Semi-Lagrangian mass-conservative advection scheme on the sphere on the reduced grid

Vladimir Shashkin
Moscow Institute of Physics and Technology 9 Institutski per. 141700 Dolgoprudniy Russia
E-mail: vvshashkin@yandex.ru

Mikhail Tolstykh
Hydrometcentre of Russia
and
Institute of Numerical Mathematics RAS, 8 Gubkina st. 119333 Moscow Russia
E-mail: tolstykh@inm.ras.ru

1. Introduction

Current climate models require efficient schemes for advection of humidity, liquid and solid water variables, and a number of chemical constituents. Semi-Lagrangian (SL) transport schemes have proved to be an efficient numerical method for treating the advection process. However, a serious disadvantage of most SL schemes is that they do not formally conserve integral invariants, in particular, total mass, which has been found to drift significantly if no corrections are applied during long integrations of the SL climate model. Finite-volume based conservative SL transport schemes have gained prominence during the recent years, but only a few of them are available for spherical geometry (global) application. The most successfull developments in the spherical geometry are presented in papers [1], [2], [3]. Current work generalizes the local conservative cascade semi-Lagrangian global advection scheme introduced in [2] to the case of latitude-longitude reduced grid (see sect. 3 for details).

2. The finite-volume based conservative SL advection schemes

Unlike traditional SL schemes, finite-volume based conservative SL schemes use grid cells rather than grid points and cell-averaged values of density rather than its grid-point values. Backward trajectories with arrival points at regular (Eulerian) grid cell corners are constructed to define departure (Lagrangian) cell. The cell averaged density value in the j-th grid cell on the time level $n+1$ is defined as follows: $\bar{\rho}_j^{n+1} = \frac{M^n(A_j^*)}{mes(A_j)}$. Here $M^n(A_j^*)$ is the mass enclosed on the time level n in A_j^* - the departure (Lagrangian) cell corresponding to the arrival cell coinciding with j-th grid cell and $mes(A_j)$ is the square of j-th grid cell. Thus, the keypoint of finite-volume based conservative SL schemes is to calculate mass enclosed in each Lagrangian cell.

3. Conservative cascade scheme on the reduced grid

Latitude-longitude reduced grid is quite similar to the regular latitude-longitude grid, the difference is that the number of points in latitude rows decreases toward the poles. So, the latitude resolution is uniform and the longitude resolution decreases towards the poles remaining uniform inside each latitude row.

In order to apply the conservative cascade scheme on the reduced grid firstly the density is redistributed from the reduced grid to the regular (full) latitude-longitude grid in conservative manner. The latitude rows of the full grid coincide with those of reduced grid, and the number of points in latitude rows of the full grid is equal to the number of grid-points in equatorial latitude row of the reduced grid. This means that series of conservative 1D remappings (the remapping technique is described in [4]) inside latitude rows should be done to obtain cell-averaged values of density on the full grid. These values are then used to estimate the masses enclosed in the Lagrangian cells via conservative cascade scheme for the regular grid. The only dissimilarity is that the number of Lagrangian cells to be treated in the 2nd remapping differs from row to row. The meridional Courant number should be less than .5 in polar regions for the scheme to works correctly (see [2] for details). However, modification obviating this restriction is going ahead.

4. Numerical Experiments

a. Solid body rotation. The scheme was tested on the solid body rotation problem (test #1 from [6]). The initial distribution was the cosine-bell. Numerical experiments were carried out on the regular grid with resolution of 1.5^0 and on reduced grid from [5] of the same maximum resolution, which have 10% less points than regular grid. The angle of solid rotation $\alpha = \frac{\pi}{2}$, the center of the bell was chosen such that distribution goes along the pole to pole direction. Full revolution required 480 steps (meridional Courant number $C_\theta = 0.5$). All other parameters were set up as in [2]. The exact solution after one revolution is just the initial distribution. Exact backward trajectories were used. The results are presented in Table 1 and on Figure 1.

b. Smooth deformational flow. The scheme was tested on the smooth deformational flow problem (see [2] for full description). All parameters except resolution and time step were chosen as in [2]. The grids were the same as in the solid body rotation test. The test problem was integrated for 3 time units (nondimensional) with 120 time steps. The results are presented in Table 2.

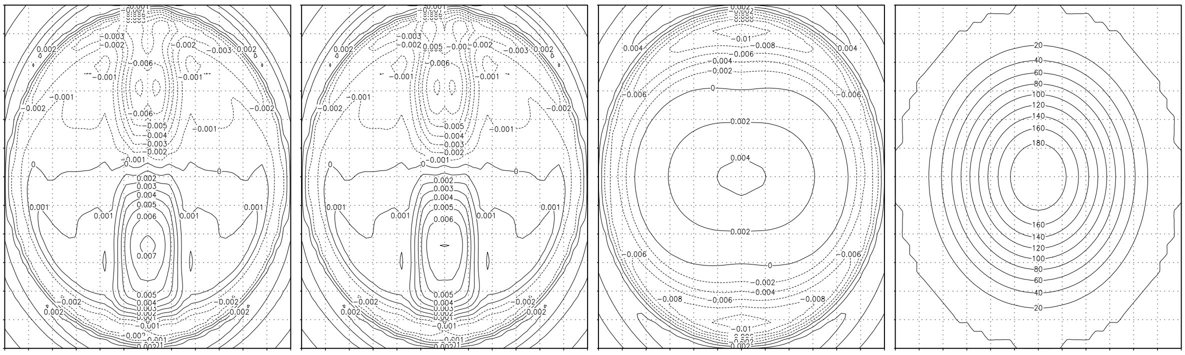

Figure 1: Error fields in solid rotation test after one revolution for conservative cascade scheme on full (a) and reduced grids (b), error field for non-conservative SL scheme (c), initial distribution (d)

Scheme	l_1	l_2	l_∞	max
Conservative cascade scheme on reduced grid	1.97E-02	1.34E-02	1,34-02	-0.4E-02
Conservative cascade scheme on regular grid	1.97E-02	1.34E-02	1,34-02	-0.4E-02
Non-conservative SL scheme	6.03E-02	3.78E-02	3,1E-02	-1.8E-02

Table 1: Error measures for solid body rotation test

Scheme	l_1	l_2	l_∞	max
Conservative cascade scheme on reduced grid	2.3E-04	6.3E-04	8.0-03	-1.45E-07
Conservative cascade scheme on regular grid	2.3E-04	6.3E-04	6.4-03	-1.45E-07

Table 2: Error measures for smooth deformational flow test

The cascade conservative SL scheme exactly preserves the mass and is also more accurate than non-conservative SL scheme.

Currently the presented scheme is being implemented in the shallow-water model on the sphere [7].

This work was supported with the Russian RFBR grant 10-05-01066.

References

1. Nair R.D., Machenhauer B. The mass-conservstive cell-integrated semi-Lagrangian advection scheme on the sphere // Mon. Wea. Rev. 2002 V. 130 P. 649-667.

2. Nair R.D. [et al.]. Efficient conservative global transport scheme for climate and atmospheric chemistry models // Mon. Wea. Rev. 2002 V. 130 P. 2059-2073.

3. Zerroukat M. [et al.]. SLICE-S: A semi-Lagrangian inherently conserving and efficient scheme for transport problems on the sphere // Quart. J. Roy. Meteor. Soc. 2004 V. 130, N 602 P. 2649-2664.

4. Colella P., Woodward P.R. The picewise parabolic method (PPM) for gas-dynamical simulations // J. Comput. Phys. 1984 V. 54 P. 174-201.

5. Fadeev R.Yu., Construction of a reduced latitude-longitude grid for a global numerical weather prediction problem. // Russian Meteorology and Hydrology, 2006, N 9, p. 5-20.

6. Williamson, D. L., Drake, J. Hack, R. Jacob, and P. N. Swartztrauber, 1992: A standard test for numerical approximation to the shallow water equations in spherical geometry. // J. Comput. Phys., 102, 211 224.

7. Tolstykh M. Vorticity-divergence semi-Lagrangian shallow-water model on the sphere based on compact finite differences // J. Comput. Phys. 2002. V. 179. P. 180-200.