# Optimal-controlled Ensemble Prediction Technique and its Application in ENSO Prediction

Xudong Liang\* and Yihong Duan Shanghai Typhoon Institute, China Meteorological Administration \*Email: liangxd@mail.typhoon.gov.cn

#### 1. Introduction

More and more researchers are focusing on how to reduce the effects of initial errors on numerical prediction while others examined the effects of chaos in the atmosphere (Lorenz 1963) and diversity of numerical models. As a result, four-dimensional variational data assimilation (4D-VAR) and ensemble prediction (EP) techniques have been developed.

The method of 4D-VAR (Lewis and Derber 1985; Talgrand and Courtier 1987) was developed to use observations at different spatial and temporal points to optimize model initial conditions while assuming a perfect model.

The EP technique (Tracton and Kalnay 1993; Toth and Kalnay 1993) is based on the assumption that very small errors in the initial conditions can induce appreciable changes in the forecast. It also assumes that the numerical models are not perfect, each model having it own skill. The main problem in EP is how to generate the perturbations. Singular vector (Tracton and Kalnay 1993) and breeding of growing modes (Toth and Kalnay 1993) are some typical methods.

In this study, we combine 4D-VAR and EP techniques as an optimal-controlled ensemble prediction technique to predict ENSO events using an imperfect model and imprecise observations.

# 2. The technique

If the prediction model is

$$\boldsymbol{X}_{t} = \boldsymbol{M}(\boldsymbol{X}_{t-1}) \tag{1}$$

where *M* is the numerical model,  $X_t$  and  $X_{t-1}$  are atmosphere states at time *t* and *t*-1 respectively, and the reality at time *t* is  $Y_t$ , the forecast error is

$$\varepsilon_t = X_t - Y_t \tag{2}$$

The error includes two parts: one is due to the error of the model itself, another is induced by initial condition errors. In other words, the numerical model can only describe part of the atmospheric variation that can be written as the inner product of the model forecast variation  $\delta X_t$  and forecast error  $\varepsilon_t$  as  $\langle \varepsilon_t, \delta X_t \rangle$ . The aim of weather forecast is to minimize the mode

 $\left\|\left\langle \varepsilon_{t},\delta X\right\rangle\right\|$ 

According to theory of 4D-VAR,

$$<_{\varepsilon_t}, \delta X_t > = <_{\varepsilon_t}, L \delta X_0 > = < L^* \varepsilon_t, \delta X_0 >,$$

where L and  $L^*$  are continuous linear operators of the model M and its adjoint respectively. In the adjoint model, this can be written as

$$\sigma = L^* \varepsilon_t$$

Meanwhile. minimize to mode  $|| < \varepsilon_t, \delta X_t > ||$  is equivalent to minimize mode  $|| \langle L^* \varepsilon_t, \delta X_0 \rangle ||$ , which can be achieved by introducing a disturbance  $W\sigma$  (where W is a weight coefficient) in the initial conditions. From the EP perspective. Wσ is the required disturbance. Here, the disturbance  $W\sigma$ differs from the disturbance in the usual EP technique because it is an optimal value controlled by observations and the model itself using the 4D-VAR technique. On the other hand, (2) indicates that model forecast error is a function of the length of forecast time. Likewise, the disturbance  $W_{\sigma}$  is a function of forecast time. Therefore, the optimal-controlled EP technique can be described as using 4D-VAR technique with a different length

of forecast time to calculate a set of disturbances and to get a set of EP members.

### 3. Model and results

simple Cane-Zebiak The air-sea coupled model (Cane et al. 1986) is used and its adjoint model is developed in this study. An optimal-controlled EP system is established based on the 4D-VAR system. Monthly-averaged sea-surface temperature anomaly (SSTA) from 1971 to 1998 from the National Centers of Environmental Prediction (NCEP) and wind field at 1000 hPa from the reanalysis data of NCEP are used. EP members are formed by setting assimilation period as 3, 6, 9, 12, 15, 18, 24, 27 and 30 months. There are 10 members including the control run (without disturbance in initial conditions) with 277 cases from June 1973 to June 1996. Each case has 18 months of NINO3 index forecast.

Figure. 1 gives the time variations of skill (correlation coefficient) of each member while Fig. 2 gives those of the control, EP and persistence forecast. In EP scheme 1, the average weight coefficient of each member is the same, while in EP scheme 2, the average weight coefficient of each member is calculated according to its skill as in Fig.1. Figure 3 is the mean square of NINO 3 index forecast error. The EP schemes have higher skill and lower errors (Figs. 2 and 3), especially in EP scheme 2. Therefore, the optimal-controlled EP technique can improve the forecasting skill evidently even using a simple numerical model.

#### References

- Cane, M.A., S.E.Zebiak, and S. C. Dolan, 1986: Experimental forecast of El Niño. *Nature*, **321**, 827-832.
- Lewis, J. M., and J. C. Derber, 1985: The use of adjoint equations to solve a variational adjustment problem with advective constraint. *Tellus*, **37A**, 97-100.
- Talgrand, O. and Courtier, P., 1987 : Variational assimilation of meteorological observations with the adjoint vorticity equation I: Theory. *Q. J. Meteor. Soc.*, **113**, 1311-1328.

- Tracton, M. S. and E. Kalnay, 1993: Operational ensemble prediction at the National Meteorological Center: Practical aspects. *Wea. Forecasting*, **8**, 379-398.
- Lorenz, E. N., 1963: Deterministic non-periodic flow. *J. Atmos. Sci.*, **20**, 103-141.
- Zoltan T. and E. Kalnay, 1993: Ensemble forecasting at NMC: The Generation of Perturbations. *Bull. Amer. Meteor. Soc.*, **74**, 2317-2330.



Fig. 1. NINO 3 forecast skill of each EP member (with different assimilation period).



Fig. 2. NINO 3 index forecast skill of EP (schemes 1 and 2), control and persistence forecasting.



Fig. 3. Mean square of NINO3 index forecast error.