**An intermediate solution between diagnostic exchange coefficients and prognostic TKE methods for vertical turbulent transport.**

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Method: It is often claimed that searching the stationary solution of a full prognostic Turbulent Kinetic Energy (TKE) scheme should in principle give a computation for vertical turbulent exchange coefficients close to that of schemes simply inferring those coefficients from the local vertical gradients of wind and potential temperature. We implicitly refer here either to the methods solving the Monin-Obukhov (MO) implicit set of equations or to those using a local Richardson number, in the wake of the Louis (1979) paper. To our knowledge however, there has never been an attempt to inverse the proposal, i.e. to find the TKE prognostic equations that would have for stationary solution some already computed exchange coefficients $K_m$ (for momentum) and $K_h$ (for energy).

This note aims at this unexplored goal. Said differently, given an already well tuned scheme delivering 'static' vertical exchange coefficients, how can one introduce in the simplest possible way the missing physical items that are advection, TKE vertical 'auto-advection' and balance between production-destruction on one hand and dissipation on the other hand? This problem could be attacked from several angles but its trademark is to search a common ground between TKE and MO-type methods. Since this is exactly what Redelsperger *et al.* (2001) (thereafter RMC01) did for creating a smooth transition between the upper-air behaviour of a 'full' TKE scheme and surface similarity laws, we elected to just adapt their method to the whole depth of the atmosphere. For this, an a-priori knowledge of the 'stationary' coefficients just replaces the choice of the MO functions. The procedure symbolically reads:

$$\widetilde{K}_m, \widetilde{K}_n \Rightarrow \widetilde{K}_* \Rightarrow \widetilde{E}, K_E, \tau_\varepsilon \tag{1}$$

$$dE/dt = f(E, \widetilde{E}, K_E, \tau_\varepsilon) \tag{2}$$

$$E \Rightarrow K_* \tag{3}$$

$$K_*, \widetilde{K}_*, \widetilde{K}_m \ \&\ \widetilde{K}_h \Rightarrow K_m \ \&\ K_h \tag{4}$$

where tilded values refer to the 'static' part of the computations and non-tilded ones to the prognostic aspects (including the last step of vertical exchange using the Equation (4) values of $K_m$ and $K_h$, something which will not be detailed here, since being unchanged in the procedure). $E$ is the TKE, $K_E$ the auto-diffusion coefficient and $\tau_\varepsilon$ the relaxation time scale of the dissipation process ($E/\varepsilon$). $K_n$ is the neutral state equivalent of $K_m$ while $K_*$ is a coefficient co-dimensional to $K_m$ and $K_n$ which we define as being the mirror image of $E$ in case the equations at neutrality would be applicable for the whole range of stability conditions. Hence our problem is split into two parts: (i) solving the problem at neutrality and (ii) finding an expression to compute $K_*$ from $K_m$ and $K_n$, knowing that we shall then finish the exercise by using:

$$K_m = K_*(\widetilde{K}_m/\widetilde{K}_*) \ \&\ K_h = K_*(\widetilde{K}_h/\widetilde{K}_*) \tag{5}$$

Coming back to the problem at neutrality, it writes (RMC01):

$$\frac{dE}{dt} = A_{dv}(E) + \frac{1}{\rho}\frac{\partial}{\partial z}\rho K_E \frac{\partial E}{\partial z} + \frac{1}{\tau_\varepsilon}(\widetilde{E} - E) \tag{6}$$

$$K_* = C_K L_K \sqrt{E} = C_K A_K \frac{l_m}{\kappa}\sqrt{E} \Leftrightarrow \widetilde{E} = \left(\frac{\kappa \widetilde{K}_*}{C_K A_K l_m}\right)^2 \tag{7}$$

$$K_E = \alpha K_* \tag{8}$$

$$\frac{1}{\tau_\varepsilon} = \frac{C_\varepsilon \sqrt{E}}{L_\varepsilon} = \frac{C_\varepsilon \kappa \sqrt{E}}{A_\varepsilon l_m} \tag{9}$$

with $\kappa$ the Karman constant and $l_m$ the static-scheme-type mixing-length for momentum, not to be confused with the TKE-type mixing-lengths $L_{K/\epsilon}$. As already hinted at, the last term of the first above equation represents a simple version of the balance between shear plus buoyancy production-destruction and dissipation, the Newtonian time scale being the one of the dissipation in the 'full' TKE formalism. In order to match these various aspects we have (RMC01, only near the surface in their case):

$$A_K = \frac{1}{\sqrt{\alpha}} \frac{\kappa}{C_K} \;\&\; A_\varepsilon = \alpha\sqrt{\alpha}\kappa C_\varepsilon \tag{10}$$

and (RMC01 again) we want $A_K = A_\varepsilon$. Then, introducing $\nu$ with

$$C_K C_\varepsilon = \frac{1}{\alpha^2} = \nu^4 \tag{11}$$

we finally get the very simple set of equations:

$$K_* = \nu l_m \sqrt{E} \Leftrightarrow \widetilde{E} = \left( \frac{\widetilde{K}_*}{\nu l_m} \right)^2, \quad K_E = \frac{l_m \sqrt{E}}{\nu}, \quad \frac{1}{\tau_\varepsilon} = \frac{\nu^3 \sqrt{E}}{l_m} \tag{12}$$

where literature-reported measurements seem to indicate a value of 0.52 as the optimal one for $\nu$.

Furthermore, studying the stability dependence of the various terms of the above equations and following again the guidelines of RMC01, it appears that $K_* = \sqrt{K_m K_n}$ is a good approximation of the 'observed' implicit behaviour of $K_*$.

Results: The scheme was implemented (in research mode) in the so-called CE version of the ALADIN NWP model. It gives either neutrality or slightly improved wind scores at the top of the PBL, leads to a reasonable distribution of $E$-values and appears rather stable. But its main advantage is probably its capacity to separate numerical stability problems (already treated, see below), choice of adequate mixing length values and the encompassing in $\widetilde{E}$ and in $K_*$ of all the needed information about the functional dependency of the production-destruction mechanisms (i.e. more sophisticated formulae for the latter could mimic the intrinsic 'physics' of complex 'full' TKE schemes, without changing the numerical framework).

Discussion: Apart from the just mentioned flexibility-modularity issue, there are other advantages in this way of formulating the problem. In case of a potentially stiff behaviour of the diffusion equations in the vicinity of the neutral state, a so-called 'anti-fibrillation' specific treatment (alike the one of Bénard *et al.*, 2000) can be applied to the computation of the tilded value, something impossible in a fully prognostic TKE scheme. The scheme is naturally discretised in a way that allows to have the $E$-values in the middle of the model layers, this facilitating a semi-Lagrangian handling of the advection term common with that of other prognostic variables, a quite important practical advantage. Of course the vertical staggering with respect to the $K$-values (at the layers' interfaces) might create a spurious vertical mode. But inspection of the last two Equations of (12) shows that the welcome local proportionality between $K_E$ and $1/\tau_\varepsilon$ allows to curb this danger through the choice of big enough values of $l_m$ (and hence of $L$) with respect to the layer's thicknesses.

The proposed scheme has a few common points with the one of Brinkop and Roeckner (1995) but, apart from its reliance on the computation of $\widetilde{K}_m$ and $\widetilde{K}_h$, (i) it allows to decouple the intensities of diffusion and auto-diffusion, (ii) it treats separately the two types of stability dependence and (iii) it does not require the change of variable in $(\sqrt{E})$ to offer a numerically stable solution.

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