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Stratosphere-troposphere coupling: use of the Fluctuation-Dissipation Theorem as a quantifier of tropospheric response

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Introduction

Some aspects of the coupling between stratosphere and troposphere can be understood as tropospheric response to stratospheric forcing, with the tropospheric circulation acting, through the coupled effects of eddies and mean flow, as an amplifier. The Fluctuation-Dissipation Theorem (FDT) is one theoretical tool available to predict the tropospheric response to forcing. The FDT provides an estimate of the linear operator relating forcing to response, based only on the statistics of the unforced tropospheric circulation, calculated from a suitable time series. The simplest prediction of the FDT, already exploited in the context of response to stratospheric forcing, is that response to forcing will be proportional to the longest correlation timescale in the unforced circulation. Potentially the FDT can provide more precise information on the structure and magnitude of the response to an arbitrary forcing. However the usefulness is limited by (a) sampling issues (i.e. the accuracy of the prediction is limited by the length of the time series of the unforced circulation) and (b) the Gaussianity assumption in the traditional form of the FDT. This poster provides quantitative analysis of (a) and presents a non-Gaussian extension of the traditional FDT -- we refer to the extension as a 'non-parametric FDT'. The usefulness of the non-Gaussian FDT as a predictor of changes in the tropospheric circulation is currently being explored.

The Fluctuation-Dissipation Theorem

Consider a dynamical system described by a state vector **X** and governed by a set of evolution equations that give d**X**/dt in terms of **X**. The dynamics are assumed to be stochastic either because of deterministic chaos or because of the inclusion of explicit stochastic forcing terms. In equilibrium the system has a smooth probability density function ρ(**X**). A steady forcing δ**F** is added to the right-hand sides of the equations for d**X**/dt. A measure of the response is the change < δ**X** > in the mean value of **X**, which is predicted by a linear operator **L** acting on δ**F**.

$$\langle \delta\mathbf{X}\rangle = \mathbf{L}\,\delta\mathbf{F} \qquad (1)$$

The Fluctuation-Dissipation Theorem (FDT) predicts the operator **L** using information only about the equilibrium state. A general statement of the FDT is

$$\mathbf{L} = -\int_0^\infty \langle \mathbf{X}(\tau)[\frac{\nabla\rho(\mathbf{X}(0))}{\rho(\mathbf{X}(0))}]^T\rangle\, d\tau = -\int_0^\infty \mathbf{\Lambda}(\tau)\, d\tau. \quad (2)$$

To evaluate the above expression for **L** requires knowledge of (i.e. estimation of) the equilibrium probability density ρ(**X**).

The Gaussian FDT

A conventional simplifying assumption is that the system is Gaussian, i.e.

$$\rho(\mathbf{x}) = \frac{1}{\sqrt{\det[2\pi\mathbf{C}(0)]}}\,\exp[-\frac{1}{2}\mathbf{X}^T\mathbf{C}(0)^{-1}\mathbf{X}]$$

implying that

$$\mathbf{L} = \int_0^\infty \mathbf{C}(\tau)\mathbf{C}(0)^{-1}\, d\tau \qquad (3)$$

where **C**(0) is the covariance matrix and **C**(τ) is the lag covariance matrix. In this estimate for **L** all required knowledge of ρ(**X**) is captured by **C**(0).

Acknowledgement: This research was supported by the UK Natural Environment Research Council Grant NE/0005051/1 to the consortium "Solar Influences on Climate" led by Professor J. Haigh and by the Natural Environment Research Council Grant NE/G003122/1 to Dr. G. Esler.

Implementation of the Gaussian FDT (GFDT)

The expression (3) has to be estimated from 'data' on the dynamical system of interest. Two important considerations are (a) the length of the dataset required to make a robust estimate of the lagged covariance C(τ) for each τ and (b) estimation of the infinite integral, e.g. by truncating at some finite upper limit *T*. Any test of the Gaussian FDT must take these points into account. Useful insight can be gained by considering a very simple model. The results below are for a linear model with 2 degrees of freedom forced by white noise.

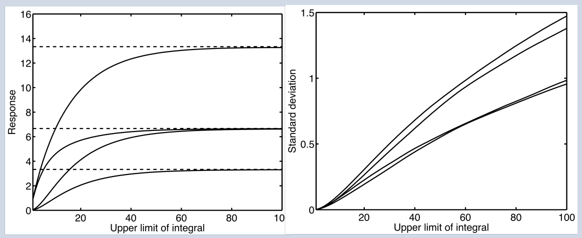

Figure 1: Response (4 elements of matrix **L**) to forcing of a linear model estimated using Gaussian FDT showing dependence on upper limit *T*. The decay time for the autocorrelation is about 16. The estimate is constructed from a simulation of length 10^5. *Left panel:* Mean estimate from 10^3 simulations. Dashed lines are the analytically calculated response. *Right panel:* Standard deviation of estimated response. [Note that the uncertainty of the estimate increases with *T* , also that standard deviation will be 10 times greater for a simulation of length 10^3.]

A non-Gaussian ('nonparametric') FDT

The expression (2) can be estimated using elementary methods of nonparametric statistics, estimating ρ(**X**) from data $\mathbf{X}_1$, $\mathbf{X}_2$, ..., $\mathbf{X}_n$ on the equilibrium system by

$$\hat{\rho}(\mathbf{X}; h, n) = \frac{1}{n}\sum_{i=1}^{n} N(\mathbf{X}; \mathbf{X}_i, h)$$

where N(**X**; $\mathbf{X}_i$,*h*) is a Gaussian centred at $\mathbf{X}_i$ with standard deviation *h*. The resulting expression for **L** depends on *h* and there must be a range of *h* for which the prediction is insensitive to the actual value of *h* and bias and uncertainty are both small.

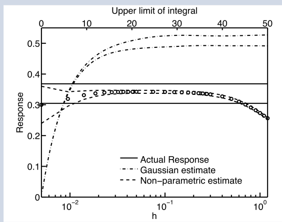

Figure 3: Test of non-parametric FDT vs GFDT for a simple 1-d nonlinear stochastic system.

$$\frac{dX}{dt} = X - 0.02X^3 + 5\xi$$

where ξ is white noise with unit variance. The horizontal lines show the simulated response (with uncertainty). The dash-dot lines show the GFDT estimate for response as a function of the upper limit *T* of the integral. The dots show the non-parametric estimate for different values of *h* with the dashed lines indicating uncertainty.

Application of the GFDT to an atmospheric model

The expression (3) can also be used to estimate **L** for a quasi-realistic atmospheric model. Results are shown here for a simple 'dynamics-only' general circulation model (T30 resolution with 20 levels). The response of the zonal mean velocity to a zonal symmetric momentum forcing is considered. The underlying dynamical system is taken to be an evolution equation for the zonal mean velocity field. This is a significant reduction from the full dynamical system defined by the model since (i) the zonal mean is taken and (ii) the full set of zonal mean dynamical fields is reduced to zonal velocity only (justified by a balance assumption). (ii) requires a suitable modification of the imposed momentum forcing (through solution of an 'Eliassen problem' (see Ring and Plumb 2008 JAS). The large dimension of the state space makes it useful to project the response onto some chosen basis function, in this case chosen to be the leading EOF. Important questions include possible choices for *T* and for truncation of the representation of the forcing in terms of EOFs and the correct formulation of the Eliassen problem.

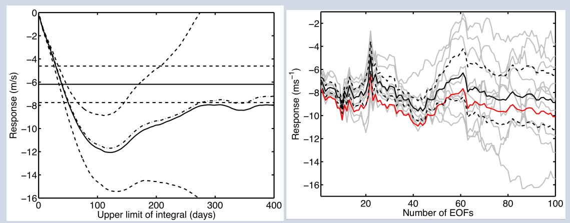

Figure 2: Predicted response in EOF1 of zonal wind to localised momentum forcing in T30L20 model. Equilibrium simulation is 10^6 days. *Left panel:* Predicted response as function of upper limit *T* including all EOFs in forcing. Solid curve is mean of prediction made using 10 individual segments each of length 10^5. Dashed curves indicate (using standard deviation) uncertainty of that set of predictions. Dot-dash curve is prediction made using full 10^6 days of simulation. Horizontal lines show simulated response (with uncertainty estimate). *Right panel:* variation of predicted response with EOF truncation applied to forcing. Grey curves show 10 predictions from individual segments, solid curve is mean and dashed curves show uncertainty. Red curve solves Eliassen problem on resting rather than 'climatological' state. [As in Figure 1 there is optimal range of *T* with large *T* implyng large uncertainty As more EOFs are included uncertainty increases. Within uncertainty there is systematic difference between predicted and simulated response. Overall conclusion is that GFDT makes significant quantitative errors (> 30% in this case).]

Synopsis

• Stratosphere-troposphere coupling involves strong two-way interactions between waves and mean flow in the troposphere. Conventional dynamical descriptions (e.g. in terms of EP fluxes) tend to be 'descriptive' rather than 'predictive'.

•The FDT offers an alternative approach to quantifying tropospheric response to stratospheric (or other) forcing. The operator predicting response in terms of perturbation can be estimated in terms of data on fluctuations in the unperturbed system.

•Estimates are fundamentally statistical and due attention needs to be paid to bias and to uncertainty. Many previous studies have not given explicit information on these.

•Assessment of the Gaussian FDT suggests that it is not an accurate estimator of response to forcing for a simple atmospheric general circulation model. This assessment takes proper account of uncertainty and therefore suggests that the Gaussianity assumption is inadequate.

•A non-Gaussian ('nonparametric') FDT can be formulated and has been tested on low dimensional systems (Cooper and Haynes 2011 JAS). Work is ongoing to extend to more realistic systems.