A 4-dimensional spectral description of ocean variability for uncertainty estimation

Massachusetts Institute of Technology

Cimarron J. L. Wortham MIT-WHOI Joint Program; contact: worthamc@mit.edu

Carl I. Wunsch Massachusetts Institute of Technology

Introduction

Effective use of observations in climate science requires an estimate of the background variability and correlation scales. Unfortunately, no single set of observations provides enough information to estimate the space- and time-covariances necessary for uncertainty calculations.

We present an empirical model of the 4-dimensional frequency-wavenumber spectrum of ocean variability, constructed to fit a range of observations including SSH, current meters, and hydrography, supplemented with numerical model results.

The covariance function is the Fourier transform of the power spectrum. Thus, the power spectrum can be used to estimate the uncertainty in the mean or trend in a set of measurements a priori.

Model spectrum

We present a model for the 3D spectrum of the surface geostrophic streamfunction:

$$\Phi(k,l,\omega,z) = I(\phi,\lambda)F(z)\frac{\exp\left(-\frac{l^2+k^2}{\delta^2}\right)}{\alpha(ck+w)^2+\beta\left((ck)^2+w^2\right)^2+\epsilon}$$

where $I(\phi\,\lambda)$ is chosen to match the observed surface EKE and c is the dominant phase speed.

The spectrum for a variable q (velocity, temperature, pressure, etc.) can be calculated based on this model spectrum:

$$P_q(k,l,\omega,z) = |\tilde{q}|^2\Phi(k,l,\omega,z)$$

using the characteristic functions

$$\tilde{p} = \rho_0 f F(z) \qquad (\tilde{u},\tilde{v}) = i2\pi(-l,k)F(z)$$

$$\tilde{w} = i2\pi\omega f G(z) \qquad \tilde{\theta} = f\frac{\partial\theta_0}{\partial z}G(z)$$

where $F(z)$ and $G(z)$~dF/dz are suitable vertical structure functions (*Zang and Wunsch*, 2001).

Vertical structure

Current meter studies have shown that currents are dominated by the barotropic and low baroclinic modes. In the ECCO2 model, the BT and first BC modes are strongly coupled. We propose a coupled BT+BC mode vertical structure.

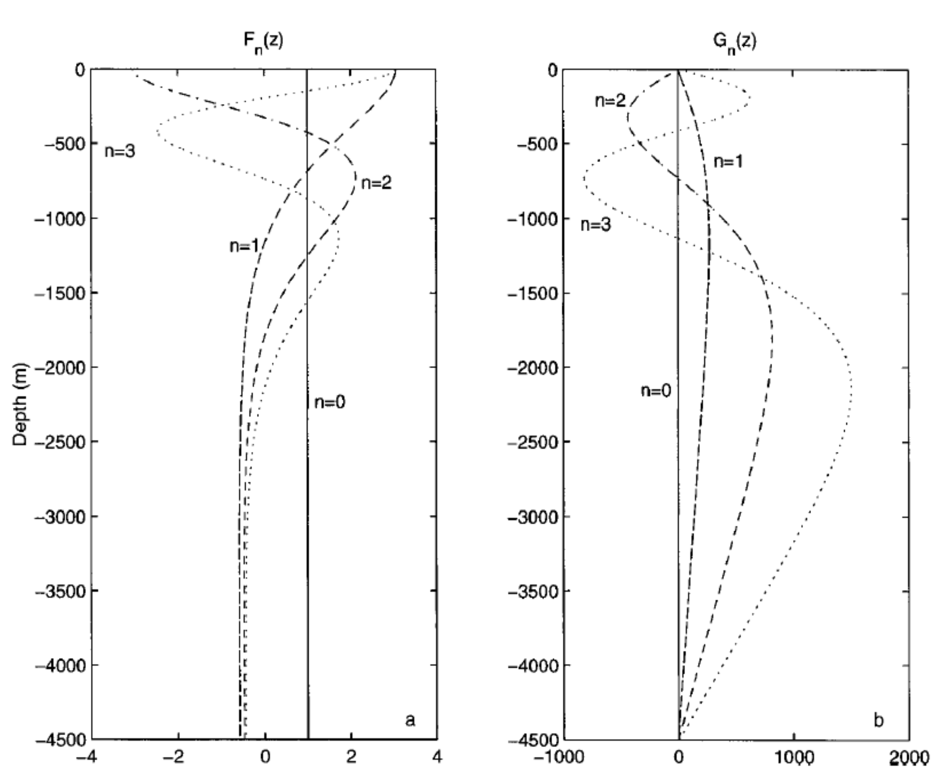
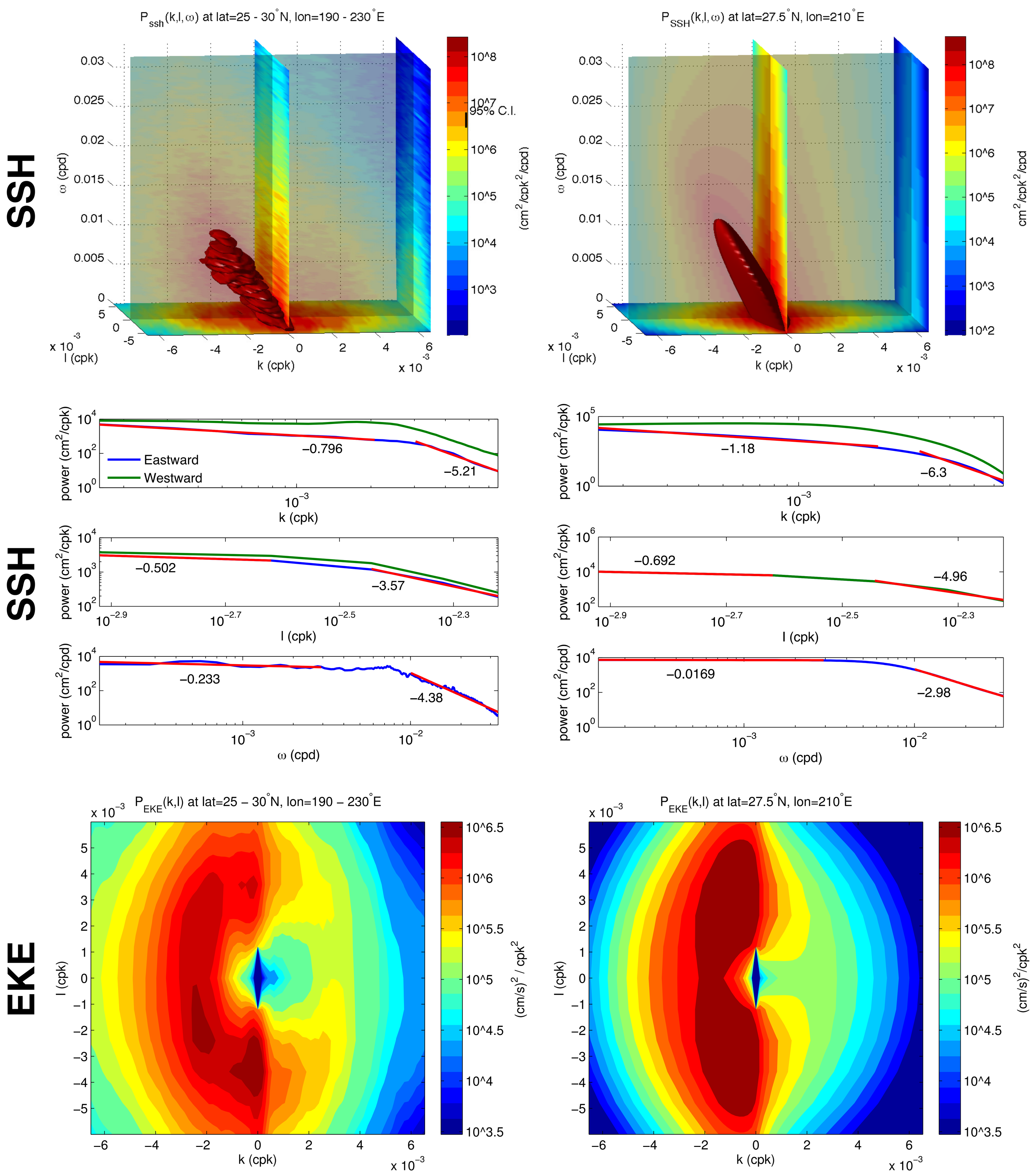


FIG. A1. The vertical eigenfunctions $F_n(z)$ and $G_n(z)$ for modes $0 \le n \le 3$ from the analytical $N(z)$ profile.

Comparison with observations

AVISO Model



Uncertainty estimate

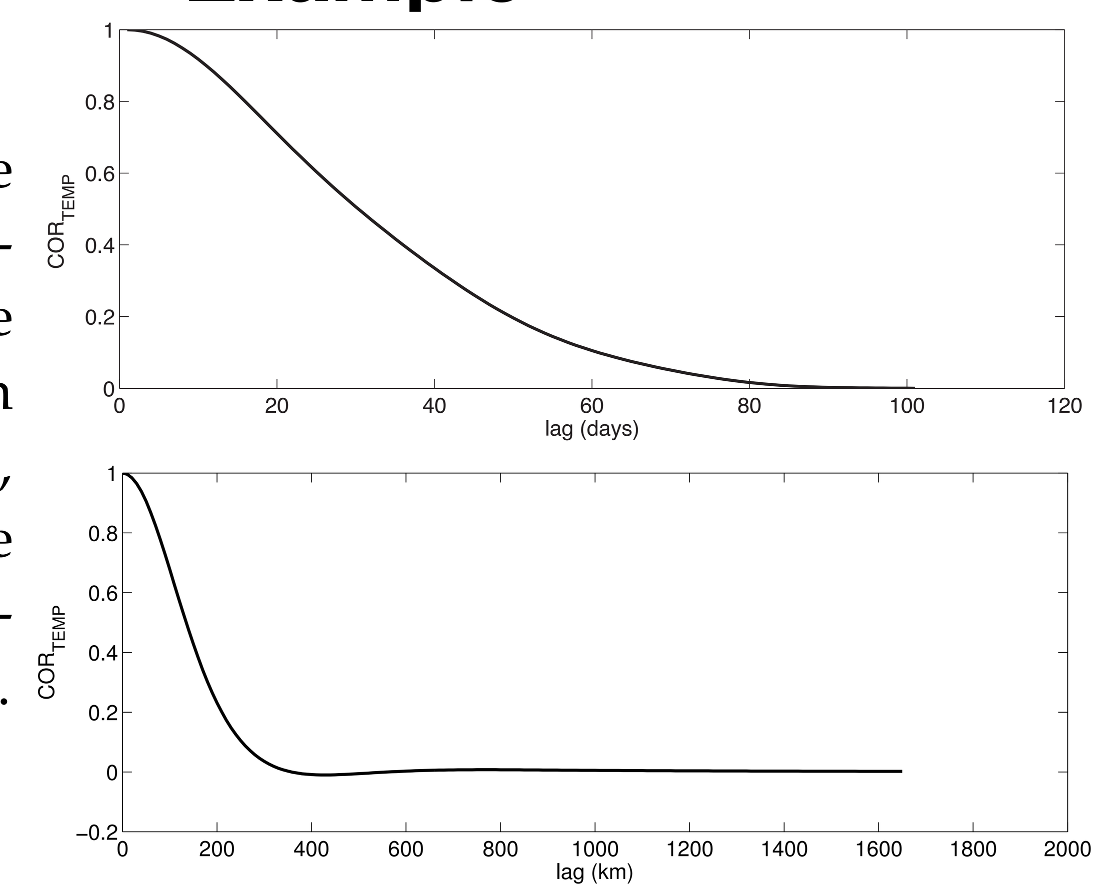
Suppose you have a set of measurements y_i of, for example, temperature in the deep ocean. The time-mean temperature is obscured by the eddy field with spatial and temporal correlation that must be accounted for in estimating the uncertainty. The expected variance is

$$\mathbf{P} = [\mathbf{E}^T\mathbf{R}_{nn}^{-1}\mathbf{E}]^{-1}$$

where $\mathbf{E}$ represents the model describing the data (time mean, or linear trend, etc.). The matrix $\mathbf{R}_{nn}$ has diagonal elements $\sigma^2 corr(\mathbf{r})$ for separation distance $\mathbf{r}$. Conveniently, the correlation is just the Fourier transform of the power spectrum.

Example

Space and time correlation functions from the model spectrum of temperature, evaluated in the central North Pacific near 2000 m.



For a hypothetical set of 10 stations spaced 150 km apart repeated for 5 years, the uncertainty in mean temperature would be ~0.015°C, about 50% larger than a naive estimate.

Conclusions

- Not only the *variance* but also the *space-* and *time-covariance functions* are important for estimating uncertainty in observed trends. However, no single experiment produces enough observations to estimate the covariance functions.
- We propose an empirical model of the frequency-wavenumber spectrum which can be used to estimate the covariance function and uncertainty. The spectrum of ocean current variability provides a convenient way to combine data from disparate sources, including satellites, moorings, and numerical models.
- Assuming geostrophy, the frequency-wavenumber spectrum of velocity, pressure, temperature, etc., can be derived from the streamfunction spectrum, and the uncertainty in a mean value or trend in observations can be estimated from this spectrum.
- Better understanding of the vertical structure is a major challenge in making use of extensive surface observations.

References

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