Mathematical Modelling Positive Carbon-Climate Feedback: Permafrost Methane Emission Case

Permafrost Methane Emission as Detector of Future Regional Arctic Climate Change

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Stefan Problem for permafrost thawing

$$\begin{cases} \dfrac{\partial T_M(x,\tau)}{\partial \tau} = a_M \dfrac{\partial^2 T_M(x,\tau)}{\partial x^2}; \quad \tau > 0; \quad x \in (0,\zeta), \end{cases}$$

$$\begin{cases} \dfrac{\partial T_m(x,\tau)}{\partial \tau} = a_m \dfrac{\partial^2 T_m(x,\tau)}{\partial x^2}; \quad \tau > 0; \quad x \in (\zeta,\infty), \end{cases}$$

Boundary Condition

$$T_M(0,\tau) = \varphi(\tau),$$

Initial Condition

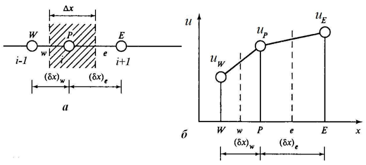
$$T_m(x,0) = f(x),$$

Condition for interphase boundary

$$T_M(\zeta,\tau) = T_m(\zeta,\tau) = T_3 = const,$$

$$\lambda_M \frac{\partial T_M(\zeta,\tau)}{\partial x} - \lambda_m \frac{\partial T_m(\zeta,\tau)}{\partial x} = Q_\phi \frac{d\zeta}{d\tau}.$$

Patankar's numerical scheme adapted for permafrost thawing

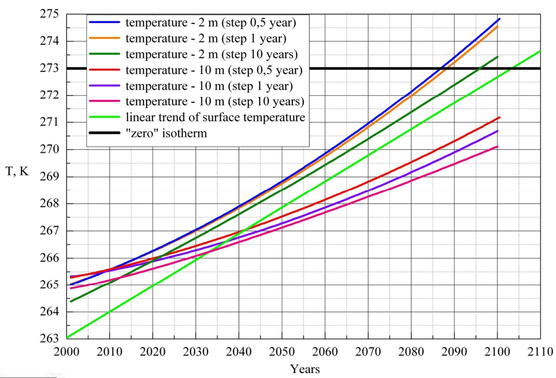


Patankar's method is the transition to discrete analog using integration of heat transfer equation in "control volume".

$$a_i u_i^{(k+1)} = c_i u_{i+1}^{(k+1)} + d_i u_{i-1}^{(k+1)} + b_i,$$

$$i = 1, 2, \ldots, n, \quad u_i^{(k+1)} = u(x, t_{k+1})$$

Predicted permafrost thermal regime for Yamal in the 21st century:



Ginzburg-Landau approach to Stefan Problem

Permafrost lake is a permafrost system where phase transition from frozen to thawing ground state is determined as the Stefan Problem and methane production and future distribution is described as the Classical Equations of Chemical Kinetics and Diffusion Equation.

Interphase boundary as a filed of finite thickness:

$$\alpha\xi^2\varphi_t = \xi^2\varphi_{zz} + a^{-1}g(\varphi)$$

Solution of this equation:

$$th\,((z - vt)/\varepsilon) + const$$

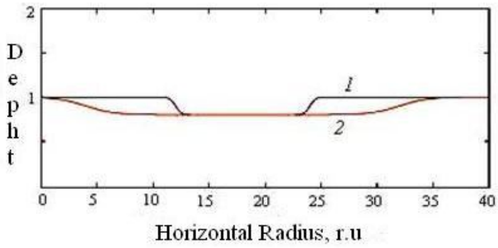
Order Parameter: $\varphi = \varphi(x,y,z,t)$

Parameter of front of thawing: $\varepsilon = a^{-1/2}\xi^{-1}$

Derivative of double-well potential :

$$g = U'(\varphi) = 0{,}5(\varphi - \varphi^3) + g_0$$

Schematic of lake growth model:



1 - initial positions of thawing fronts; *2* - final positions of thawing fronts.

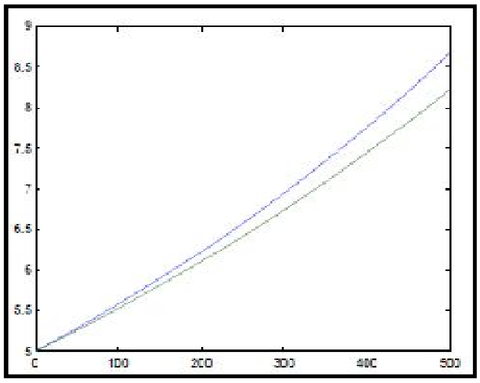
Velocity of front of thawing is proportional mean curvature:

$$v(x,y,z,t) = \delta - \mu k(x,y,z,t)$$

Equation of dynamic s of permafrost lake radius :

$$\frac{dR}{dt} = \delta - \mu R^{-1}.$$

Predicted permafrost lake methane mass in atmosphere:



Vertical axe: the methane mass in atmosphere in 10^{18} mg, horizontal axe : time in years. The blue and green curves show growth without and with tundra lake influence, respectively, time interval 500 years.

Extended Goody's radiative-convective atmospheric model

$$\mathbf{v}_t + (\mathbf{v}\cdot\nabla)\mathbf{v} = \sigma\Delta\mathbf{v} - \gamma\nabla P + f(x,y,z,t) + \sigma_1(\theta - \Theta_0)\mathbf{z},$$

$$\theta_t + (\mathbf{v}\cdot\nabla)\theta + \omega\Gamma = \Delta\theta - 3\alpha\theta + Q$$

$$\nabla\cdot\mathbf{v} = 0$$

$$C_t + (\mathbf{v}\cdot\nabla)C = d\Delta C - b_0^2 C$$

Boundary Condition for velocity

$$\mathbf{v}(x,y,z,t)\big|_{z=0} = \mathbf{v}(x,y,z,t)\big|_{z=h} = 0$$

Boundary Condition for temperature

$$\theta_z(x,y,z,t)\big|_{z=0} = \theta_z(x,y,z,t)\big|_{z=h} = 0$$

Boundary Condition for methane concentration

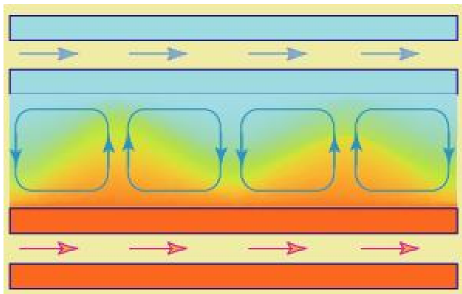
$$C_z(x,y,z,t)\big|_{z=h} = 0$$

$$C_z(x,y,z,t)\big|_{z=0} = -\mu(x,y,\theta(x,y,0,t))$$

Absorption radiation coefficient

$$\alpha(C) \approx \alpha_0 + \alpha_1 C$$

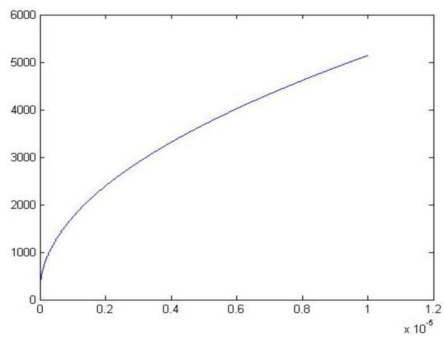
Rayleigh–Bénard convection



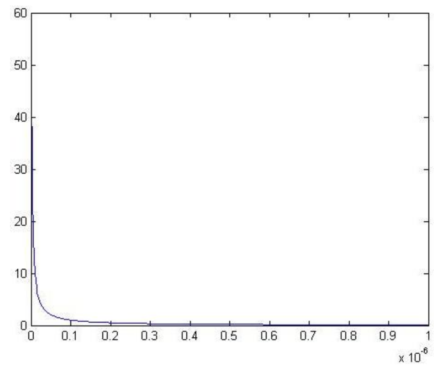
Formula of critical level of methane emission from permafrost to atmosphere

$$\beta_c = \frac{\alpha_0}{\alpha_1 T_0}\sqrt{\frac{b_0}{d}}$$

Depend of critical emission coefficient from methane diffusion in atmosphere:



Depend of critical emission coefficient from global warming potential :



Conclusions

- We see that methane emission can precede both gradually and catastrophically. Thus, by incorporating this rigorous mathematical approach into different climate models, we can describe the positive feedback for permafrost-climate system in detail to come nearer to a theory of Arctic Armageddon.
- We propose a generalization of classical Goody model taking into account methane emission effects. For the case of the methane emission by Siberian lakes we describe an asymptotical approach that allows us to obtain an expression for the methane flux via the temperature and fluid fields. By a variational approach we compute an expression for a shift by bifurcation parameter induced by the methane emission. We show that there is possible a tipping point in atmosphere dynamics resulting from methane emission.

The Research Council of Norway