A PDF-based method for computing the sedimentation effects within a full prognostic microphysical scheme

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Motivation: When performing cloud and precipitation microphysical computations in a system where all hydrometeors are prognostic, including thus the local quantities of precipitating drops or flakes, the problem arises on how to account for the differences in vertical velocity between those falling species and the total mass of the atmospheric particle. The difficulty is not so much the differential downwards vertical advection of drops/flakes in itself, but the need to treat in a physically-sound way its interactions 'along the trajectory' with other significant microphysical processes (auto-conversion, collection, evaporation-melting, to name only the main ones). To our knowledge, all solutions proposed up to now for solving this complex problem rely on an algorithmic interlacing between dynamical-like advection steps and locally treated microphysics equations. The advection steps may be of the Eulerian type (in which case a time-iterative procedure is needed to avoid violating the CFL condition) or of Lagrangian type (in which case the iterative part of the computation consists in double vertical loops twinning the 'passing by' drops/flakes with the local properties determining auto-conversion, collection, …). In any case these methods are rather expensive. The purpose of this note is to propose a radically differing approach to the problem, which shall eliminate the need to do iterative computations while keeping a high level of accuracy, thanks to the use of a probability distribution function (PDF) framework.

Method:
One may look at the above problem from the angle that advective methods require a unique (mean) fall speed ($\boldsymbol{w_0}$) for any type of precipitating hydrometeor. Let one however assume that this is replaced by a spectrum of fall speeds going from zero to infinity, $\boldsymbol{w_0}$ now being the mass weighted average $\boldsymbol{w}$ for this spectrum. For a given time interval (the model's time step in any concrete application) this spectrum may easily be converted into one of 'reachable distances along the vertical'. At that stage one is apparently in an even worse situation than previously, with an infinity of trajectories to handle. But one may go one step further and convert mathematically (with some hypotheses, see below) the 'distance-related spectrum' into a probability of leaving a given part of the atmospheric slab (down to the bottom of the considered model layer in any concrete application).

Starting from a basic PDF $\boldsymbol{P_0}$, for the probability of one steadily falling hydrometeor to cross the considered layer within one time step, one needs at the bottom of this layer three 'effective' PDFs for falling species:

- Already present in the layer at the beginning of the time step [PDF $\boldsymbol{P_1}$];
- Coming from the layer above [PDF $\boldsymbol{P_2}$];
- Locally produced during the time-step by auto-conversion, collection and melting (or destroyed by evaporation and melting) [PDF $\boldsymbol{P_3}$].

In principle, the various steps leading to these three functions are only meaningful if the resulting implicit local PDFs at the bottom of one given layer are homothetic so that they may be recombined into a single 'entering PDF' at the top of the next layer below. This property can only be warranted if the $\boldsymbol{P_0}$ function is of the decreasing exponential type, e.g. $P_0(Z) = \exp(-Z) = \exp(-\delta z / (w_0\, \delta t))$, with $\boldsymbol{Z}$ a kind of inverse mean Courant number for the sedimentation process.

It is however our practical experience that starting directly from $\boldsymbol{P_1}$, $\boldsymbol{P_2}$ and $\boldsymbol{P_3}$ functions (without homothetic shape assumption) can also provide good results, provided their derivation obeys some logic, for instance that of mimicking a Lagrangian sedimentation scheme.

For a given $\boldsymbol{P_0}$, while the choice of $\boldsymbol{P_1}$ is in principle straightforward (it should be the average of $\boldsymbol{P_0}$ for a vertical range varying from zero to the full layer's depth), the choices for $\boldsymbol{P_2}$ and $\boldsymbol{P_3}$ must take into account the time dimension within the time-step. Hence their choice may ultimately depend on the way one wishes the various microphysical processes to interact (sequentially or in parallel; with a stationary target for an implicit discretisation or not; etc.). It is out of the scope of this note to deal with this issue, but some care is of course needed to make the choices of $\boldsymbol{P_2}$ and $\boldsymbol{P_3}$ compatible with the other underlying hypotheses of the microphysical scheme.

Remarks:

- The mathematical similarity (in the choice of $\boldsymbol{P_0}$) with the Marshall-Palmer (M-P) law indicates a correct physical behaviour (there are more small drops likely to stay in one layer than big ones that shall anyhow leave it) but it should not be misinterpreted as an identity. In the M-P case it is the slope of the distribution that varies with rain intensity, here it is the total number of drops, in first approximation.
- In principle, the mean fall speed should stay constant when going from one layer to the next one below. But the heuristic approach to make $\boldsymbol{w_0}$ depend on the precipitation intensity (alike in nature) works fine, contrary to the advective case where it would be a likely source of numerical noise. But this does not identify our basic PDF with the M-P law; it just makes it a little bit closer in practice. Note however that both functions need the existence of a 'deus ex machina' process within the complexity of the full microphysical computations in order to see their shapes preserved.
- The concrete application of the method advocated here is strongly related to the tendency ⇔ flux divergence framework of the Green-Ostrogradsky theorem. For instance it allows a direct insertion of any microphysical 'local' computation within the flux-conservative equation framework of Catry et al. (2007).
- When using infinite fall speeds, the above scheme automatically degenerates into a 'classical' diagnostic scheme for precipitating species, whatever may be the handling of non-sedimentation microphysical processes.

Results: After successful pre-operational tests, the scheme presented here was introduced in the ALADIN-CE operational NWP application at CHMI (Prague) on 30/1/07. The set-up uses the decreasing exponential choice for $\boldsymbol{P_0}$, something that may take the name of 'statistical sedimentation scheme', and relies on a sequential split-implicit treatment of microphysical processes. The same version of the scheme is under trial in the services of several ALADIN partners, with promising results. At Météo-France, a version mimicking the operational Lagrangian sedimentation treatment and using an interactive microphysical algorithm is undergoing parallel tests, with a view to obtain operationally unchanged meteorological results, but at a significantly reduced computing cost.

Catry, B., J.-F. Geleyn, M. Tudor, P. Bénard and A. Trojakova, 2007. *Tellus A*, **59**, pp. 71-79.