A FORMAL APPROACH FOR SMOOTHING ON VARIABLE-RESOLUTION GRID
Part III: Filtering the vectors on the polar grid

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It is common practice to filter the fields (or sometimes their tendencies) in order to remove high wavenumbers that otherwise would affect the accuracy of a climate model. Generally, these damping methods are applied to variables such as temperature, pressure and humidity, and if filtering is needed for momentum, it is often applied to the corresponding scalar quantities, such as streamfunction and velocity potential, or vorticity and divergence. In this study we proceed to the filtering of the wind vectors themselves. The convolution filter developed by Surcel and Laprise (2010) and adapted for scalar variables on the polar grid (Surcel and Laprise 2011) is now generalized for vector fields.

When the convolution is applied to vectors such as the horizontal wind, care has to be taken to use a representation of the vector components relative to the same reference system, chosen here to correspond to the application point. As the polar grid used in this paper is an intermediate step to the application of the filter on a spherical latitude-longitude stretched grid, the representation of the vector components is made by analogy with the spherical grid.

On a polar grid defined by (r,λ), with r the radius and λ the azimuth angle, the horizontal wind is defined related to the local coordinate system, as $\mathbf{V}_h = (u, v) = \left(r\dfrac{d\lambda}{dt}, -\dfrac{dr}{dt} \right)$, where (u, v) correspond to the "zonal" and "meridional" wind components (using the terminology on the sphere), with the sign convention that u is positive eastward and v is positive northward.

The filter is applied simultaneously for both wind components and the convolution is applied successively in radial and azimuthal directions. Following the meteorological tradition, the wind components are defined relative to a locally orthogonal reference system whose base vectors change with location (only with longitude in fact for the polar grid). Therefore the application of the filter operator requires representing the wind components contributing to the convolution at a point in the same coordinate system as that point. For each point $P_0(r_i,\lambda_j)$ where the convolution filter is applied for (u,v), we transform all wind vectors in the neighbouring points $P(r_k,\lambda_l)$ contributing to the convolution, i.e. those for which their distance is within the chosen truncation distance for the convolution. The wind components at point $P(r_k,\lambda_l)$ are expressed in the coordinate system relative to the application point P_0 as follows:

$$\begin{pmatrix} u \\ v \end{pmatrix}_{P_0} (k,l) = \begin{bmatrix} \cos(\lambda_l - \lambda_j) & -\sin(\lambda_l - \lambda_j) \\ \sin(\lambda_l - \lambda_j) & \cos(\lambda_l - \lambda_j) \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix}_P (k,l).$$

We note that the conversion only involves the longitude angle, not the radial distance, thus no transformation is required for points aligned on the same meridian.

To verify the efficiency of the convolution filter we define test wind fields by constructing rotational and divergent motions using the Helmholtz theorem for two-dimensional vector field $\mathbf{V}_h$ as

$$\mathbf{V}_h = \mathbf{V}_R + \mathbf{V}_D = \mathbf{k} \otimes \nabla\psi + \nabla\chi$$

where ψ is the streamfunction and χ the velocity potential. We then employ scalar test functions, for use as streamfunction or velocity potential, and we develop analytically the corresponding zonal and meridional wind components in polar coordinates. We use a signal corresponding to either a pure rotational or divergent large-scale motion, and then add to it a small-scale noise that is also either rotational or divergent.

The filter's ability for application to vectors was tested first on a uniform polar grid and we checked the performance of the filter around the pole. To verify the performance of the convolution filter we represent a large-scale wind field, considered as analytical solution, a perturbed wind field created by adding a noise to the analytic wind field, and the filtered wind field that must be identical with the analytical solution if the filter works properly.

In Fig. 1 (right panel), a streamfunction represented by a double cosine with wavelengths of 20,000 km defines a purely rotational large-scale wind field. To this large-scale field a small-scale divergent wind noise with wavelength of 500 km is added (middle panel). The convolution filter uses a weighting function that keeps unchanged all signals with wavelengths larger than 3,000km and removes all signals with wavelengths smaller than 800km. The convolution is calculated for truncation distance of 1,100 km. The filtered field (right panel) shows that the large-scale signal is preserved and the noise removed. For this test the large-scale field was located specifically such as to have not zero winds at the pole. Numerically the pole is considered as (r_1, λ_j) with $j = 1, \ldots, m$ and the convolution filter is applied there as for all other grid-points. The tests revealed that the convolution filter works properly in the vicinity of the pole, and the large-scale fields are recovered without distortions near the pole.

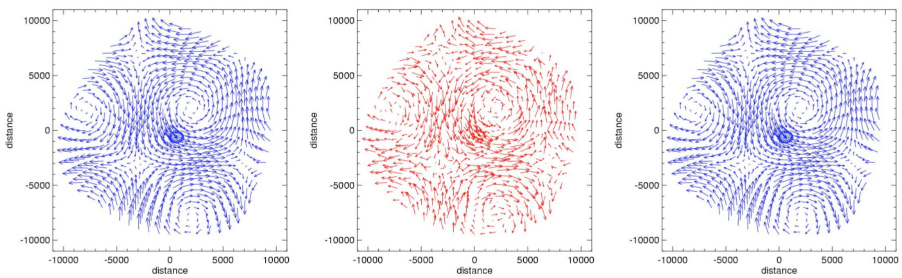

Figure 1: A large-scale rotational wind field (left panel) is perturbed by a small-scale divergent wind field (middle panel). The filtered field is represented in the right panel.

A similar experiment was repeated on a polar stretched grid. A test field wind composed from a large scale purely divergent wind developed using a velocity potential in form of cylindrical harmonic with radial wavenumber 1 and azimuthal wavenumber 2 (Fig. 2 left panel) was perturbed by a rotational noise developed using a streamfunction in form of double cosine with wavelength 400km (Fig. 2 middle panel). The filter uses a weighting function that keeps unchanged all signals with wavelengths larger than 3,000km and removes all noises with wavelengths smaller than 600km. These parameters correspond to a smoother spectral response than in the first example and thereafter necessitate a truncation distance of only 900 km to remove the noise. Because the filter is applied outside the uniform high-resolution region and to better display the effect of the filter in the stretching zones, we present only the test-function outside the high-resolution zone. In Fig.2 (right panel) we show the filtered function; visually we note that the convolution filter is able to remove the noise and after the application of the filter the large scale signal is recovered. No deformations were noted around the high-resolution domain and the filter works properly in the stretching zones as well as around the pole.

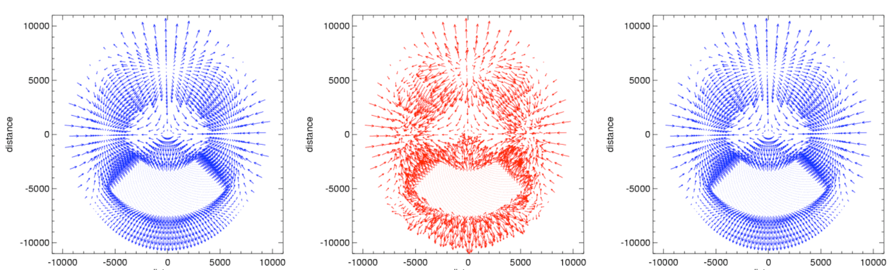

Figure 2: A large-scale divergent wind field (left panel) is perturbed by a small-scale rotational wind field (middle panel). The convolution filter is applied outside the uniform high resolution region and the filtered field is represented in the right panel.

The present study shows that with appropriate definition constraints, and representing the winds components for all points contributing to the convolution relative to the same reference system as the application point, we were able to remove small-scale noise superimposed on large-scale signals.

Reference

Surcel, D., and R. Laprise, 2010: A General Filter for Stretched-Grid Models: Application in Cartesian Geometry. *Mon. Wea. Rev.* doi: 10.1175/2010MWR3531.1.

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