An Economical Approach to Four-dimensional Variational Data Assimilation

Bin Wang ^{1*}, Juanjuan Liu¹, Bing Lu¹

1. LASG, Institute of Atmospheric Physics, Beijing, China

*. Email: wab@lasg.iap.ac.cn

Ensemble-based 4DVar (En4DVar) includes the advantages of both standard 4DVar and EnKF. Currently, there have been efforts in the development of En4DVar family (e.g., Hunt et al., 2004; Qiu and Chou 2006; Liu et al. 2008; Tian et al. 2008; Zhang et al. 2009; Wang et al. 2010). One of the representative approaches in this family is the Dimension-Reduced projection 4DVar (DRP-4DVar) proposed by Wang et al (2010). B matrix used in DRP-4DVar is not only implicitly evolved within the time window but also explicitly developed from window to window. The implicit flow-dependent feature is derived from the equivalence between standard 4DVar and this approach in theory, while the explicit flow-dependence is decided by the estimation of B matrix using a number of initial-condition-reliant historical forecast samples.

The implementation of DRP-4DVar mainly includes three steps. The first step is to establish projection matrices using historical forecasts. In operational application, the historical forecasts are generally composed of two parts. One is the 72-hour forecasts initiated at the time 24 and 48 hours prior to the analysis time, and the other is an analog forecast selected from the past years. They are used to generate initial perturbations (IPs) by simply subtracting the background from them, and to produce simulated observations (SOs) and simulated observation increments (SOIs) using the observation operator and the basic state of SO. A quality control is included to select those weighted (or normalized) SOIs (WSOIs) derived from the 72-hour forecasts, which are significantly correlated with the weighted observation increment (WOI). The analog forecast is used to produce a higher-quality IP sample, whose corresponding WSOI is more significantly correlated with the WOI. The second step is to minimize the cost function in the sample space (or the coefficient space). After IP and its corresponding WSOI are projected onto the sample space using the projection matrices established at the first step, the original minimization of standard 4DVar in the control variable space whose dimension is about 10^{6} - 10^{8} can be easily completed in a 10^{1} - 10^{2} -dimension sample space in this approach. The optimal solution to the minimization can be explicitly obtained, and does not require iterative procedure with adjoint technique. At this step, the construction of B matrix is very important. We use the IP samples generated at the first step to estimate B matrix to ensure its global flow-dependent feature. After it is projected from the control variable space onto the sample space, it becomes a low-dimensional and constant matrix. The third step is localization. Because the ensemble is composed of far fewer members, many spurious correlations between observation locations and model grids may appear. Thus, a localization technique as the one in EnKF is included to ameliorate the spurious long-range correlations. The details of DRP-4DVar can be found in Wang et al. (2010).

In the past two years, DRP-4DVar has been evaluated with assimilation experiments. Fig. 1 shows the results of assimilation-forecast experiments of typhoon. They are conducted with the Advanced Regional Eta Model (AREM), a limited area, hydrostatic primitive equation model, with 32 eta layers for all grid meshed with horizontal resolution of 37 km. Fengshen (2008 06TC) is chosen to complete two group experiments. One is control experiment (CTL) initialized at the analysis time (0000 UTC 24 June 2008) with $1^{\circ}\times1^{\circ}$ National Centers for Environmental Prediction (NCEP) Final (FNL) Global Tropospheric Analyses. The other is DRP-BDA (DRP-4DVar-based bogus data assimilation) experiment that assimilates bogus surface pressures calculated according to Fujita formula into the first guess through DRP-4DVar. The assimilation window is 20 minutes, in which the same bogus surface pressures are used as observations every 2 minutes. Figure 1 gives the forecast of the 36-h track and intensity from CTRL and DRP as well as observations from 0000 UTC 24 to 0012 UTC 25 June 2008. CTL presents a track error of 39 km at the start of the window and of 452 km after

36-hour forecast (Fig. 1b). In contrast, DRP-BDA greatly improved the prediction of movement. The track error is only 10 km at initial time and 32 km after 36 hours (Fig. 1b). Furthermore, it produces a much better forecast landing time and a central sea level pressure closer to the observed than CTL (Fig. 1c).



Fig. 1 Forecasts of Typhoon Fengshen for the entire 36-hour forecast period from 0000 UTC 24 June to 0012 UTC 25 June 2008 at 3-hr intervals: (a) track (km), (b) track error (km), and (c) central sea level pressure (hPa).

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