Velocity reconstruction by radial basis functions in a triangular staggered C grid

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1 The ICON project

The ICON project is a joint development effort of MPI-M and DWD to achieve a unified climate and NWP model using geodesic grids with local grid refinement. The model under development in the ICON project will use the fully elastic, nonhydrostatic Navier-Stokes equations, which provide a framework that is sufficiently general for meteorological applications on most scales relevant to numerical weather prediction and climate simulation.

2 Velocity reconstruction

The proposed horizontal discretization uses the triangular staggered C grid approach. A full description of the horizontal discretization can be found in [2] and [3].
Vector radial basis function (RBF) interpolation is used to reconstruct a uniquely defined velocity field $\vec{v}$ from the velocity components v_i normal to the cell sides.
The interpolation function $\vec{s}$ for an arbitrary point $\vec{x}$ on the sphere is a linear combination of the unit vectors $\vec{n}_i$, that are normal to cell edges, multiplied by the RBF kernels Φ

$$\vec{s}(\vec{x}) = \sum_{j=1}^{N} c_j \cdot \Phi(\|\vec{x} - \vec{x}_j\|) \cdot \vec{n}_j \qquad (1)$$

using N data points $\vec{x}_j$ that satisfy the interpolation constraints

$$v_i = \vec{s}(\vec{x}_i) \cdot \vec{n}_i, \quad \forall i = 1, ..., N \qquad (2)$$

Hence the problem can be written as

$$v_i = \sum_{j=1}^{N} c_j \cdot \Phi(\|\vec{x}_i - \vec{x}_j\|) \cdot \vec{n}_j \cdot \vec{n}_i \quad \forall i = 1, ..., N \qquad (3)$$

and reduced to solve the linear system $\mathbf{A} \cdot \vec{c} = \vec{d}$ with

$$a_{ij} = \Phi(\|\vec{x}_i - \vec{x}_j\|) \cdot \vec{n}_j \cdot \vec{n}_i$$
$$\vec{c} = \left(c_1, \cdots, c_N\right)^T$$
$$\vec{d} = \left(v_1, \cdots, v_N\right)^T$$

The matrix $\mathbf{A}$ is symmetric positive definit and the system can easily be solved by

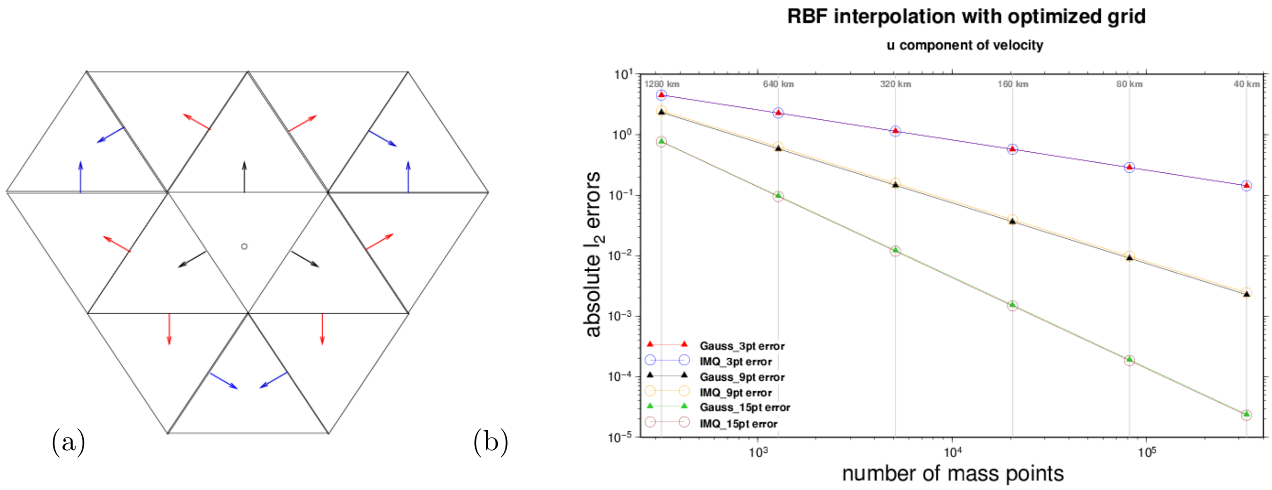

Figure 1: (a) 3 (black), 9 (red) and 15 (blue) point stencil (b) Convergence of zonal velocity component with different kernels and stencils

Cholesky decomposition.
In a triangular C grid setting the 3, 9 or 15 nearest edge centers are the natural choice for the N data points (see figure 1(a)).
Two kernels are investigated, Gaussian (GAU) and inverse multiquadric (IMQ).

$$\text{GAU:} \qquad \Phi(r) = e^{-(\frac{r}{\epsilon})^2} \tag{4}$$

$$\text{IMQ:} \qquad \Phi(r) = (1 + (\frac{r}{\epsilon})^2)^{-\frac{1}{2}} \tag{5}$$

where $r = \|\vec{x} - \vec{x}_j\|$ and ϵ is a scaling factor set to 0.9 (GAU) resp. 1 (IMQ) in the following example. More details of the mathematical background of the vector reconstruction by RBF can be found in [1].
Figure 1(b) shows a convergence plot for the zonal component of the reconstructed velocity. The error in the center of the triangles is calculated by comparing the interpolated velocity to the initial state of test case 6 of [4] (Rossby-Haurwitz wave number 4) in these points.
The outcome demonstrates that there are no relevant differences between the two RBF kernels, but the interpolation order depends on the choosen stencil. 1^{st} order can be achieved by a 3 point, 2^{nd} order by a 9 point and 3^{rd} order by a 15 point stencil.

References

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