S2S predictions are inherently probabilistic.

Does one need ultra-high resolution models to correctly represent the associated forecast probability distributions?

Or can lower resolution models with a combination of deterministic and stochastic parameterizations be adequate for this purpose?
A hierarchy of simplified anomaly models
from nonlinear GCMs (top) to linear stochastically forced models (bottom)

\[
\frac{dx}{dt} = A(x) + P(x) + R
\]

resolved parameterized unparameterized

\[
\approx A(x) + (1 + r)P(x)
\]

\[
\approx \{A_0 x + (S_{0A} + S_{1A} x)\xi_A\} + (1 + r)\{P_0 x + (S_{0P} + S_{1P} x)\xi_P\}
\]

\[
\approx Lx + b\eta_1 + (E x + g)\eta_2
\]

\[
\approx Lx + S\eta
\]

1. Approximate \( R \sim r P(x) \) where \( r \) is a random number \([-1,+1]\)

2. Approximate chaotically nonlinear \( A(x) \) and \( P(x) \) as linear terms plus noise

3. Combine terms

4. Ignore state-dependent noise \( E x \)

If these approximations are good for S2S scale dynamics, then one may not need ultra-high resolution models for S2S predictions
And indeed, for *seasonal* tropical SST predictions, a Low-Order (28-component) model of the form

\[
\frac{dx}{dt} = Lx + S\eta
\]

where \( L \) and \( S \) are estimated through Linear Inverse Modeling (LIM, Penland and Sardeshmukh 1995), has similar skill to that of the operational NMME models (Newman and Sardeshmukh 2017).

The curve on the **BLUE SEMICIRCLE** on the left, and the bottom panel on the right, indicate the potential LIM skill.

The **BLUE SEMICIRCLE** shows the perfect-model skill trajectory in any chaotic dynamical system in which the forecast signals are on average orthogonal to the forecast noise.
For subseasonal (Week 2) predictions, stochastic parameterizations of the form \((1 + r)P(x)\) in a relatively low-resolution (T254) version of the NCEP/GFS model leads to both a marked reduction in the rms error of ensemble-mean forecasts and an increase in the ensemble spread, and thus have a positive impact on both the deterministic and probabilistic skill. (Sardeshmukh, Wang, Compo, and Penland, 2018)

Results are for 80-member ensemble forecasts for 80 separate forecast cases in Jan-Mar 2016.
In fact, individual ensemble members of the T254 GFS forecasts with stochastic parameterizations have comparable skill to that of the 6x higher resolution T1534 individual forecasts of the operational GFS model for this period.

Note that the rms errors of both the individual T254 and T1534 forecasts are much larger than those of the ensemble mean T254 forecasts. This is another advantage of lower resolution models, since it is much cheaper to generate large forecast ensembles with them.
But can lower resolution models adequately represent the skew and heavy tails of sub-seasonal anomaly distributions, and therefore extreme anomalies on S2S scales?
The PDFs of daily atmospheric variations are not Gaussian. They are generally skewed and heavy tailed, and in a distinctive way. This has large implications for extreme weather statistics.

Skewness \( S = \frac{< x^3 >}{\sigma^3} \) and

Kurtosis \( K = \frac{< x^4 >}{\sigma^4} - 3 \)

of wintertime daily anomalies \( x \) of 250 mb Vorticity in the 140-yr 20th Century Reanalysis (Compo et al 2011)

These distinctive non-Gaussian properties are captured by a general class of so-called Stochastically Generated Skewed (“SGS”) probability distributions that are associated with anomaly dynamics of the form

\[
\frac{dx}{dt} = Lx + b\eta_1 + (Ex + g)\eta_2
\]

Sardeshmukh, Compo, Penland (2015)

Note the parabolic inequality \( K \geq 3/2 S^2 \)

Note that the crossover point where \( p(x) = p(-x) \) lies between 1.4\( \sigma \) and 1.7\( \sigma \)
ECMWF models used in the ATHENA project (Jung et al 2012) capture the Skewness $S$ and Kurtosis $K$ of daily 200 mb Vorticity in DJF at both the low T95 (~ 120 km) and high T2047 (~ 6 km) resolutions remarkably well.

**NOTE** that the color scale for kurtosis is (1.5*x*x) the color scale for the skewness to highlight the consistency of both the patterns and magnitudes of $S$ and $K$ with the “SGS” probability distribution theory.
Consistent with the simple theory of “SGS” probability distributions, the K-S curve for 200 mb vorticity is almost identical for different model resolutions!

This result raises the important issue of whether one needs ultra-high resolution models to represent the non-Gaussian \textit{shapes} of the observed PDFs, given that even a low-resolution T95 (~100 km) model can already capture them, and given that even in the T95 model the non-Gaussianity is effectively due to “SGS” anomaly dynamics.
Since these approximations are good for S2S dynamics, one may not need ultra-high resolution models for S2S predictions. Lower resolution (~ 25-50 km) models with “scale-aware” deterministic and stochastic parameterizations may suffice.