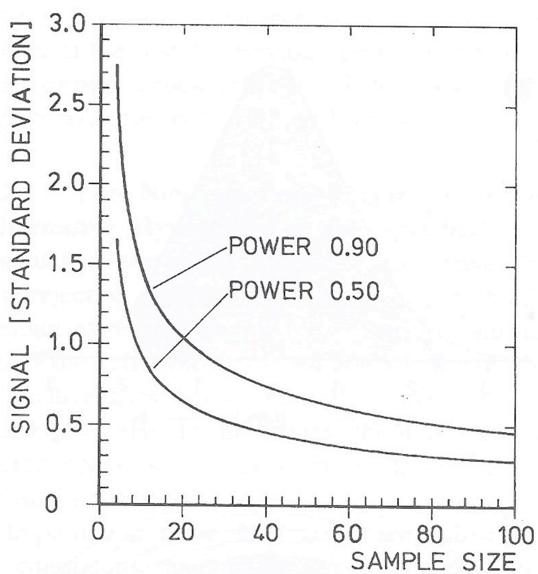


Figure 6.3: Signal-strength $\delta = \frac{\mu_Y - \mu_X}{\sigma}$ for which H_0: $\mu_Y = \mu_X$ is rejected with probability 50% or 90% at the 5% significance level, shown as a function of n, the number of realizations of each $\mathbf{X}$ and $\mathbf{Y}$. It is assumed that $\mathbf{X} \sim \mathcal{N}(\mu_X, \sigma)$ and $\mathbf{Y} \sim \mathcal{N}(\mu_Y, \sigma)$. [404]

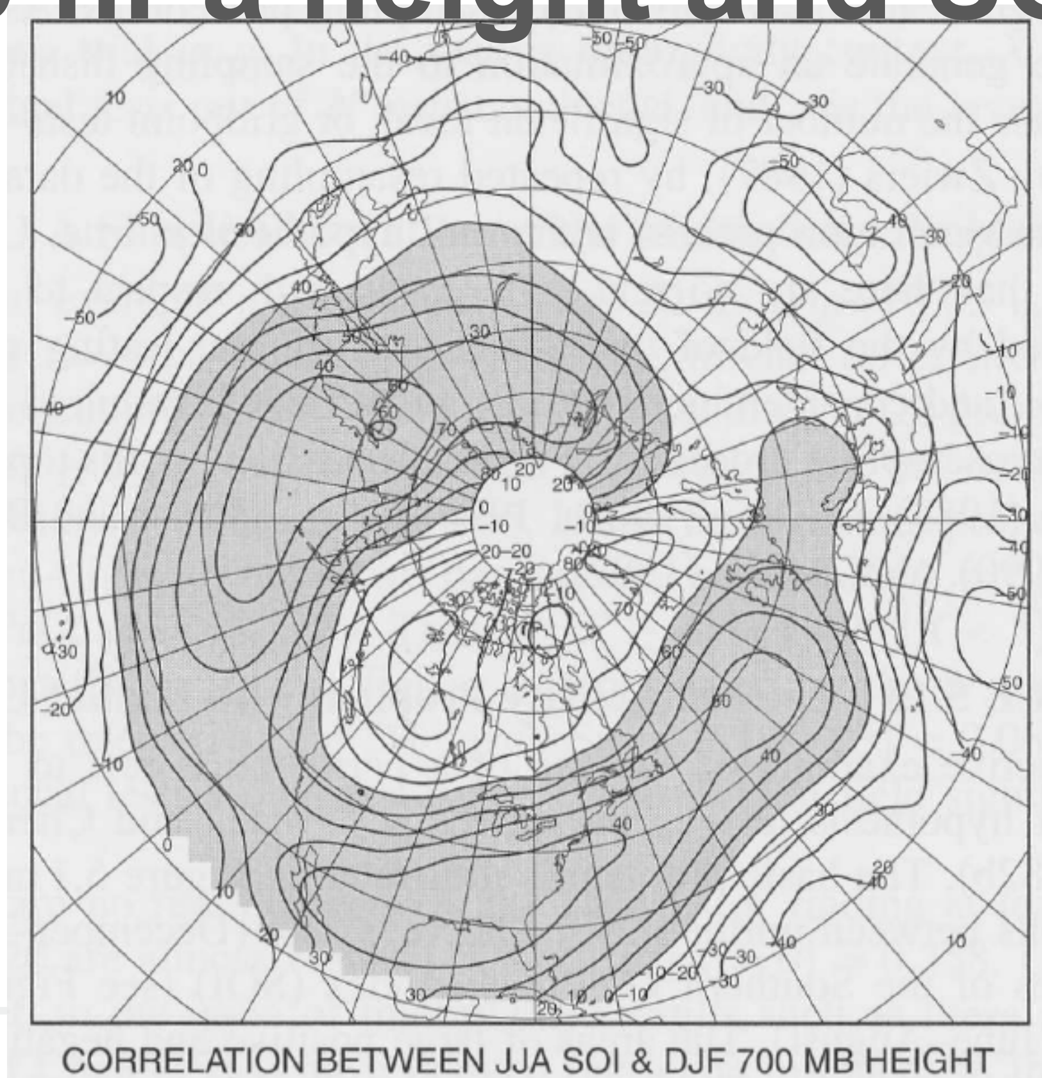
Test of a hypothesis

- Null hypothesis H_0
- Alternative hypothesis H_a
- Two outcomes of a test
  - Reject H_0: we have strong evidence that H_0 is false (but does not imply acceptance of H_a)
  - Failure to reject H_0: evidence in the sample not inconsistent with H_0 (but does not imply acceptance of H_0)
- Only consider the case without H_a

Multiple testing

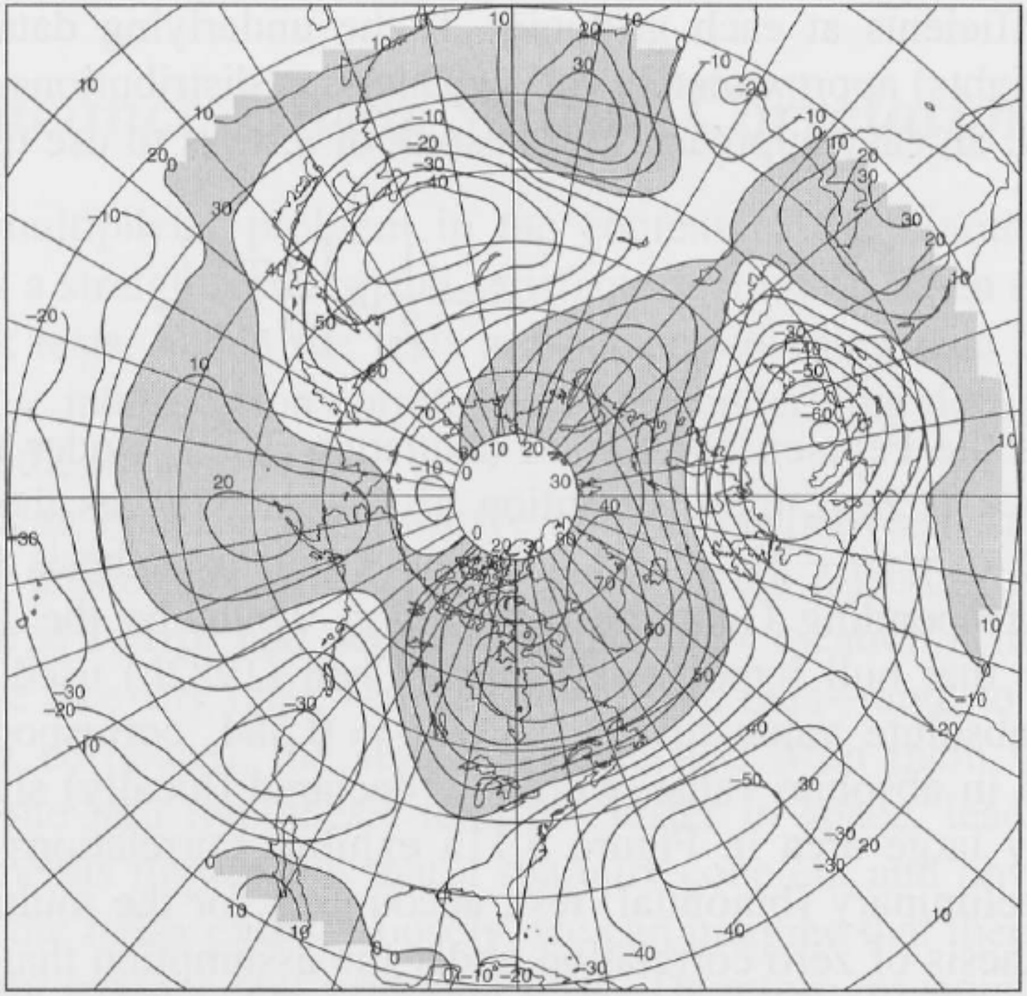
- At multiple locations
- On multiple variables of the same system
- False rejection with a predefined probability (at the significance level) for each test➔ more tests mean more possible passed tests by chance
- Local significance and global (field) significance
- Example based on Livezey and Chen (1983), methods applicable to trend estimate

700 hPa height and SOI



Chen 1981

700 hPa height and noise



CORRELATION BETWEEN NOISE AND DJF 700 MB HEIGHT

Livezey and Chen 1983

Global significance: independent tests

- False rejection ***expected*** by chance (at *p* probability)
- Probability of x out of *N* falsely passed tests follow a binomial distribution

$$P_r = \{X = x\} = \binom{N}{x} p^x (1-p)^{N-x}, x = 0,1,..,N$$

$$\binom{N}{x} = \frac{N!}{x!\,(N-x)!}$$

- With a limited number of tests, false rejection rate is ***greater*** than the nominal rate defined by the local significance
- How many rejections are needed to claim a global significance?
- The significance levels for local and global may differ

Probability of exact M over 30 passed tests

M	percentage (M/30*100)	p
0	0.0%	0.215
1	3.3%	0.339
2	6.6%	0.259
3	10%	0.127
4	13.3%	0.045
5	16.6%	0.012

ROBERT E. LIVEZEY AND W. Y. CHEN

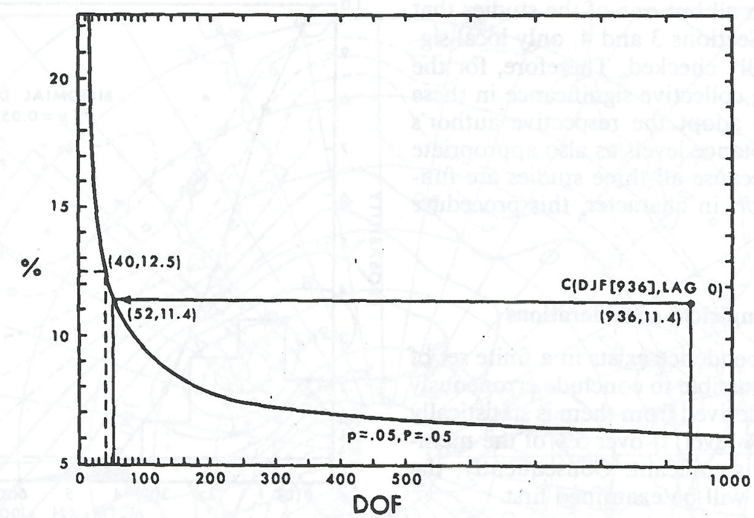


FIG. 3. Estimated percent of independent 95% ($p = 0.05$) significance tests passed that will be equalled or exceeded by accident 5% ($P = 0.05$) of the time versus the number of independent tests N (labeled "DOF" for "degrees of freedom"). The curve is based on the binomial distribution. The plotted point and coordinate lines and points refer to the significance test of Chen's experiment described in the text.

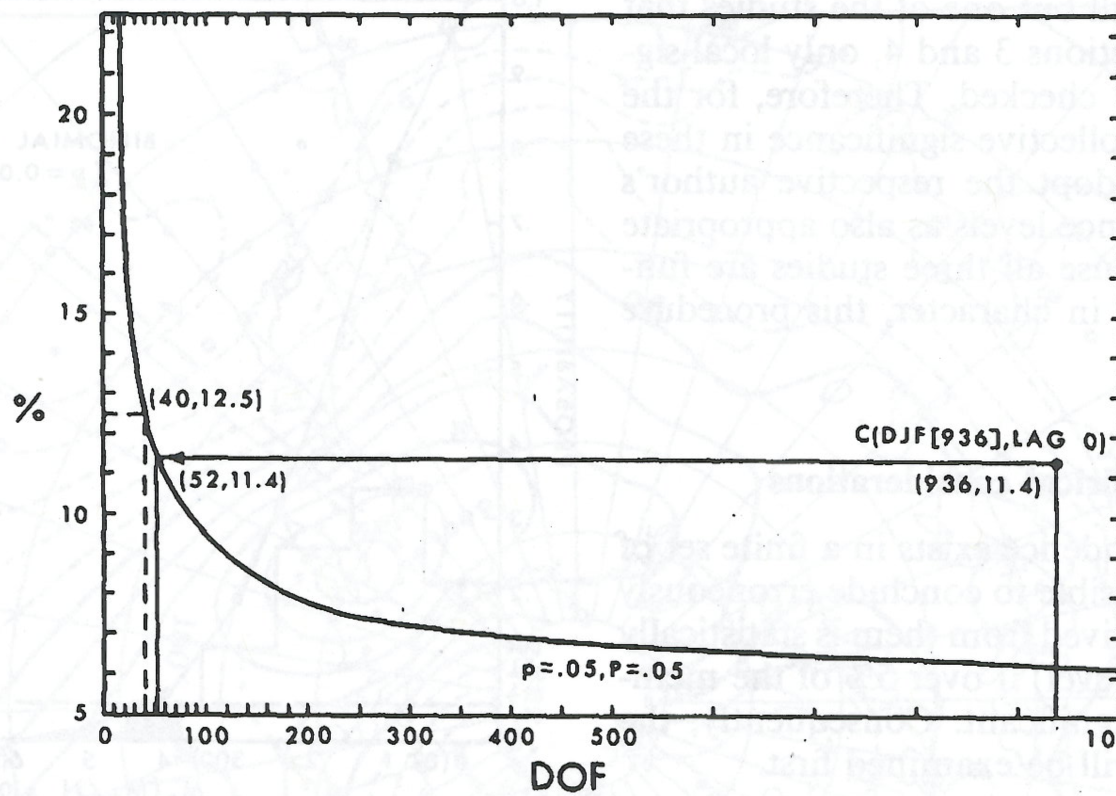
Global significance

- At p=0.05, there could be 14.1% or more passed tests in 30 tests
- Or one needs to obtain more than 14.1% passed test to claim global significance at the 5% level
- It takes more than 1000 independent tests in order for the proportion of passed tests close to (but still slightly higher than) the nominal level

Multiple tests: non-independent

- Multiple tests are very often not independent
- Estimate the proper number of degrees of freedom, use the results for the independent tests
- Monte-Carlo simulation

Estimate DoF



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M-C simulation, more details

- Repeatedly generate random variables to mimic the SOI index
  - Random noise
  - Block Bootstrap to consider serial correlation
  - AR process
- Compute the correlation between 700 hPa height and the generated “soi” indices, and fraction that locally significant correlation has been detected
- The fraction corresponding to the pre-defined global significance level is the threshold value with which the correlation with real SOI should be compared

There are likely more land regions where the number of heavy precipitation events has increased than where it has decreased (IPCC AR5 SPM) ---

How do we know?

--- A worked example

Data collection and consideration

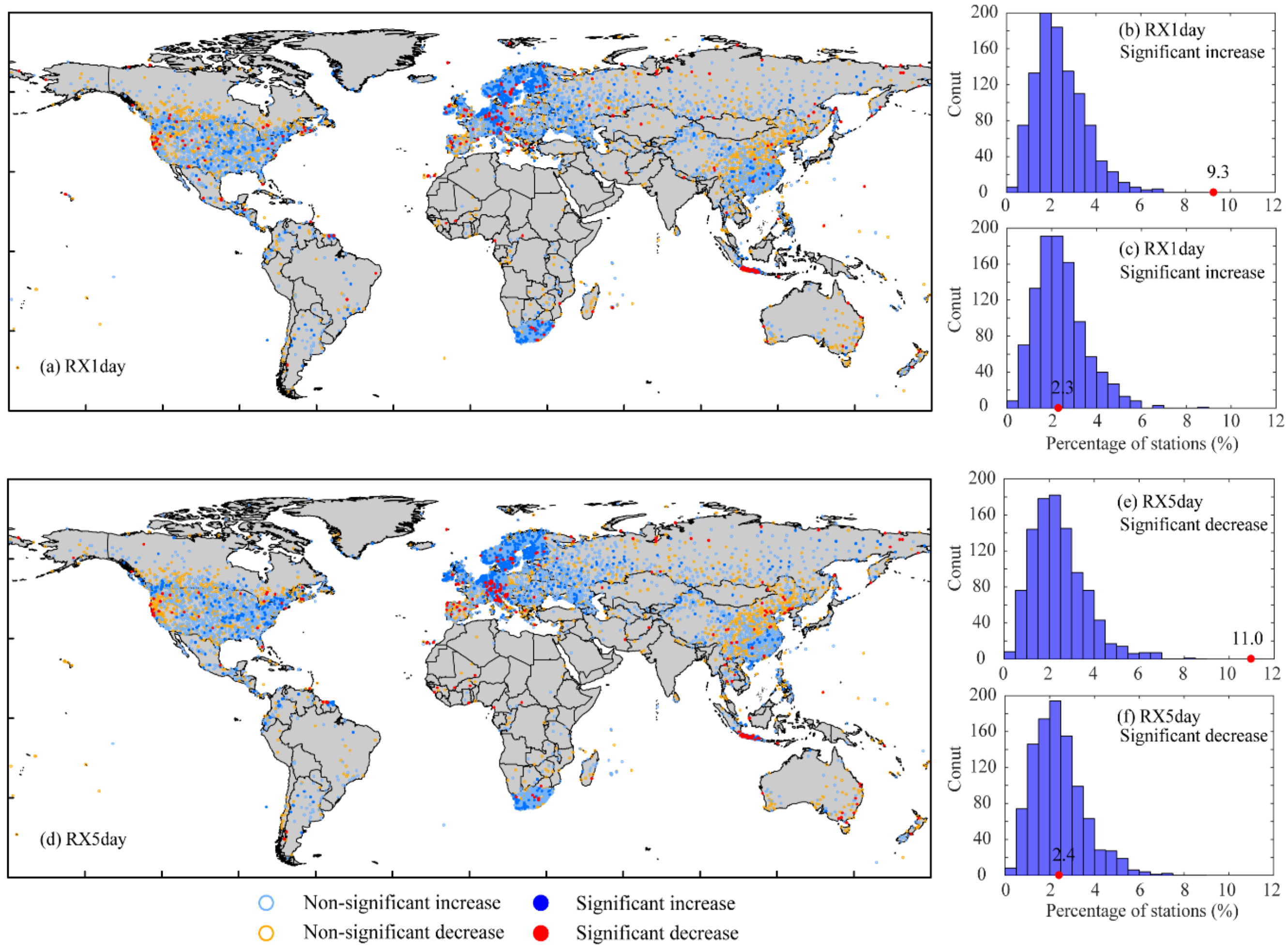
- Data collection
- Consideration of data quality and homogeneity
- Missing values

Selection of methods

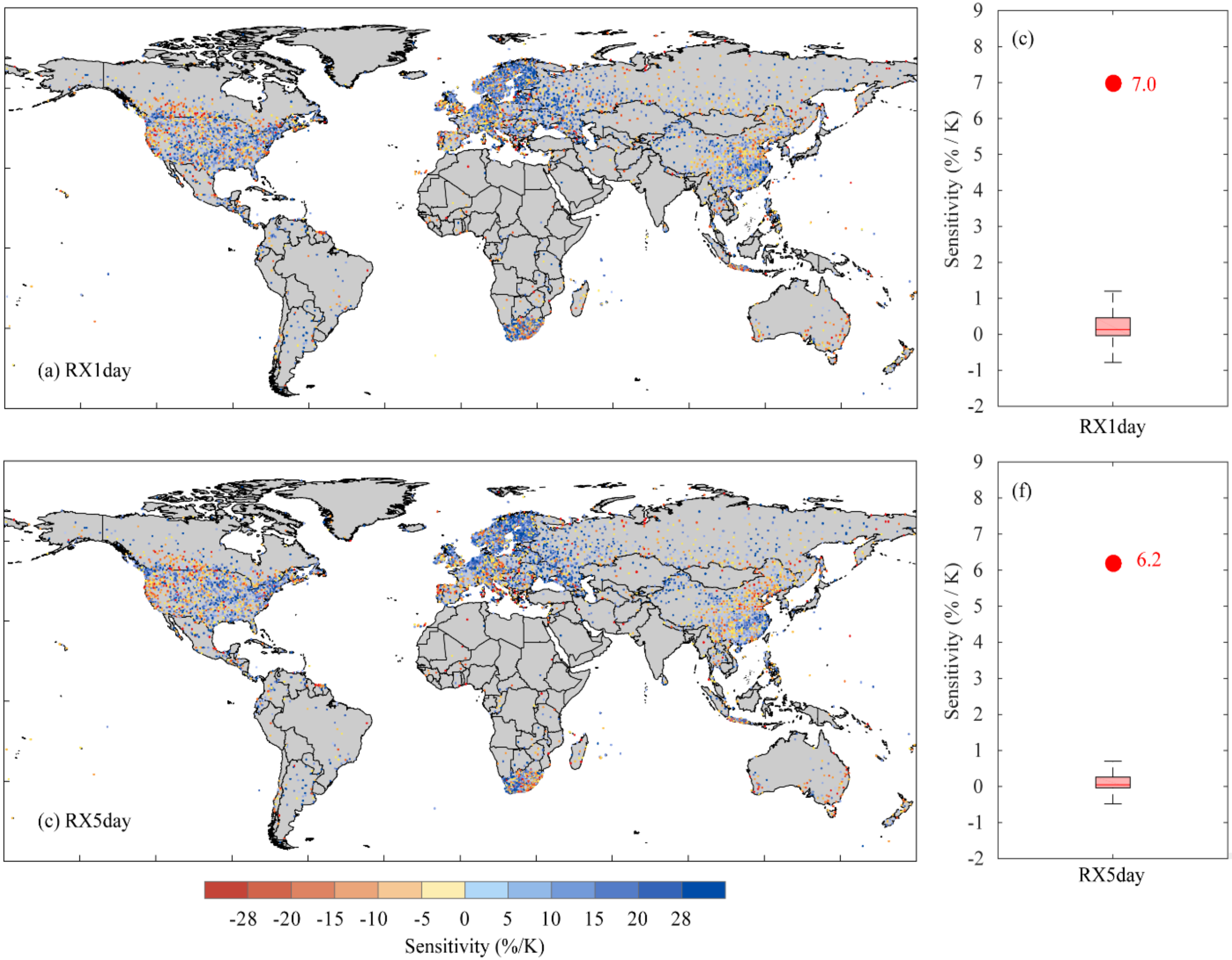
- Mann-Kendall test for statistical significance of trends
- Bootstrap to determine field significance
- GEV fit to determine prcp sensitivity

What we have learnt

- No significant trends in most stations
- Percentage of stations with statistically significance increase trend larger than expected by chance
- Percentage of stations with statistically significance decrease trend is not different from that by chance
- Conclusion: 1) Difficult to detect a trend at individual locations; 2) Evidence of heavy precipitation intensification at the global scale



Is there an association between annual maximum 1-day precipitation and global mean temperature?



Sun et al. 2019 in preparation