Background Error Covariances in a Quasigeostrophic Reduced Rank Kalman Filter

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The purpose of this work is, in continuation of Beck and Haas (2001), to study the benefits on analysis quality of including *dynamical* (i.e., state–dependent) background information in a simple, yet reasonably realistic environment. The impact of flow–dependent background error covariances is studied within a four–dimensional variational data assimilation (4D–Var) system based on the quasigeostrophic model of Marshall and Molteni (1993), carrying 1449 degrees of freedom. This 4D–Var system allows for assimilating a given set of synthetic observations in cycling experiments with different specifications for the background error covariance matrix B. Specifically, B may be kept either static, or fully dynamic as the entire analysis error covariance matrix is carried forward in time according to the Extended Kalman Filter (EKF) equations (Ehrendorfer and Bouttier 1998). Further, B may be specified as a combination of dynamically evolved analysis error covariances and static background according to the theory developed for the Reduced Rank Kalman Filter (RRKF). The implementation of the RRKF closely follows the formulation at the European Centre for Medium–Range Weather Forecasts (see, Fisher 1998; Fisher and Andersson 2001). The basic idea of the RRKF is that the dynamical propagation of the background errors is only applied in a subspace of relatively small dimension (say, k) that is defined by the so–called Hessian singular vectors (HSV; Barkmeijer et al. 1998). Preliminary results from four cycling experiments are presented that are designed to investigate the performance of the RRKF – in different configurations (i.e., different dimension of the subspace k) – in comparison to either a static background formulation, or to the EKF. Each of the cycling experiments covers 12 assimilation intervals with a window length of 12 hours each. These preliminary results suggest that the performance of this 4D–Var system is sensitive to the specification of the background.

As an example, Fig. 1 shows the analysis and the forecast error (fc; as a function of lead time), for a static B (solid curve), an RRKF formulation with k=10 (dotted) and k=100 (dashed) HSVs, and the full EKF (chain–dashed) corresponding to an RRKF with k=1449 HSVs. Errors are measured in terms of the total energy (TE) metric (see, Ehrendorfer 2000), in units of J / kg. Each of the curves is the mean over the 12 subsequent assimilation intervals. Thinned observations on an irregular grid are sampled every 6 hours from a "truth run" with pre–specified observation error variances. It is this "truth run" that has been used in the computation of the forecast (and analysis) errors. Hence, these errors can be quantified exactly. Fig. 1 suggests that incorporating dynamical background error information is beneficial in terms of reducing analysis and subsequent forecast errors. Such a benefit is also expected from theoretical considerations (e.g., Fisher 1998).

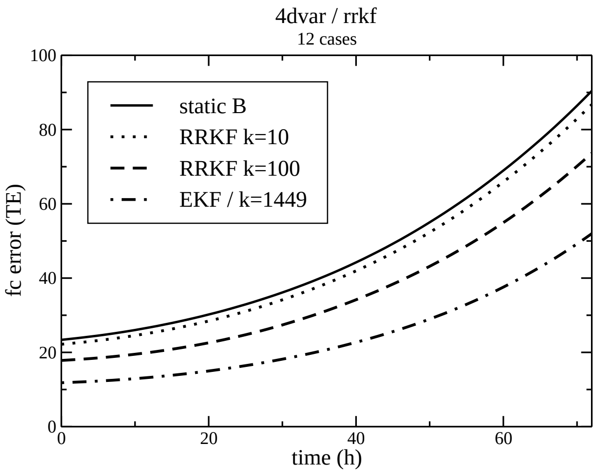

Fig. 1: Performance of the 4D–Var system in terms of analysis and fc error as a function of lead time.

However, it is also evident that a significant improvement is only achieved if O(100) HSVs are used. Finally, it is necessary to emphasize that these results are dependent on various other specifications of the 4D-Var system studied here, and are thus of a very preliminary nature. Additional investigation is necessary to further confirm the benefits of a dynamical B in data assimilation.

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