

# Understanding the zonally symmetric component of the flow from potential vorticity inversion

Aarnout J. van Delden and Yvonne B.L. Hinssen

Institute for Marine and Atmospheric research Utrecht (IMAU), Utrecht University, Utrecht, The Netherlands

a.j.vandelden@uu.nl

## Introduction

The dynamical mechanisms that maintain the zonal mean (longitudinally averaged) state have not been fully identified. The "potential vorticity (PV) viewpoint" of the general circulation, as advocated by Hoskins (1991), can make a significant contribution to identifying these mechanisms for two reasons: (1) PV-conservation under adiabatic circumstances makes it possible to identify specific diabatic processes with specific features of the PV-field, and (2) piecewise PV-inversion links these PV-features to features in wind velocity, temperature and pressure.

Identifying, from a PV-viewpoint, the mechanisms that maintain the zonal mean state involves a research program that consists of 3 steps.

In step 1 the theory is described. This theory consists of defining a hemispheric scale reference PV-distribution, which is associated with the state of rest, and identifying the zonal mean PV-anomalies that induce the zonal mean flow (figs. 2 & 4). Piecewise PV-inversion is applied to the PV-anomaly distribution in order to connect specific features of this PV-anomaly distribution to features in the wind- and temperature-field (fig. 5), such as the jets and the tropopause inversion layer. In step 2 the seasonal cycle of the zonal mean PV-anomaly distribution is diagnosed from observational data. Finally, in step 3, step 2 is repeated using numerical model output in order to identify diabatic sources of PV (fig. 6).

## Variables

$u$ : zonal mean zonal wind  
 $\phi$ : latitude  
 $\theta$ : potential temperature  
 $a$ : radius of the Earth  
 $p$ : pressure  
 $g$ : acceleration due to gravity  
 $f$ : Coriolis parameter  
 $\zeta$ : relative vorticity  
 $\sigma$ : isentropic density  
 $Z$ : potential vorticity

## Definitions in isentropic coordinates

Zonal mean relative vorticity:  $\zeta = \frac{u \tan \phi}{a} + \frac{\partial u}{\partial y}$

Isentropic density:  $\sigma = -\frac{1}{g} \frac{\partial p}{\partial \theta}$

Zonal mean potential vorticity:  $Z = \frac{\zeta + f}{\sigma}$

Reference potential vorticity:  $Z_{ref} = \frac{f}{\sigma_{ref}}$

Reference isentropic density:  $\sigma_{ref} = \frac{\int \sigma \cos \phi d\phi}{\int \cos \phi d\phi}$

## Non-dimensional anomalies

Reference state + anomalies:

$$Z = Z_{ref} + Z^*$$

$$\sigma = \sigma_{ref} + \sigma^*$$

$$\zeta = f + \zeta^*$$

Non-dimensional anomalies

$$Z^* \equiv \frac{Z - Z_{ref}}{Z_{ref}}; \sigma^* \equiv \frac{\sigma - \sigma_{ref}}{\sigma_{ref}}; \zeta^* \equiv \frac{\zeta - f}{f}$$

Relation between anomalies

$$Z^* = \zeta^* - (1 + Z^*)\sigma^*$$

See figures 2 & 3.

## Potential vorticity inversion equation

$$\frac{\partial}{\partial y} \left( \frac{\partial u}{\partial y} - \frac{u \tan \phi}{a} \right) + \frac{Z}{g} \frac{\partial}{\partial \theta} \left( \rho f_{loc} \theta \frac{\partial u}{\partial \theta} \right) = \frac{df}{dy} - \sigma \frac{\partial Z}{\partial y}$$

$$f_{loc} \equiv f + \frac{2u \tan \phi}{a}$$

R.h.s. of PV-inversion (forcing)=0 if  $Z = Z_{ref}$  and  $\sigma = \sigma_{ref}$

Given the COSPAR International Reference Atmosphere (CIRA) monthly mean, zonal mean potential vorticity, the solution of this equation for the northern hemisphere (10° to 90°lat, earth's surface to 2250 K) yields the zonal wind,  $u$  (figures 4 & 5).

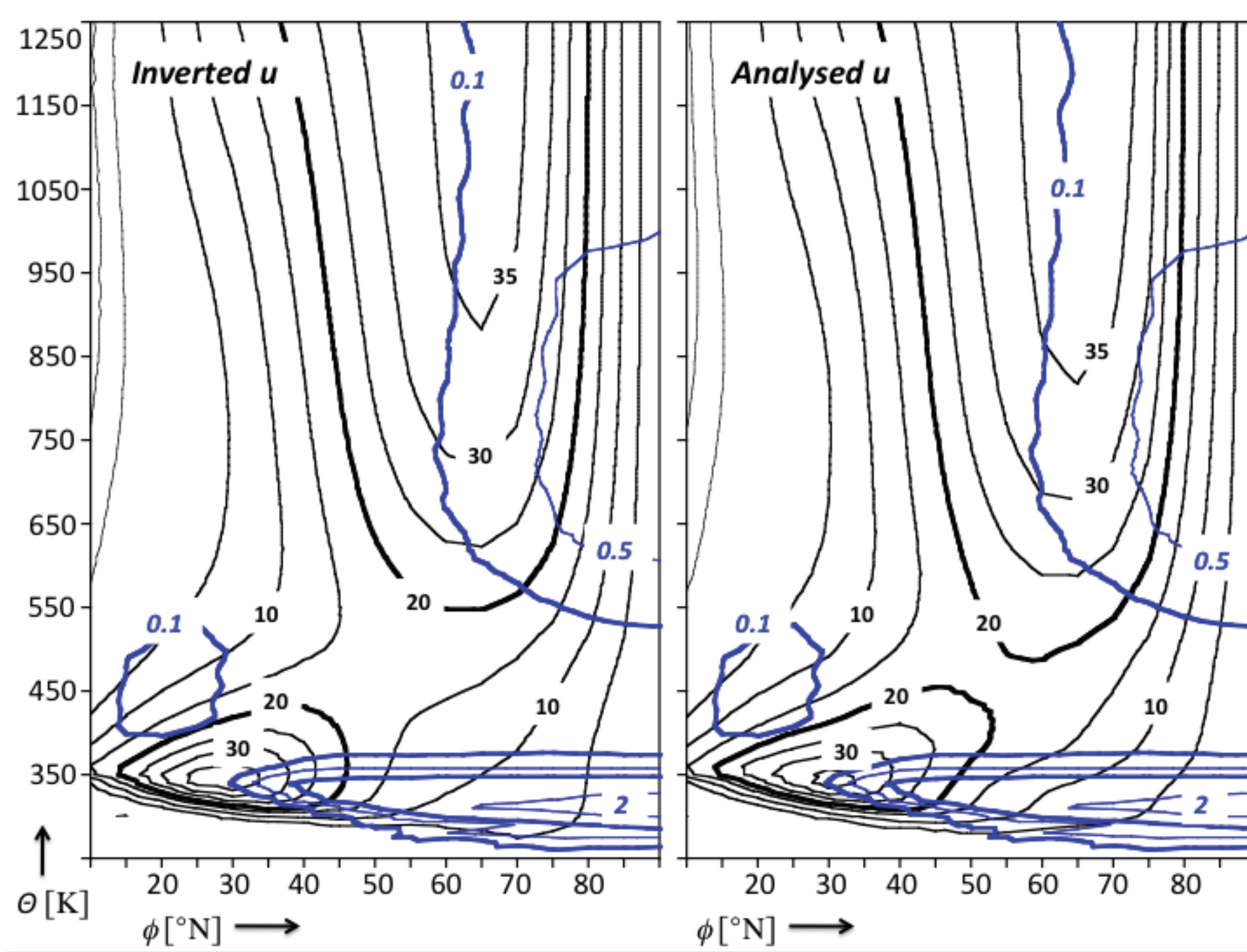


FIGURE 4. The zonal average, January average zonal wind velocity as a function of potential temperature and latitude (black contours, labeled in m/s) in January derived from PV-inversion (left panel) and according to the CIRA-analysis (right panel). The potential vorticity anomalies that "induce" this wind field are shown in blue (only positive value are contoured). For more details of the structure of these anomalies, see figure 2. Labels are given in non-dimensional units. The isopleths corresponding to 0.1 and 1 non-dimensional unit are drawn thick; the isopleths corresponding to 0.5, 2 and 3 non-dimensional units are drawn thin.

## References

- Fleming, E. L., S. Chandra, J. J. Barnett, and M. Corney, 1990: Zonal Mean Temperature, Pressure, Zonal Wind, and Geopotential Height as Functions of Latitude. *Advances in Space Research*, 10, No. 12, 11-59.
- Hoskins, B.J., 1991: Towards a PV- $\theta$  view of the general circulation. *Tellus*, 43AB, 27-35.
- Plumb, R.A., and J. Eluszkiewicz, 1999: The Brewer-Dobson Circulation: Dynamics of the tropical upwelling. *J. Atmos. Sci.*, 56, 868-890.

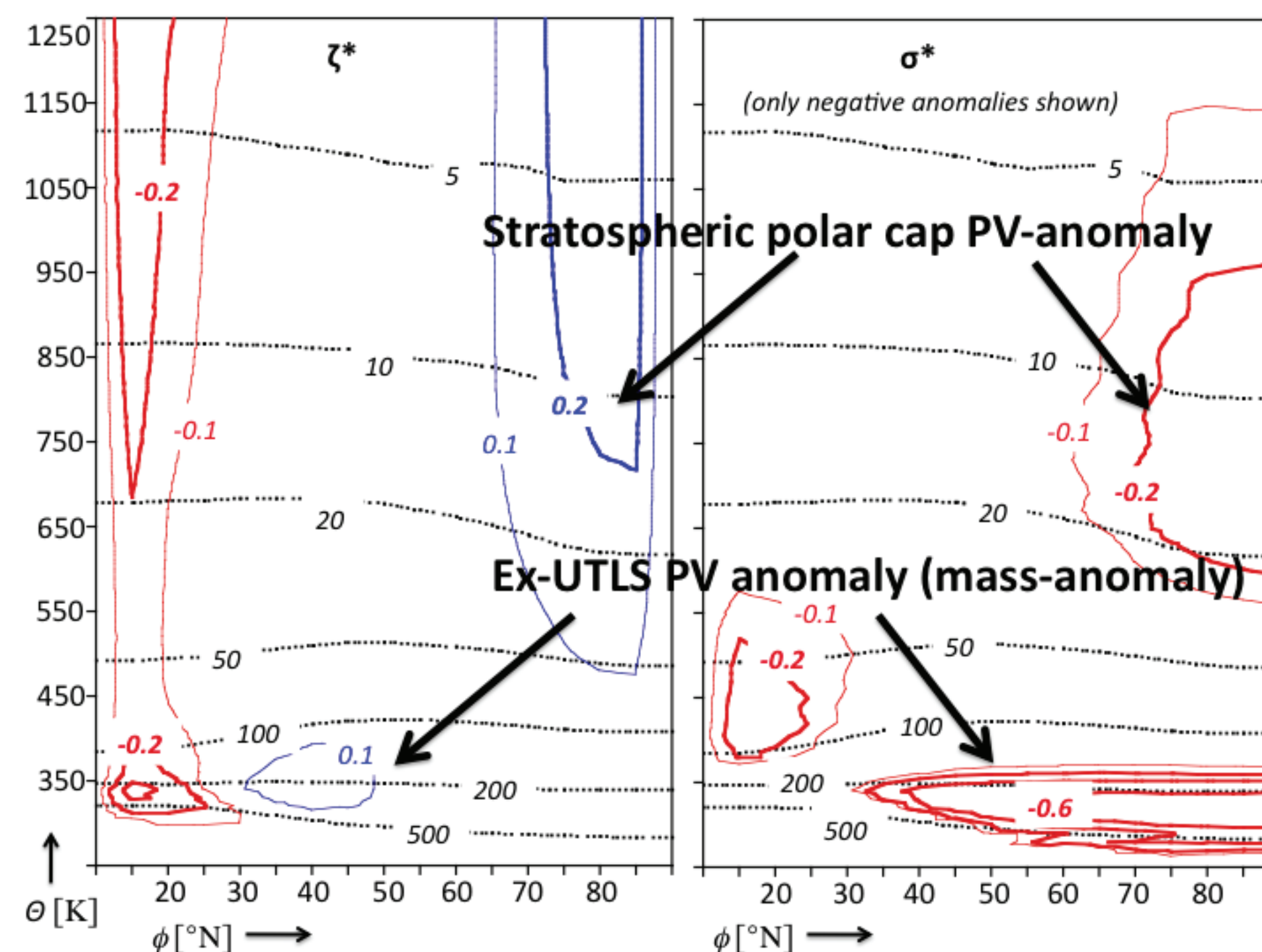


FIGURE 3. The normalized relative vorticity (left panel) (labeled in non-dimensional units) and the normalized isentropic density (right panel) (labeled in non-dimensional units) in January (CIRA). Also shown is the pressure field (dashed lines, labeled in hPa). The Ex-UTLS PV-anomaly is manifest strongly as an isentropic density- (or mass-) anomaly. This fact should be taken into account when imposing the lower boundary condition when applying the technique of piecewise PV-inversion (figure 5).

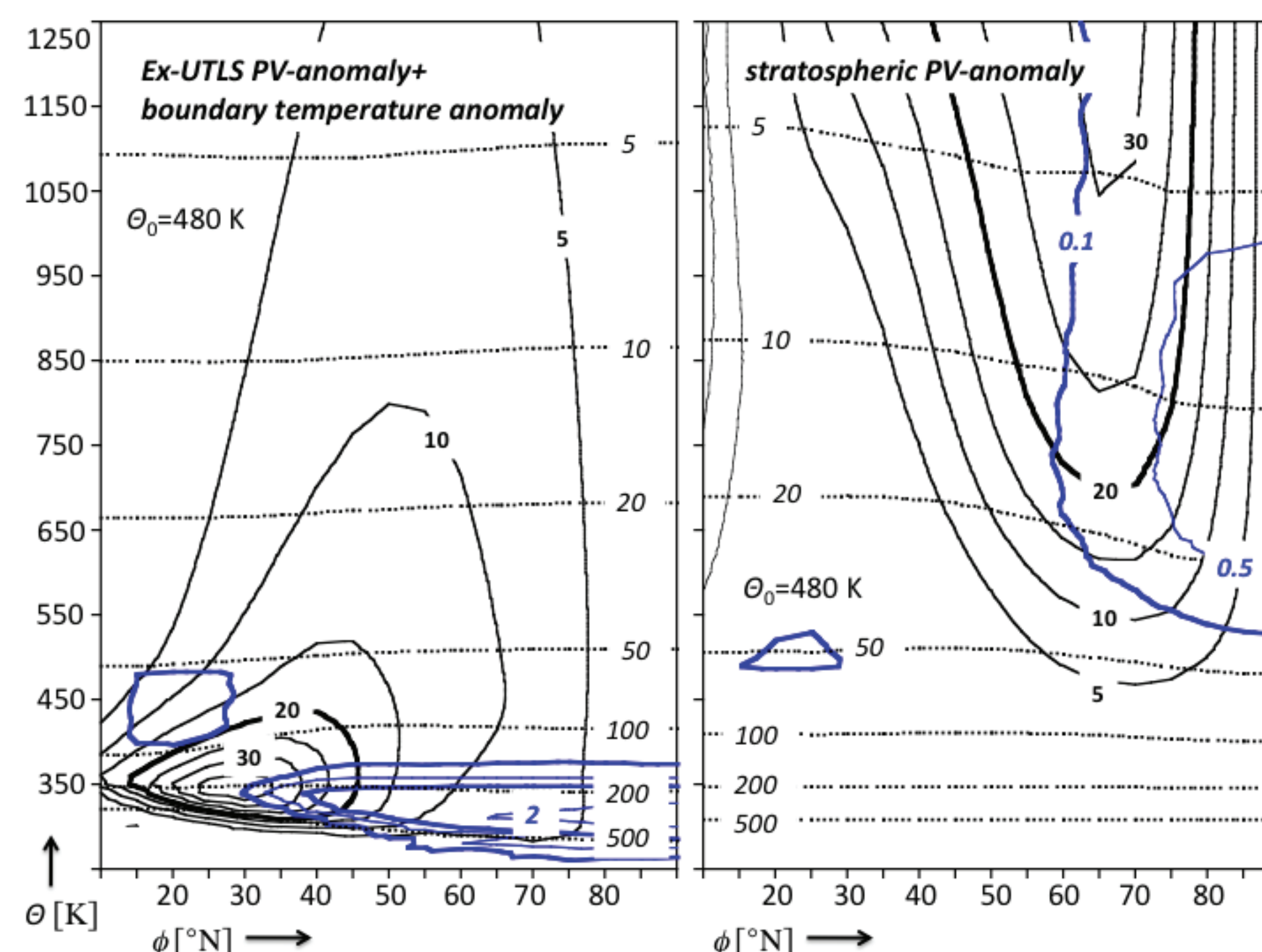


FIGURE 5. The zonal average, January average zonal wind velocity (black contours labeled in m s<sup>-1</sup>) and pressure (dashed lines, labeled in hPa) as a function of potential temperature and latitude, derived from piecewise PV-inversion. Left panel shows the result when the Ex-UTLS PV-anomaly and the surface temperature anomaly are retained (i.e. Z<sup>\*</sup>=0 for  $\theta > 480$  K). The right panel shows the result when only the polar cap upper stratospheric PV-anomaly is retained (i.e. Z<sup>\*</sup>=0 for  $\theta \leq 480$  K and the thermal wind at the lower boundary,  $\Delta u = 0$ ). The PV-anomalies that are retained in the inversion are shown in blue and labeled in non-dimensional units.

## More information about this research:

<http://www.staff.science.uu.nl/~delde102/research.php>

## Note:

The model-results shown in figure 6 are preliminary. A paper is in preparation on the results of piecewise PV-inversion and the seasonal cycle of PV (figures 1 to 5).

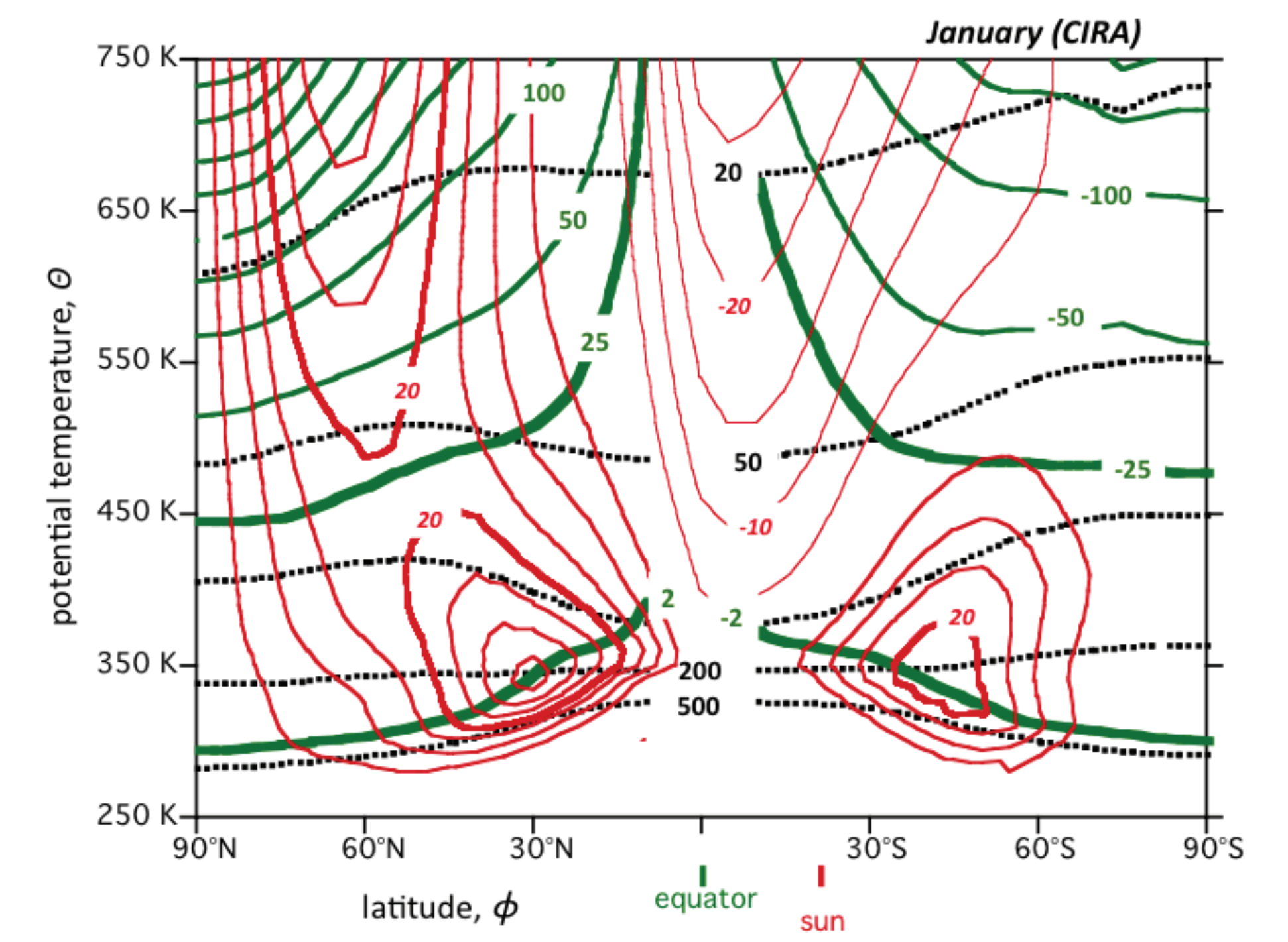


FIGURE 1. The zonally averaged, monthly average zonal wind,  $u$  (red contours, labeled in m s<sup>-1</sup>), the potential vorticity,  $Z$  (green contours, labeled in PVU; interval is 50 PVU for absolute values greater than 50) and pressure,  $p$  (black dashed contours, labeled in hPa) as a function of potential temperature and latitude for January according to the COSPAR International Reference Atmosphere (CIRA) (Fleming et al., 1990).

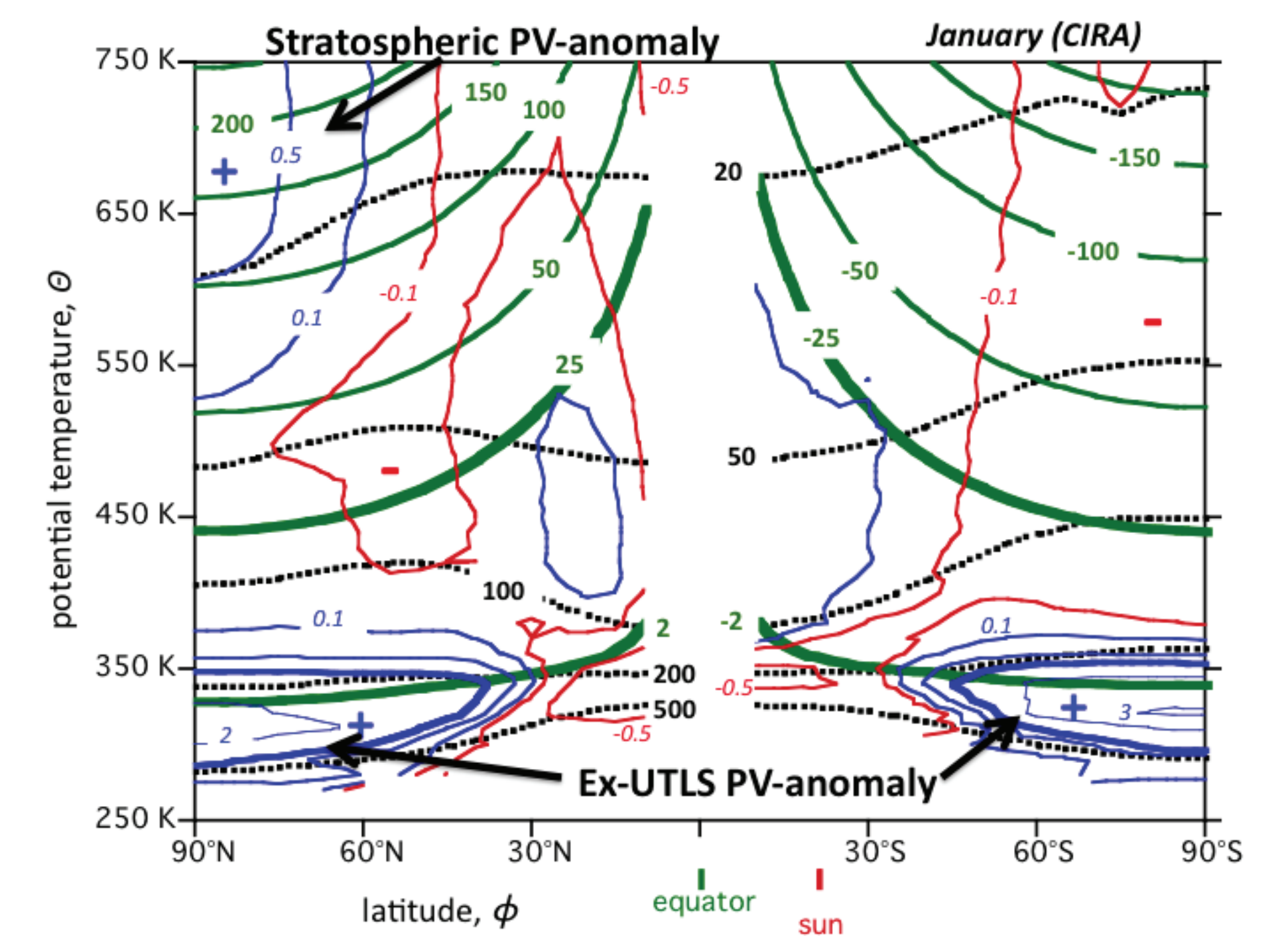


FIGURE 2. The zonal average distribution of  $Z_{ref}$  (green contours; labeled in PVU),  $Z^*$  (blue: positive; red: negative; labeled in non-dimensional units) and pressure,  $p$  (black dotted; labeled in hPa) as a function of latitude and potential temperature for January to the CIRA. Plus- and minus signs indicate maxima and minima in  $Z^*$ , respectively, or positive and negative PV-anomalies. The monthly average overhead position of the sun is indicated in red below. Contours equatorward of 10° latitude and below the 650 hPa-level are not drawn. The thick green line corresponds to  $Z_{ref}=2$  PVU, which can be interpreted as the "reference dynamical tropopause".

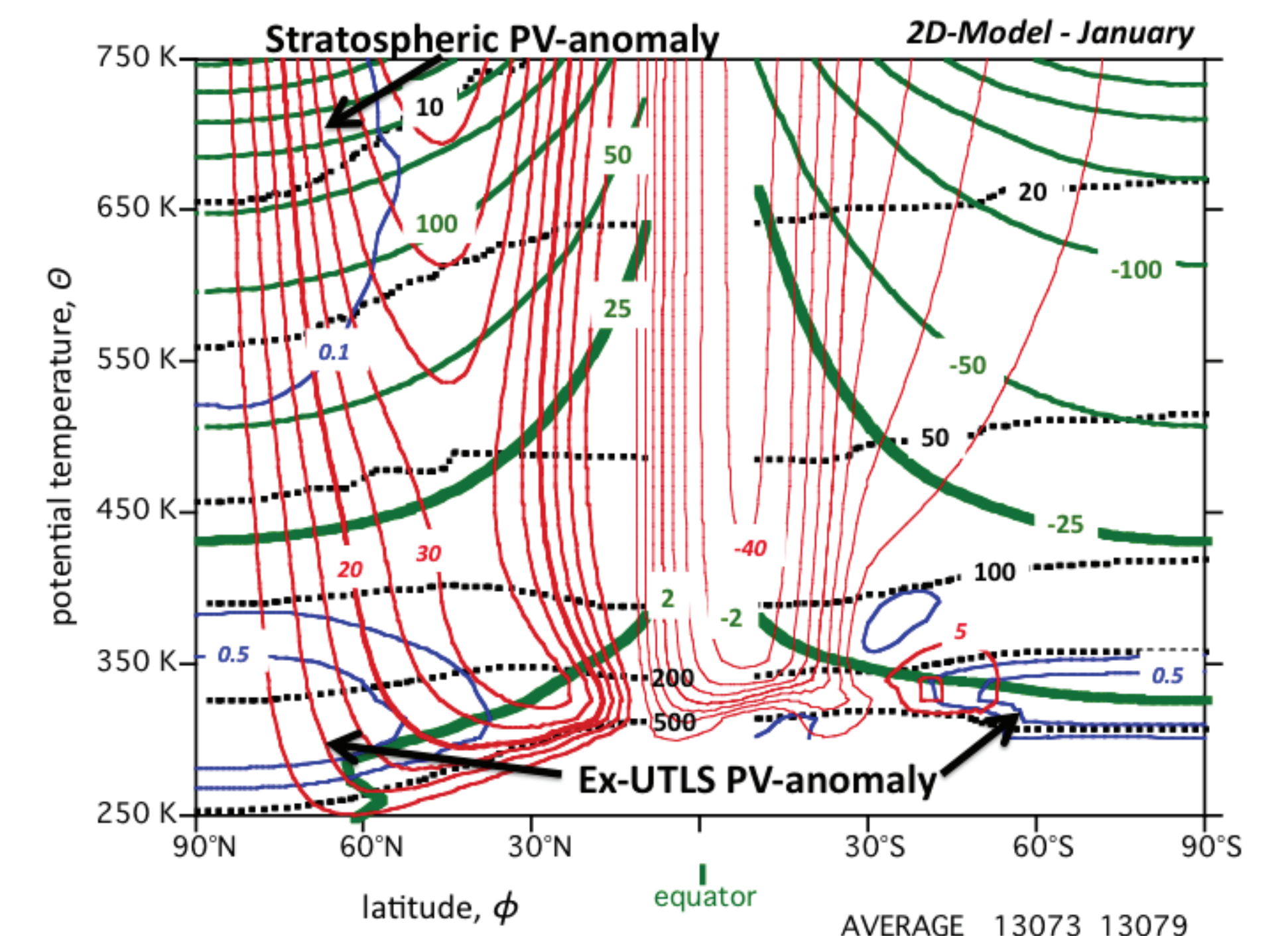


FIGURE 6. The zonal average distribution of  $u$  (red, labeled in m s<sup>-1</sup>),  $Z_{ref}$  (green, labeled in PVU),  $Z^*$  (blue, labeled in non-dimensional units; only positive PV anomalies are shown) and pressure,  $p$  (black dashed, labeled in hPa) as a function of latitude and potential temperature according to a 2D-model primitive equation model for January. Contours of  $Z_{ref}$  and  $Z^*$ , equator-ward of 10° latitude and below the 650 hPa-level, are not drawn. The thick green line corresponds to  $Z_{ref}=2$  PVU, which can be interpreted as the "reference dynamical tropopause". The model an explicit radiation parametrization and includes a crude water cycle. This model result should be compared with the analysis of observations shown in figures 1 and 2. The Ex-UTLS PV-anomalies in both hemispheres are too weak, but qualitatively the modelled PV-distribution looks realistic. Wave drag (parametrized as in Plumb and Eluszkiewicz, 1999) and a strong tropical water vapour greenhouse effect are the key processes that determine the PV-distribution in the UTLS.

## Summary of most important results

The zonally symmetric flow is determined by two PV-anomalies, and by the temperature at the Earth's surface. The first PV-anomaly is located in the stratosphere. It exhibits a strong seasonal cycle: in winter it is positive, while in summer it is negative. The second PV-anomaly, the "Ex-UTLS PV-anomaly", coincides approximately with the extra-tropical tropopause (fig. 2), and is manifest strongly as a mass-anomaly (fig. 3). This complicates the application of the lower boundary condition in piecewise PV-inversion. Nevertheless, piecewise PV-inversion (fig. 5) reveals clearly that the "Ex-UTLS PV-anomaly" induces the year round westerly winds in the troposphere and the lower stratosphere, including the subtropical jet.