# Development of a hybrid terrain-following vertical coordinate for JMA Non-hydrostatic Model

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## 1 Introduction

The Japan Meteorological Agency (JMA) has been operating the JMA Non-hydrostatic Model (NHM) with a horizontal resolution of 5km since March 2006. The governing basic equations of NHM are the fully compressible equations and written in flux form. A time splitting method is used and the terms responsible for the sound and gravity waves are treated implicitly in the vertical direction and explicitly in the horizontal direction. The governing equations are transformed into a spherical curvilinear orthogonal coordinate and the vertical terrain-following coordinate (Gal-Chen and Somerville 1975).

The terrain-following transformation is linear and written in as follows:

$$z = \zeta + z_s \left(1 - \frac{\zeta}{z_T}\right),$$

where $\zeta$ is the transformed vertical coordinate, $z_s$ is the surface height, and $z_T$ is the model-top height.

This transformation has some advantages. A treatment of the lower boundary condition of this transformation is quite simple. Since $\zeta$ is linearly related to $z$, one-dimensional physical process such as atmospheric radiation and cumulus convection scheme will be implemented easily.

Since the coefficient of $z_s$ of this transformation is not zero except at the model top, the constant-$\zeta$ levels are not flat even in the upper atmosphere and this non-orthogonal property would be a disadvantage. The horizontal pressure gradient term and the horizontal advection term are split into the horizontal and vertical derivative. Since the vertical grid spacing of NWP models is generally large in the upper atmosphere, the error of the vertical difference would cause errors of the pressure gradient force and the advection.

To reduce above disadvantage, a new hybrid vertical terrain-following coordinate which is based on the same approach as the $\eta$ coordinate (Simmons and Burridge 1981) is implemented. It is transformed using following equation

$$z = \zeta + z_s f(\zeta).$$

The new transformation has the same advantages. As $f(\zeta)$ is getting close to zero, the constant-$\zeta$ levels become flat. Therefore the selection of the appropriate function can reduce the disadvantage. The function $f(\zeta)$ should satisfy $f(0) = 1$ and $f(z_T) = 0$ because of the boundary condition. The function $f(\zeta)$ must be second differentiable because the Christoffel's symbols require it and $f'(\zeta) > -1/z_s$ to make the transformation monotone.

## 2 Momentum Equations

The original momentum equations of NHM are as follows (Saito et al. 2006):

$$\frac{\partial U}{\partial t} + \frac{m_1}{m_2}\left(\frac{\partial P}{\partial x} + \frac{\partial G^{\frac{1}{2}} G^{13} P}{G^{\frac{1}{2}} \partial \zeta}\right) = -ADV_1 + R_1,$$

$$\frac{\partial V}{\partial t} + \frac{m_2}{m_1}\left(\frac{\partial P}{\partial y} + \frac{\partial G^{\frac{1}{2}} G^{23} P}{G^{\frac{1}{2}} \partial \zeta}\right) = -ADV_2 + R_2,$$

$$\frac{\partial W}{\partial t} + \frac{1}{m_3 G^{\frac{1}{2}}}\frac{\partial P}{\partial \zeta} = -ADV_3 + R_3.$$

Here $U, V$ and $W$ represent the momentum components and $P$ the pressure perturbation. $ADV$ and $R$ are the advection terms and residual terms including the buoyancy term, respectively. Subscripts $1, 2$ and $3$ correspond to the $x, y$ and $\zeta$ components, respectively. Symbols $m_1$ and $m_2$ are the map factors while $m_3$ is not a map factor but a constant introduced for definition of momentum. $G^{\frac{1}{2}}, G^{13}$ and $G^{23}$ are metric tensors and given by

$$G^{\frac{1}{2}} = \frac{\partial z}{\partial \zeta}, \quad G^{\frac{1}{2}} G^{13} = -\frac{\partial z}{\partial x}, \quad G^{\frac{1}{2}} G^{23} = -\frac{\partial z}{\partial y}.$$

To introduce the hybrid vertical coordinate, above equations are rewritten by using tensor analysis as follows:

$$\frac{\partial U}{\partial t} + \frac{m_1}{m_2}\left(\frac{\partial P}{\partial x} + \frac{\partial G^{13} P}{\partial \zeta}\right) = -ADV_1 + R_1,$$

$$\frac{\partial V}{\partial t} + \frac{m_2}{m_1}\left(\frac{\partial P}{\partial y} + \frac{\partial G^{23} P}{\partial \zeta}\right) = -ADV_2 + R_2,$$

$$\frac{\partial W}{\partial t} + \frac{1}{m_3}\frac{\partial}{\partial \zeta}\left(\frac{P}{G^{\frac{1}{2}}}\right) = -ADV_3 + R_3.$$

Here, $G^{\frac{1}{2}}, G^{13}$ and $G^{23}$ are written as follows:

$$G^{\frac{1}{2}} = 1 + z_s f'(\zeta),$$

$$G^{\frac{1}{2}} G^{13} = -f(\zeta)\frac{\partial z_s}{\partial x}, \quad G^{\frac{1}{2}} G^{23} = -f(\zeta)\frac{\partial z_s}{\partial y}.$$

Only the pressure gradient terms are modified by the introduction of the hybrid coordinate. The above equations correspond to the original equations if $G^{\frac{1}{2}}$ is independent of $\zeta$. This means that the original equations are available for the original coordinate transformation (Gal-Chen linear transformation). Though the pressure gradient terms are modified, the computational cost of the hybrid coordinate is almost the same as that of the original coordinate.

## 3 Experiment results and conclusions

Idealised advection experiments with the original and hybrid coordinates are carried out to evaluate the computational error. The number of grid points is $301 \times 7 \times 50$ with a horizontal resolution of 1 km. A bell-shaped mountain with a height of 3000 m and a x-direction width of 50 km is placed at the centre of the domain. Initial potential temperature field and wind field are horizontally uniform and $\partial\theta/\partial z = 3$ K/km, $u = v = 0$ m/s ($z < 10000$ m) and $u = 2.5$ m/s and $v = 0$ m/s ($z > 12000$ m). A moisture mass with a width of 50 km and a thickness of 6000 m is placed 108 km west of the centre of the domain, at an altitude of 16000 m. This means that it will pass just over the mountain at the forecast time of 12 hours. The time step is 20 s and the time integration is carried out up to 24 hours. A fourth-order advection scheme with a flux correction scheme is used.

The following function is selected,

$$f(\zeta) = \frac{c\left\{1 - \left(\dfrac{\zeta}{z_T}\right)^n\right\}}{c + \left(\dfrac{\zeta}{z_T}\right)^n}, \quad c = \frac{\left(\dfrac{z_l + z_h}{2z_T}\right)^n}{1 - 2\left(\dfrac{z_l + z_h}{2z_T}\right)^n},$$

where $z_T = 21600, z_l = 1000, z_h = 11000$ m and $n = 3$. The coefficient of $z_s$ at the centre of the domain is shown in Fig. 1. The coefficient by the hybrid coordinate is almost zero at $z = 16000$ m while that by the original coordinate is about 0.3.

The result of the experiment with the original coordinate is shown in Fig. 2 and that with the hybrid coordinate is shown in Fig. 3. The moisture masses at $t = 0, 12$ and 24 h are drawn from left to right, respectively. The shape of the moisture mass in the hybrid coordinate is well-preserved while that in the original coordinate is remarkably deformed.

The hybrid vertical coordinate is implemented into NHM without the increase of the computational cost. This coordinate can be also used as the Gal-Chen vertical coordinate if $f(\zeta) = 1 - \zeta/z_T$. The hybrid coordinate and the new transformation function shown in above will be in operation in May 2007.

## 4 Acknowledgements

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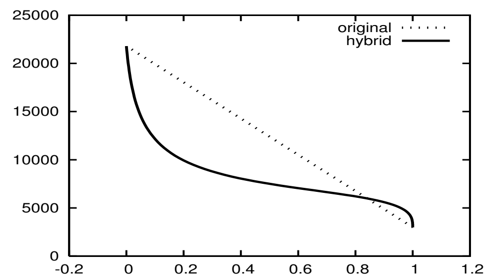


Figure 1: The coefficient of $z_s$ by the hybrid transformation function (solid line) and the original transformation function(dotted line) The x-axis is the coefficient and the y-axis is the height.

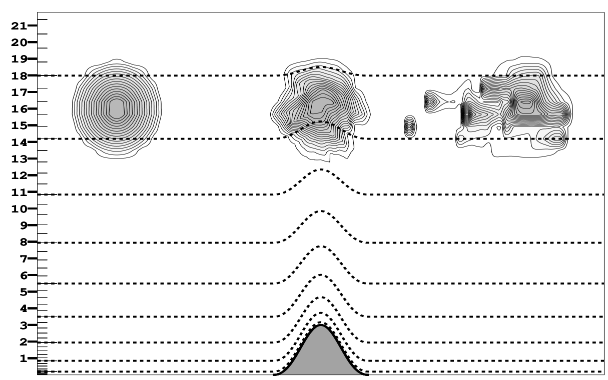

Figure 2: The result of the advection test with the original vertical coordinate. The moisture masses at $t = 0, 12$ and 24 h are drawn from left to right, respectively.

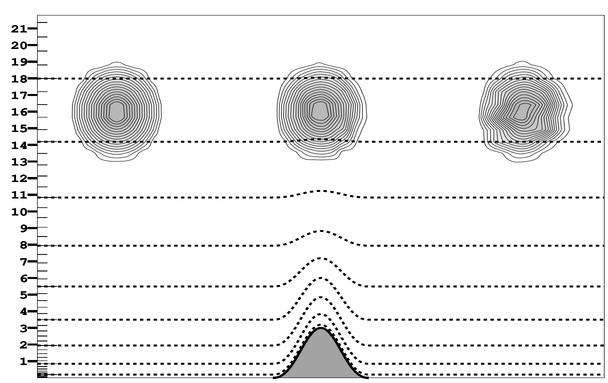

Figure 3: Same as in Figure 2 but with the hybrid vertical coordinate