Time-splitting of advection in the JMA Nonhydrostatic Model

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The Japan Meteorological Agency (JMA) started an operational run of a 10 km horizontal resolution mesoscale NWP model in March 2001. The model, MSM, is a high-resolution version of the JMA's operational regional spectral model, where the hydrostatic equilibrium is assumed. As for its initialization, mesoscale 4DVAR has been introduced in March 2002. Meanwhile, JMA will replace MSM with a nonhydrostatic model in March 2004. Development of an operational nonhydrostatic model for regional NWP (NHM) has been underway, based on the Meteorological Research Institute/Numerical Prediction Division unified nonhydrostatic model (MRI/NPD-NHM; Saito et al., 2001; http://www.mri-jma.go.jp/Dep/fo/mrinpd/INDEXE.htm).

Among the three dynamical cores of MRI/NPD-NHM, the split-explicit time integration scheme (HE-VI scheme) is used for operation, considering the computational efficiency on the distributed memory parallel computer in the JMA's NWP system. The HE-VI scheme of MRI/NPD-NHM treats sound waves in the short time step, but has no special treatment for gravity waves. For operational purpose, it is crucial to stabilize the gravity wave modes and remove the dependency of the maximum time step on the atmospheric static stabilities.

Saito (2002) proposed a time splitting scheme of gravity waves for NHM where the computation of the buoyancy terms and vertical advection of the reference potential temperature are treated in the short time step in the HE-VI scheme. This scheme stabilizes the gravity waves, however, in case horizontal wind is strong, it is not necessarily effective to use a large time step in the leap-frog time integration. A new time splitting scheme of advection has been developed to improve the computational stability of NHM.

In the HE-VI scheme of NHM, the forward time integration

$$\frac{U^{\tau+\Delta\tau}-U^{\tau}}{\Delta\tau}+\frac{\partial P^{\tau}}{\partial x}+\frac{\partial G^{\frac{1}{2}}G^{13}P^{\tau}}{G^{\frac{1}{2}}\partial z^{*}}=-(ADVU+RU), \qquad (1)$$

$$\frac{V^{\tau+\Delta\tau}-V^{\tau}}{\Delta\tau}+\frac{\partial P^{\tau}}{\partial y}+\frac{\partial G^{\frac{1}{2}}G^{23}P^{\tau}}{G^{\frac{1}{2}}\partial z^{*}}=-(ADVV+RV), \qquad (2)$$

are used for horizontal momentums, and the backward time integration

$$\frac{W^{\tau+\Delta\tau}-W^{\tau}}{\Delta\tau}+\frac{1}{mG^{\frac{1}{2}}}\frac{\partial P^{\beta}}{\partial z^{*}}+\frac{g}{mC_m^{\ 2}}P^{\beta}=\frac{1}{m}BUOY-(ADVW-RW), \qquad (3)$$

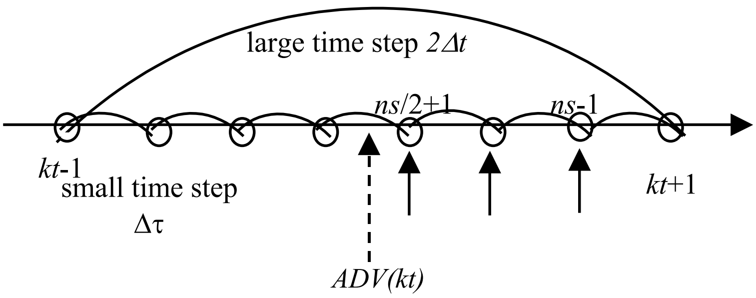
$$\frac{P^{\tau+\Delta\tau}-P^{\tau}}{\Delta\tau}+C_m^{\ 2}(-PFT+m^2(\frac{\partial U^{\gamma}}{\partial x}+\frac{\partial V^{\gamma}}{\partial y})+m\frac{\partial}{\partial z^{*}}[\frac{1}{G^{\frac{1}{2}}}\{W^{\beta}+m(G^{\frac{1}{2}}G^{13}U^{\gamma}+G^{\frac{1}{2}}G^{23}V^{\gamma})\}]$$
$$-PRC)=dif.P, \qquad (4)$$

are employed for vertical momentum and pressure. In NHM, advection terms are originally computed in the long time step Δt. It is expected that computing advection terms in short time step $\Delta\tau$ contributes to improve the computational stability. However, this choice is not acceptable because the higher order advection scheme with the modified scheme is expensive and spoils the merit of HE-VI scheme.

In the new splitting scheme, we fully evaluate higher-order advection terms with the modified scheme at the center of the leap-frog time step, and then adjust the lower-order (second-order) components at each short time step in the later half of the Leap-frog time integration as

$$ADV = ADV(kt) - ADVL(kt) + ADVL^{\tau}. \qquad (5)$$

Here, *ADV(kt)* is the higher order advection with the modified scheme at time step *kt*, *ADVL(kt)* and *ADVL* is the lower-order advection component at *kt* and each short time step $\Delta\tau$, respectively. This adjustment is done from (*ns-1)*/2+1 to *ns*-1, where *ns* is the ratio of 2Δt and $\Delta\tau$.



Time splitting of advection terms. Case of *ns* = 7.

In order to split the gravity waves, equations for the potential temperature and vertical momentum are rewritten as

$$\frac{\theta^{\tau+\Delta\tau} - \theta^{\tau}}{\Delta\tau} = -(ADV\theta - ADVL\theta + ADVL\theta^{\tau}) + \frac{Q}{c_p \pi} + dif.\theta = ADVL\theta - ADVL\theta^{\tau} + \left[\frac{\partial\theta}{\partial t}\right], \quad (6)$$

$$\frac{W^{\tau+\Delta\tau} - W^{\tau}}{\Delta\tau} + \frac{1}{mG^{\frac{1}{2}}}\frac{\partial P^{\beta}}{\partial z^{*}} + \frac{g}{m{C_m}^2}P^{\beta} = \frac{1}{m}\frac{\rho G^{\frac{1}{2}}\theta^{\tau+\Delta\tau}(1+0.61Q_v)(1-Q_c-Q_r-Q_i-Q_s-Q_g)}{\theta_m}g$$
$$-(ADVW - ADVLW + ADVLW^{\tau} - RW). \qquad (7)$$

As in Saito (2002), the third term of *r.h.s.* of (6) is given by a tentative time integration in the cloud microphysical process.

This time splitting scheme enables the model to use 40 sec. for Δt of the leap-frog scheme in the simulation of 10 km horizontal resolution, which (roughly speaking) corresponds to 80 sec. in the second order Runge-Kutta time integration scheme. Additional computational cost for time splitting of advection is less than 10 % of the total computation time when the model includes cloud microphysics.

References.

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