Permafrost Methane Emission as Detector of Future Regional Arctic Climate Change

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Mathematical Modelling Positive Carbon–Climate Feedback: Permafrost Methane Emission Case

Stefan Problem for permafrost thawing

\[
\begin{align*}
\frac{\partial T_m(x,t)}{\partial t} + \frac{\partial}{\partial \tau} \left( \frac{T_m(x,t) - T_{\infty}}{\kappa} \right) &= 0, \\
\frac{\partial T_m(x,t)}{\partial \tau} &= \frac{\partial}{\partial \tau} \left( \frac{T_m(x,t) - T_{\infty}}{\kappa} \right), \\
T_m(x,0) &= \phi(\tau), \\
\lim_{\tau \to t} \frac{\partial T_m(x,\tau)}{\partial \tau} &= \lim_{\tau \to t} \frac{\partial T_m(x,\tau)}{\partial \tau},
\end{align*}
\]

Boundary Condition

\[ T_m(0,\tau) = T_{\infty}, \]

Condition for interphase boundary

\[ T_m(\zeta,\tau) = T_m(\zeta,\tau) = \text{const}, \]

Initial Condition

\[ T_m(x,0) = f(x), \]

Ginzburg-Landau approach to Stefan Problem

Permafrost lake is a permafrost system where phase transition from frozen to thawing ground state is determined as the Stefan Problem and methane production and future distribution is described as the Classical Equations of Chemical Kinetics and Diffusion Equation.

Interphase boundary as a field of finite thickness:

\[ a^2 \frac{\partial ^2 \phi}{\partial \tau^2} + \frac{\partial \phi}{\partial \tau} = a^{-1} g(\phi) \]

Solution of this equation:

\[ \int (z + \varepsilon \tau) + \text{const} \]

Order Parameter: Parameter of front of thawing:

\[ \phi = \phi(x,y,z,t), \]

Derivative of double-well potential:

\[ g = U'(\phi) = 0.5(\phi - \phi^2) + \phi_0 \]

Extended Gooijer’s radiative-convective atmospheric model

\[ \begin{align*}
\vec{v} \cdot (\nabla \vec{v}) &= \alpha \nabla \phi - \gamma \nabla P + f(x,y,z,t) + \sigma_i (\theta - \Theta_i) \vec{z}, \\
\theta + (\vec{v} \cdot \nabla) \vec{\theta} &= \Delta \vec{\theta} - 3\alpha \vec{\theta} + \vec{Q}, \\
\nabla \cdot \vec{v} &= 0, \\
C_1 + (\vec{v} \cdot \nabla) \vec{C} &= d\vec{C} - \vec{b} \cdot \vec{C}.
\end{align*} \]

Boundary Condition for velocity

\[ \vec{v}(x,y,z,t) \bigg|_{\tau = 0} = \vec{v}(x,y,z,t) \bigg|_{\tau = \infty} = 0 \]

Boundary Condition for temperature

\[ \theta_i(x,y,z,t) \bigg|_{\tau = 0} = \theta_i(x,y,z,t) \bigg|_{\tau = \infty} = 0 \]

Boundary Condition for methane concentration

\[ C_1(x,y,z,t) \bigg|_{\tau = 0} = 0, \]

\[ C_2(x,y,z,t) \bigg|_{\tau = 0} = -\mu(x,y,0(x,y,0,t)) \]

Absorption radiation coefficient

\[ \alpha(C) \approx \alpha_0 + \alpha C \]

Rayleigh–Bénard convection

\[ \frac{dR}{dt} = \delta - \mu R^{-1}. \]

Conclusions

We see that methane emission can proceed both gradually and catastrophically. Thus, by incorporating this rigorous mathematical approach into different climate models, we can describe the positive feedback for permafrost-climate system in detail to come nearer to a theory of Arctic Armageddon.

We propose a generalization of classical Gooijer model taking into account methane emission effects. For the case of the methane emission by Siberian lakes we describe an asymptotical approach that allows us to obtain an expression for the methane flux via the temperature and fluid fields. By a variational approach we compute an expression for a shift by bifurcation parameter induced by the methane emission. We show that there is possible a tipping point in atmosphere dynamics resulting from methane emission.