A minimalistic damped harmonic oscillator framework for assessing decadal climate predictability in the Subpolar North Atlantic

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The Subpolar North Atlantic (SPNA) stands out for its high decadal predictability of heat content variability[1]. At the same time, climate models exhibit oscillatory behaviour on decadal to multi-decadal time scales in that region. We show that a damped harmonic oscillator model driven by North Atlantic Oscillation (NAO) variability successfully mimics the behaviour of more complex climate prediction models and exhibits similar hindcast performance during the period 1980 to present. In line with previous studies that emphasise the role of ocean dynamics, the performance of the analytical model drops if the ocean heat transport – represented as SPNA heat content tendency – is unitialised. The model's resonance characteristic further suggests that the amount of predictable internal variability is conditional and strongly depends on the recent frequency history of the atmospheric forcing. It is therefore likely that the extended period of predominantly positive NAO prior 1995 led to enhanced SPNA predictability in subsequent years and decades. The simulated variability during the period of interest is not very sensitive to the spin-up length and synchronises rather quickly – within few decades – when NAO forcing is applied. This confirms the utility of using atmospheric re-analysis products to synchronise the ocean in prediction models. Amongst other applications, the damped harmonic oscillator framework may help to investigate the limits of SPNA decadal predictability under the assumption that the atmospheric forcing itself is not predictable and to better understand inter-model prediction differences.

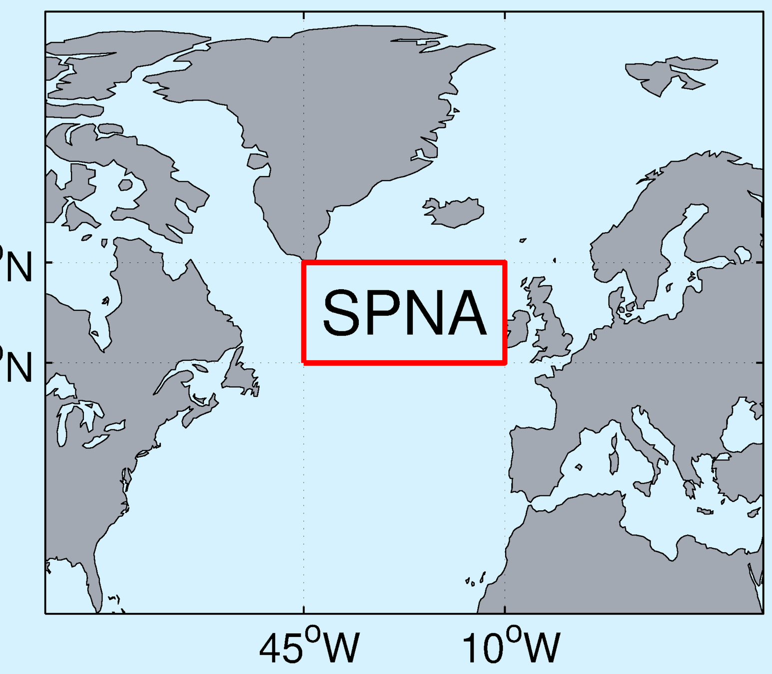

Fig 1 | **Subpolar North Atlantic region**[1]

1. Introduction

SPNA (Fig. 1) climate is characterised by

1. decadal-scale climate variability driven by atmospheric NAO variability[1]
2. prediction models show large benefit from initialisation[2]
3. initialisation of meridional transports thought to be key[1] (Fig. 2)
4. ESMs show quasi-bidecadal oscillatory behaviour[3] (Fig. 3)

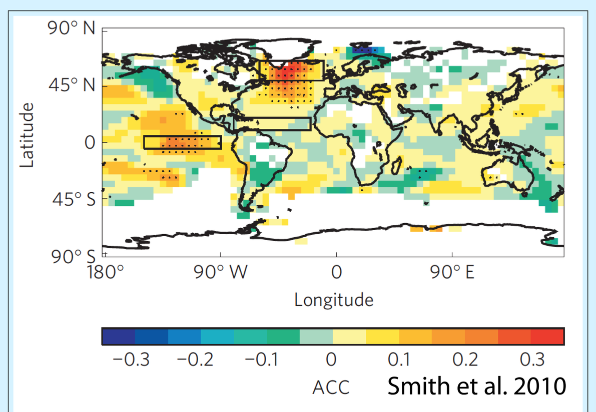

Fig 2 | **Impact of initialisation on correlation skill score**[2]. Skill difference between DePreSys and NoAssim for five-year (June-November) SAT.

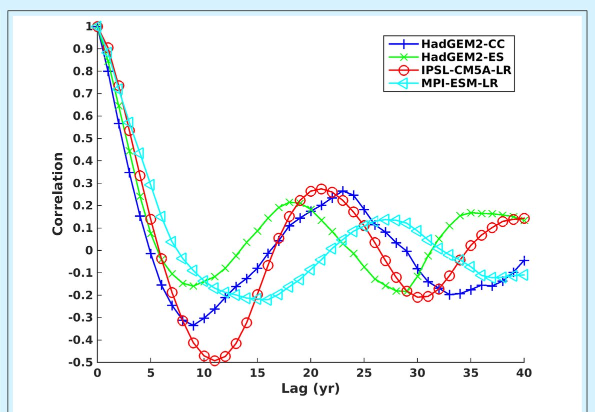

Fig 3 | **Auto-correlation of SPNA temperature in CMIP5 models.** Box-averaged annual 0-1000m temperature from CMIP5 piControl experiment.

This study aims at designing the **simplest conceptual/analytical model that captures 1–4** and applying it to the following problems:

- How important is meridional heat transport initialisation?
- How does the ability of ESMs to produce quasi-oscillatory behaviour affect their ability to hindcast SPNA trends?
- How is SPNA predictability conditionalised by the forcing history? Given the recent forcing evolution, is SPNA variability more or less predictable in the near-future? How much forcing history need to be considered to successfully initialise the meridional heat transport?

We argue that a **damped harmonic oscillator forced with NAO variability** is a natural candidate for such a task. Here we

- compare reanalysis and hindcast results of the damped harmonic oscillator to results from a dynamical climate prediction model and from a thermal inertia AR1 model
- perform a sensitivity experiment where the meridional ocean heat transport is initialised to zero

2. Damped Oscillator Model (DOM)

Second-order Linear Differential Equation

$$y'' + 2\zeta\omega_0 y' + \omega_0^2 y = -NAO_{normalised-index}$$

y	SPNA temperature
y'	SPNA temperature tendency (≡meridional heat transport)
ζ	damping coefficient := 0.05 (65% energy loss per oscillation)
ω_0	resonance frequency := $2\pi/25$ yr^{-1} (approx. average from Fig. 3)

Spectral transformation of NAO forcing

To be able to analytically integrate the model, we perform a spectral transformation of the NAO time-series using the discrete cosine series

$$NAO(t) = \sum_{k=0}^{N} \frac{2}{N} X_k \cos\left(\frac{\pi k}{N} t + \frac{\pi k}{2N}\right) \equiv -\sum_{k=0}^{N} B_k \cos(\omega_k t + \phi_k)$$

where $X_k = \sum_{n=0}^{N-1} NAO_n \cos\left(\frac{\pi k}{N} n + \frac{\pi k}{2N}\right)$, $B_k = \frac{-\pi k}{N}$, $\phi_k = \frac{\pi k}{2N}$

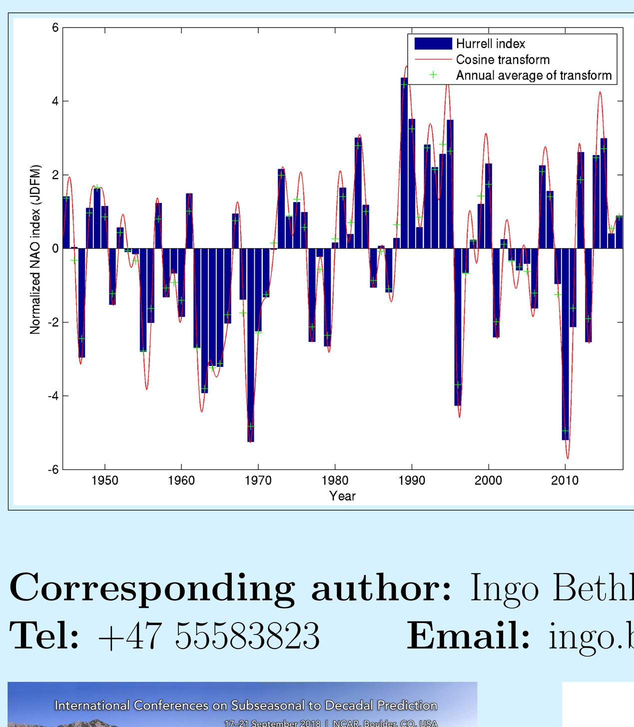

Fig 4 | **Cosine transform of NAO time-series.** Station-based Hurrell winter (djf) NAO index (bars), cosine transform at high temporal resolution (curve) and averaged over winters (crosses).

2. Damped Oscillator Model (DOM) – cont.

Analytical solution – underdamped case

$$y(t) = -A_1 e^{-\zeta\omega_0 t} \sin\left(\sqrt{1-\zeta^2}\omega_0 t\right) + A_2 e^{-\zeta\omega_0 t} \cos\left(\sqrt{1-\zeta^2}\omega_0 t\right) + \sum_{k=0}^{N} \frac{B_k}{Z_k \omega_k} \cos(\omega_k t + \phi_k + \mathrm{atan}(2\omega_k\omega_0/(\omega_0{}^2 - \omega_k{}^2)))$$

$$y'(t) = -A_1\zeta\omega_0 e^{-\zeta\omega_0 t} \sin\left(\sqrt{1-\zeta^2}\omega_0 t\right) + A_1\sqrt{1-\zeta^2}\omega_0 e^{-\zeta\omega_0 t} \cos\left(\sqrt{1-\zeta^2}\omega_0 t\right) - A_2\zeta\omega_0 e^{-\zeta\omega_0 t} \cos\left(\sqrt{1-\zeta^2}\omega_0 t\right) - A_2\sqrt{1-\zeta^2}\omega_0 e^{-\zeta\omega_0 t} \sin\left(\sqrt{1-\zeta^2}\omega_0 t\right) - \sum_{k=0}^{N} \frac{B_k}{Z_k} \sin(\omega_k t + \phi_k + \mathrm{atan}(2\omega_k\omega_0/(\omega_0{}^2 - \omega_k{}^2)))$$

where

A_1, A_2	integration constants computed from $y(t_0) := y_0$, $y'(t_0) := y'_0$
Z_k	$\sqrt{2(\omega_0\zeta)^2 + (\omega_0^2 - \omega_k^2)^2/\omega_k^2}$

The **forced inhomogenous solution** readily provides the **DOM reanalysis**.

A_1 and A_2 are calculated by inversion from the initial conditions y_0 and y'_0. The reanalysis is initialised with $y(1864) := 0$, $y'(1864) := 0$. The hindcasts use either (i) the same A_1 and A_2 as the reanalysis, (ii) recomputed A_1 and A_2 to match the start condition y_0 of EN4-analysis or (iii) recomputed A_1 and A_2 with $y'_0 := 0$ (no heat transport).

The **homogenous solution** (=inhomogenous without last term) is used for the **hindcasts**, **implying NAO:=0**.

3. Thermal Inertia Model (AR1)

Differential equation

$$y' + \frac{1}{\tau} y \sim -NAO$$

Analytical solution

$$y(t) = -e^{-t/\tau} \sum_{k=0}^{N} \frac{2}{N} X_k \frac{1/\tau}{1/\tau^2 + \omega_k{}^2} (\cos(\phi_k) + \omega_k \tau \sin(\phi_k))) + \sum_{k=0}^{N} \frac{2}{N} X_k \frac{1/\tau}{1/\tau^2 + \omega_k{}^2} (\cos(\omega_k t + \phi_k) + \omega_k \tau \sin(\omega_k t + \phi_k))$$

where $\tau := 1$ yr is close to optimal fit (Fig.5)

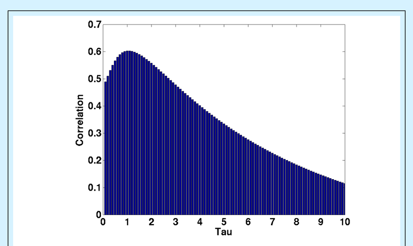

Fig 5 | **Correlation of AR1-analysis with EN4-analysis as function of τ.**

4. Results

The damped oscillator model (DOM) forced with historical NAO variability successfully captures inter-decadal variations in observed SPNA temperature (Fig. 6c vs 6b, bars).

DOM produces hindcasts results (Fig. 6c, solid lines) that largely resemble the results from a dynamical prediction model[4,5] (Fig. 6d) and compare favorably to the observed variability (Fig. 6b, bars).

Correcting the initial DOM temperature state at hindcast start has little impact on hindcast performance (Fig. 6b vs 6c, solid lines). However, initialising the temperature tendency to zero strongly degrades the hindcast results (Fig. 6b+c, stippled lines).

The thermal inertia AR1 model driven with NAO variability successfully captures some of the rapid interannual shifts (Fig. 6e vs 6b, bars) but shows poor performance over extended periods (e.g. 2000-2010) and shows overall poor hindcast performance (Fig. 6e, solid lines).

5. Summary

Good agreement with a dynamical model supports the idea that SPNA climate variability behaves primarily like a damped harmonic oscillator forced by NAO variability.

Rapid shifts are not well captured, hinting that there is more to SPNA variability than the damped oscillator dynamics investigated here.

The DOM results confirm a major role of ocean circulation and meridional ocean heat transport, giving rise to SPNA predictability even if the atmospheric forcing cannot be predicted.

The DOM predicts start of SPNA warming trend that peaks in 2025.

6. Future work

- use of stochastic or predicted NAO forcing in DOM predictions to model prediction spread/uncertainty
- characterise resonance behaviour of ESMs with idealised ocean runs
- investigate transient resonance behaviour with NAO wavelet analysis
- perturbed parameter ensemble with multiple resonance frequencies
- address failure of DOM to capture rapid shifts

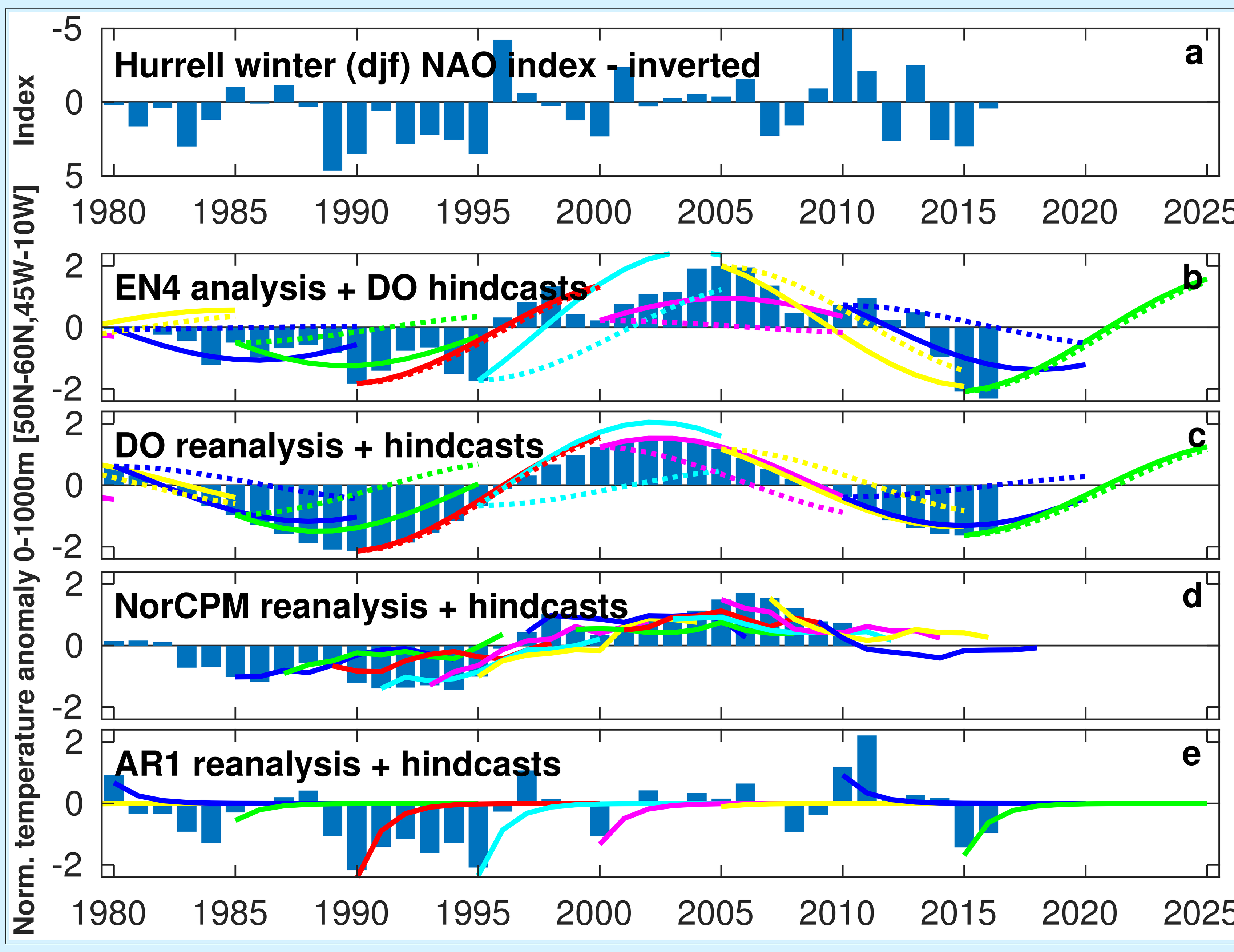

Fig 6 | **SPNA reanalysis and hindcast comparison. (a)** Station-based Hurrell winter (djf) NAO index (hPa), detrended and inverted. **(b-e)** Box-averaged [50°N-60°N, 45°W-10°W] upper ocean (0-1000m) temperature anomaly. **(b)** EN4 analysis (bars), damped oscillator hindcasts with initial state nudged to EN4 (solid curves) and with tendency initialised to zero (dotted curves). **(c)** Same as b but using the damped oscillator reanalysis instead of EN4 analysis. **(d)** NorCPM.v1[4,5] reanalysis using anomaly assimilation of SST and TS profile observations (bars) and NorESM hindcasts initialised from it (solid curves). **(e)** Reanalysis (bars) and hindcasts (solid curves) from the AR1 thermal inertia model.

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References

[1] Yeager et al., 2017: A Decadal Prediction Case Study: Late Twentieth-Century North Atlantic Ocean Heat Content. *J. Clim.*
[2] Smith et al., 2010: Skilful multi-year predictions of Atlantic hurricane frequency. *Nat. Geosci.*
[3] Swingedouw et al., 2015: Bidecadal North Atlantic ocean circulation variability controlled by timing of volcanic eruptions. *Nat. Comm.*
[4] Counillon et al., 2016: Flow-dependent assimilation of sea surface temperature in isopycnal coordinates with the Norwegian Climate Prediction Model. *Tellus A*
[5] Wang et al., 2017: Optimising assimilation of hydrographic profiles into isopycnal ocean models with ensemble data assimilation. *Ocean Model.*