Diagnosis of data assimilation systems

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**JMA: Japan Meteorological Agency.

Contents

1. Observation impact estimation

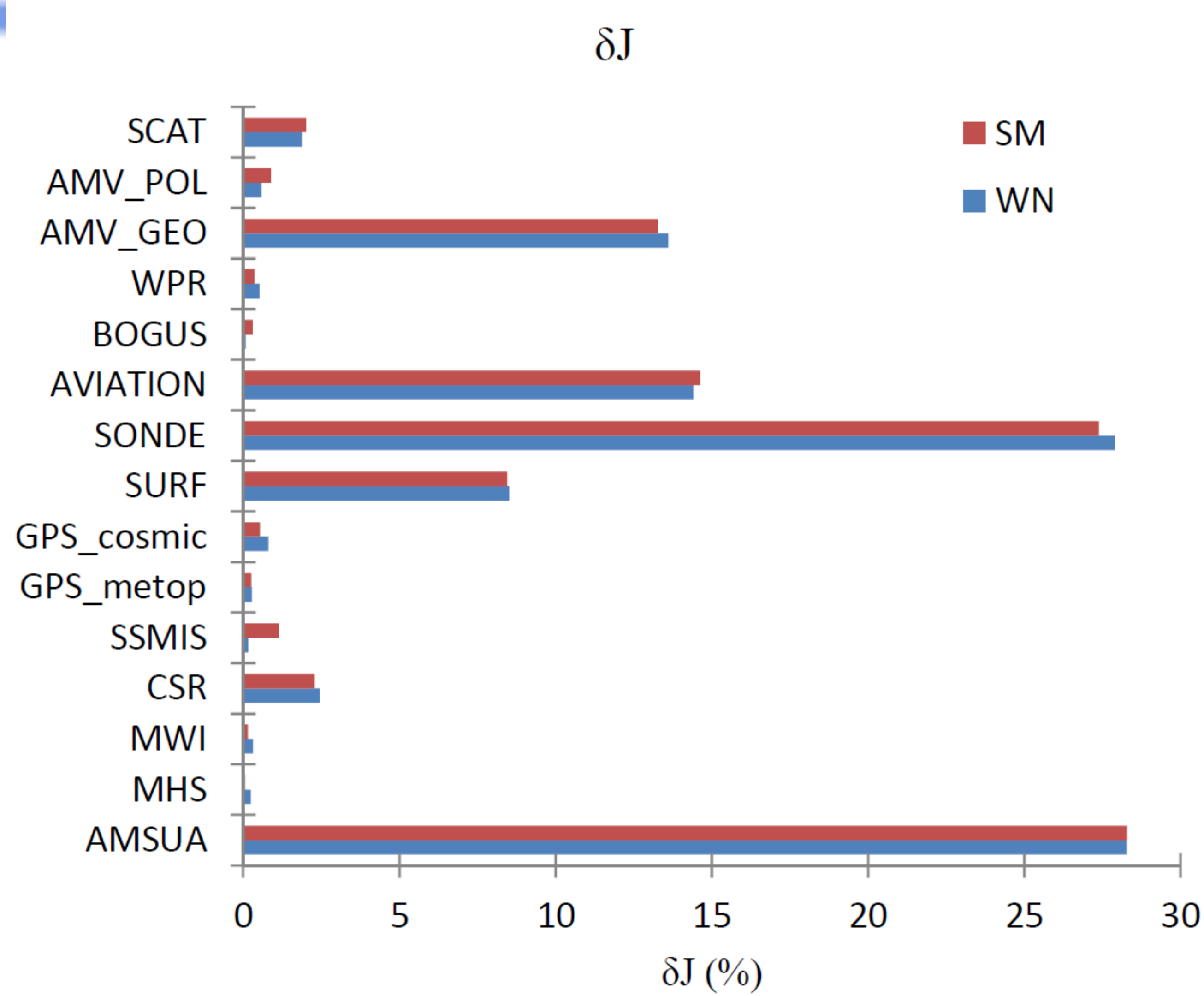
Two types of observation impact

- Basic definition
  - The observation impact is defined as “the variations of analyses and forecasts caused by changes of observation data”.
- Non-linear observation impact
  - This is the observation impact that has no limitations on the changes of observation data, so the changes includes perturbations in observation data values, and additions of observation datasets.
  - Estimation methods:
    - ✓ OSE (observing system experiment).
- Linear observation impact
  - This is the observation impact that has an limitation on the changes of observation data, which is Kalman gain is invariant.
  - Estimation methods are
    - ✓ ADJ-based method
      - Langland and Baker (2004), Errico (2007), Cardinali (2009), Tremolet (2008)
    - ✓ TL-based method
      - Ishibashi (2011)
    - ✓ DFS
      - Cardinali et al (2004), Desroziers et al (2005)

These two observation impacts are different quantities, so, in general, they cannot work as a proxy for each other.

ADJ-based estimation in JMA global 4D-Var

- JMA global 4D-Var
- Evaluation periods are:
  Summer: Aug 2010,
  Winter: Jan 2010.
  * 00UTC analyses only.
- Using dry total energy norm.
- Forecast error evaluation time is 15hours.



TL(Tangent linear)-based method

$$\mathbf{x} = \arg\min(J(\mathbf{x})),$$

$$\mathbf{d} \Rightarrow \underset{operator}{\left[4D - Var\right]} \Rightarrow \delta\mathbf{x}$$

K

Figure 1. Schematic plot of the variational DAS. We can consider that the variational DAS is an operator, an input of the operator is a departure vector and an output is an analysis increment vector.

$$\mathbf{d}_{P+Q} \Rightarrow \left[4D - Var\right]_{P+Q} \Rightarrow \delta\mathbf{x}_{P+Q}$$
$$\mathbf{d}_{Q} \Rightarrow \left[4D - Var\right]_{Q} \Rightarrow \delta\mathbf{x}_{Q}$$

K'

Figure 2. Schematic plot of the difficulty of the linear observation impact estimation in the variational DAS. We cannot estimate the linear observation impact of the dataset P in the DAS that assimilates datasets P and Q as the differences of the analysis increments from the other DAS that assimilates only the dataset Q.

Ishibashi (2011) QJRMS

Formulation of TL-based method

- Linear decomposition of analysis increments

$$\delta x_i = \sum_{r=1}^{N} K_{i,r} d_r \equiv \sum_{r\in Z} K_{i,r} d_r = \sum_{r\in P} K_{i,r} d_r + \sum_{r\in Q} K_{i,r} d_r$$

Two datasets P and Q

$$\equiv \delta x^{*P} + \delta x^{*Q},$$

Here, we introduced partial increment vectors (PIVs), as follows;

$$\delta x_i^{*P} \equiv \sum_{r\in P} K_{i,r} d_r \ ; \quad \delta x_i^{*Q} \equiv \sum_{r\in Q} K_{i,r} d_r$$

The PIV represent a linear observation impact of each dataset.
However, PIV calculation requires explicit construction of **K**.

- Linear decomposition of departures

$$\delta x_i = \delta x^{*P} + \delta x^{*Q}$$

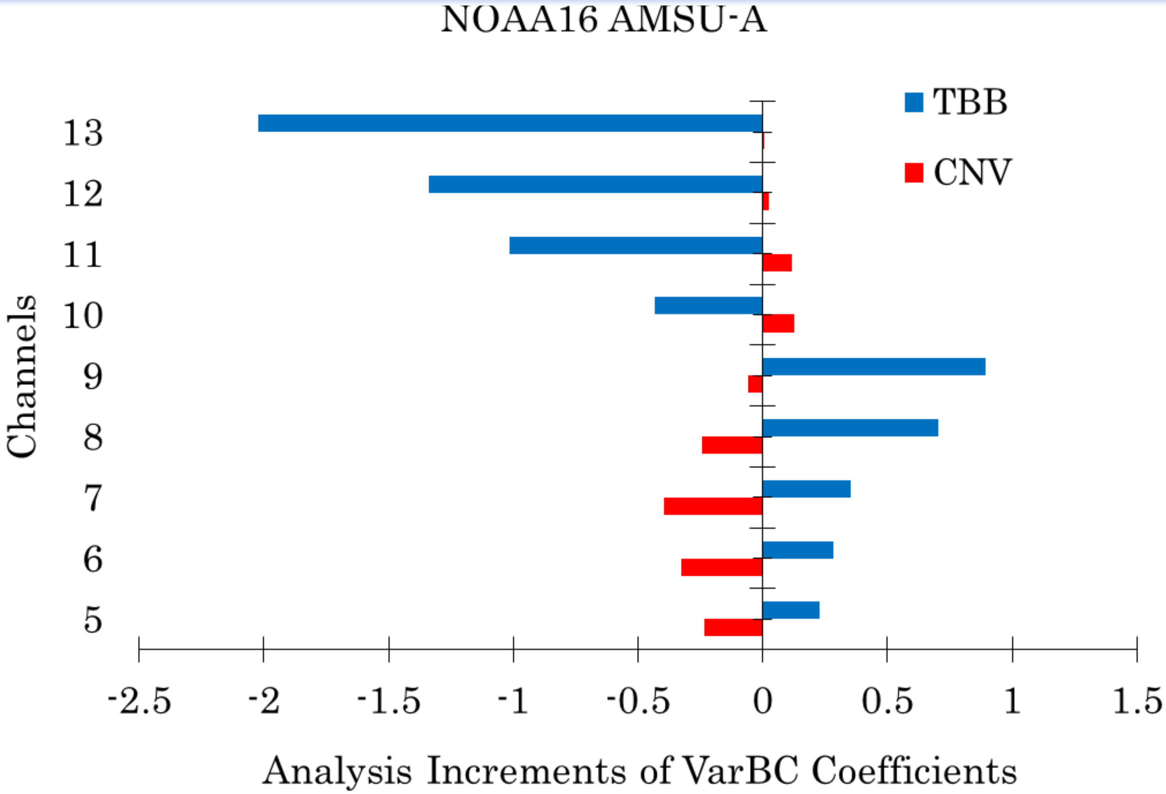
$$\equiv \sum_{r\in Z} K_{i,r} d_r^{*P} + \sum_{r\in Z} K_{i,r} d_r^{*Q}$$

We can calculate PIVs by using PDVs without explicit construction of **K** in 4D-Var.

Here, we introduced partial departure vectors (PDVs), as follows;

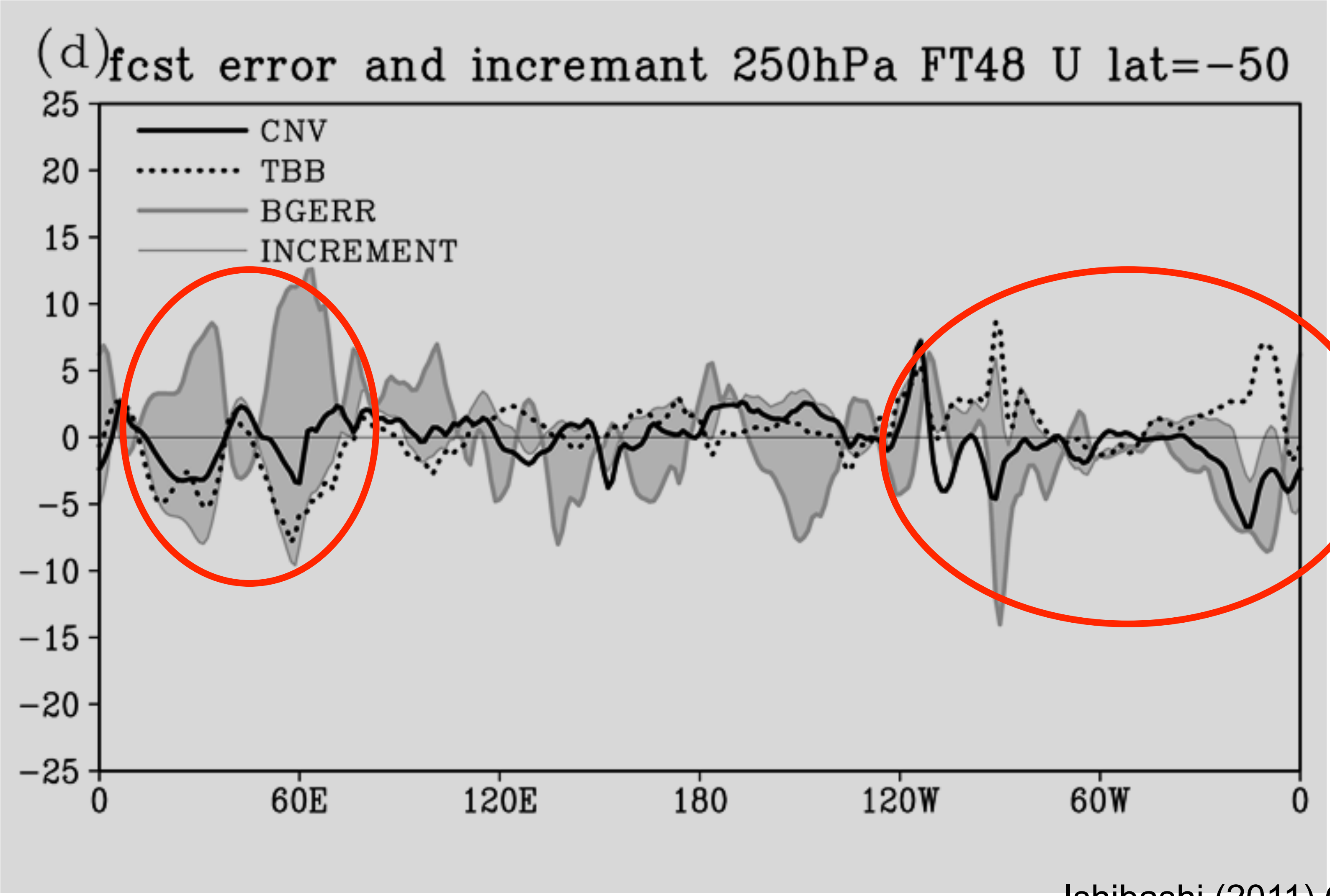
$$d_r^{*P} \equiv \begin{Bmatrix} d_r & r\in P \\ 0 & r\notin P \end{Bmatrix} ; \quad d_r^{*Q} \equiv \begin{Bmatrix} d_r & r\in Q \\ 0 & r\notin Q \end{Bmatrix}$$

TL-based method



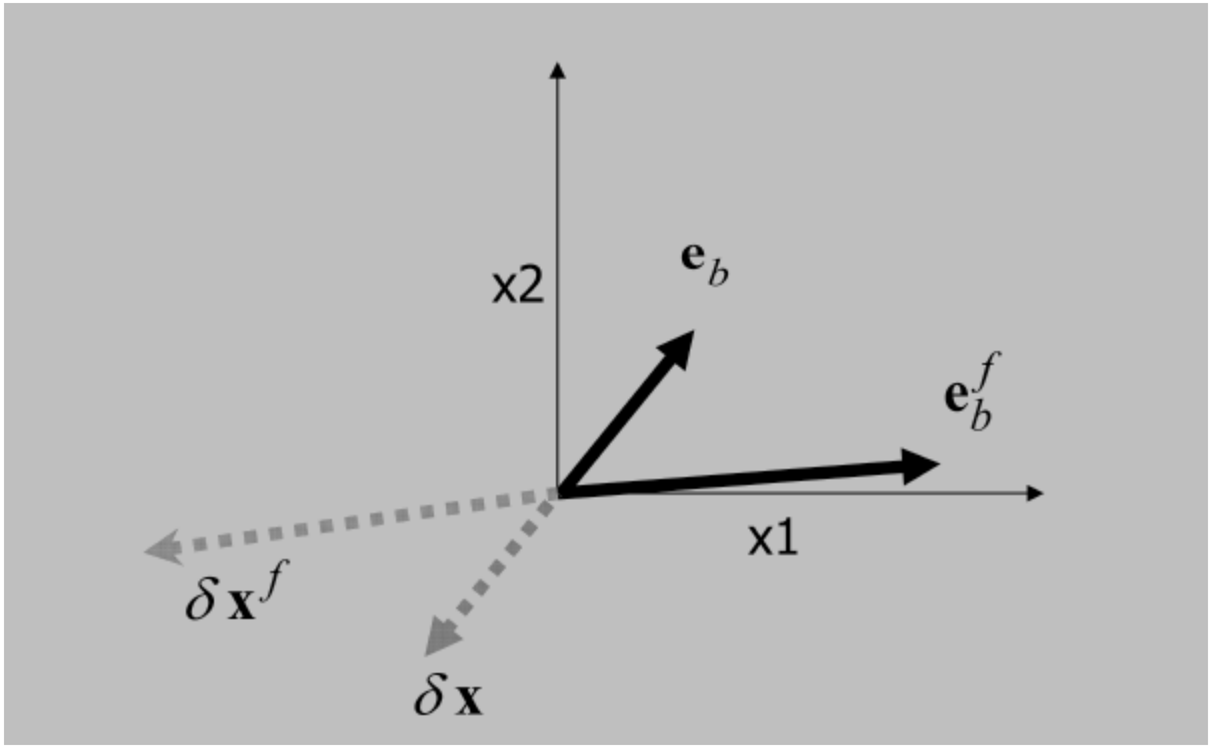
- This figure shows the CNV-PIV and the TBB-PIV for VarBC (variational bias correction) variables of the AMSU-A sensor of the NOAA16 satellite.
- We can find finite contribution from the CNV.
- This result suggests the existence of a stability effect of the CNV for the VarBC variables (Auligné et al., 2007) at least qualitatively.

TL-based method



Ishibashi (2011) QJRMS

TL-based method



Ishibashi (2011) QJRMS

2. Covariance matrix optimization

Relationships between observation impact estimations and covariance optimizations

- Two types of error covariance matrix optimization methods.
  1. Expectation-based method
     - This method optimizes error covariance matrices based on the theoretical relation ships;

       $$2E[J_o] = Tr[\mathbf{I} - \mathbf{HK}] \qquad 2E[J_b] = Tr[\mathbf{KH}]$$

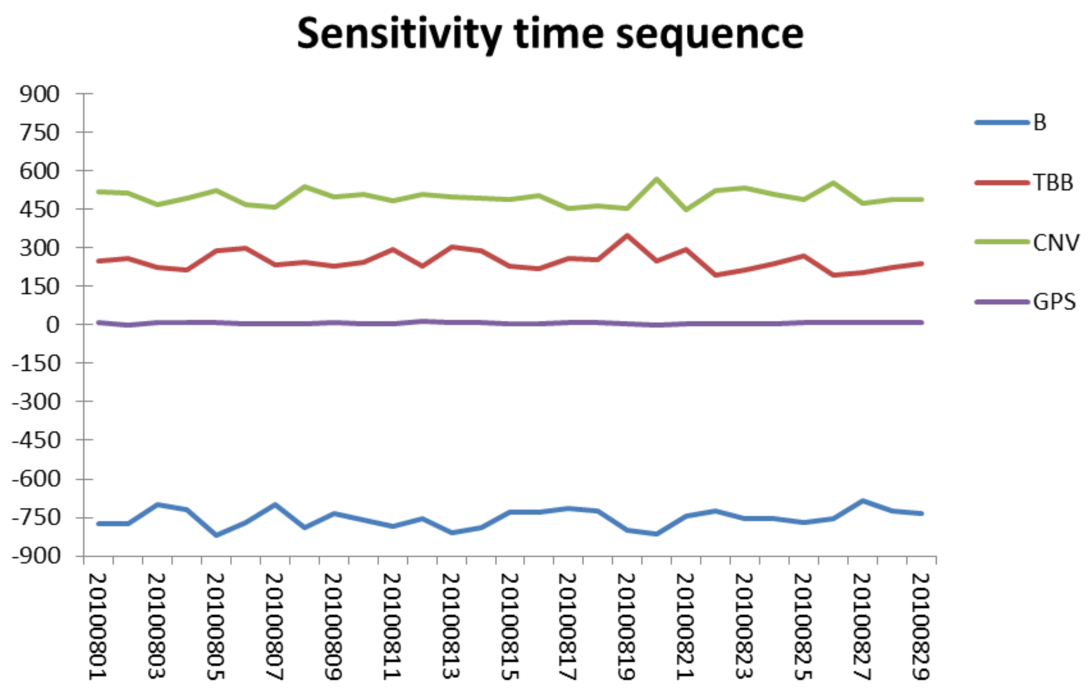
     - Desroziers and Ivanov (2001), Desroziers et al (2005), and Chapnik et al (2004, 2006)
  2. Sensitivity-based method
     - This method uses sensitivity of forecast errors respect to covariance matrices;

       $$\frac{\partial J}{\partial \mathbf{R}}, \frac{\partial J}{\partial \mathbf{B}}$$

     - Daescu (2008) , Daescu and Todling (2010).
- Each optimization methods include a linear observation impact estimation.
  1. Expectation-based method includes DFS calculation.
  2. Sensitivity-based method includes ADJ-based estimation.

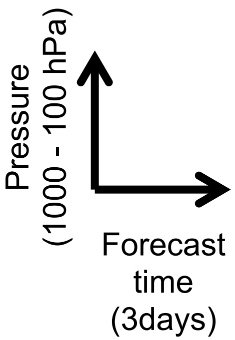
Here, Let's see the sensitivity-based method

Diagnoses of the JMA global 4D-Var

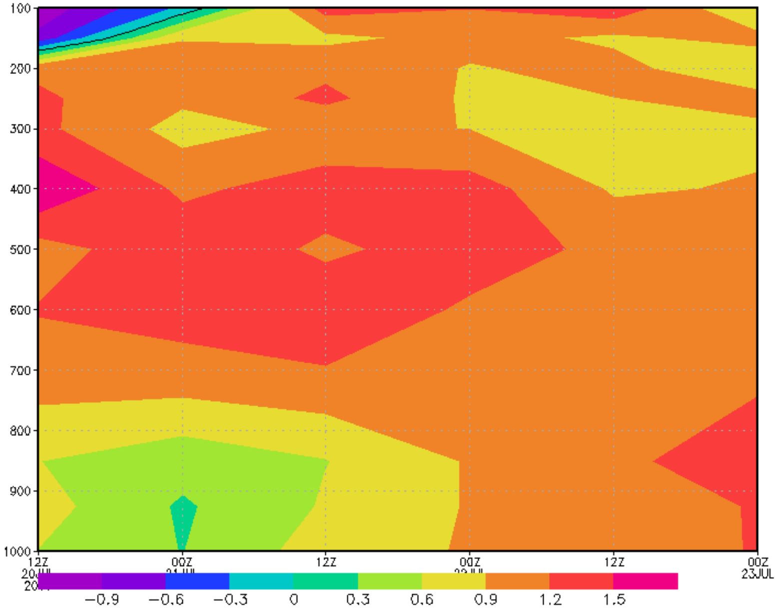


- Sensitivity calculation results in August 2010.
- Using dry total energy norm with 15hr forecasts.
- The results show that **B** is too small and **R** is too large in average.

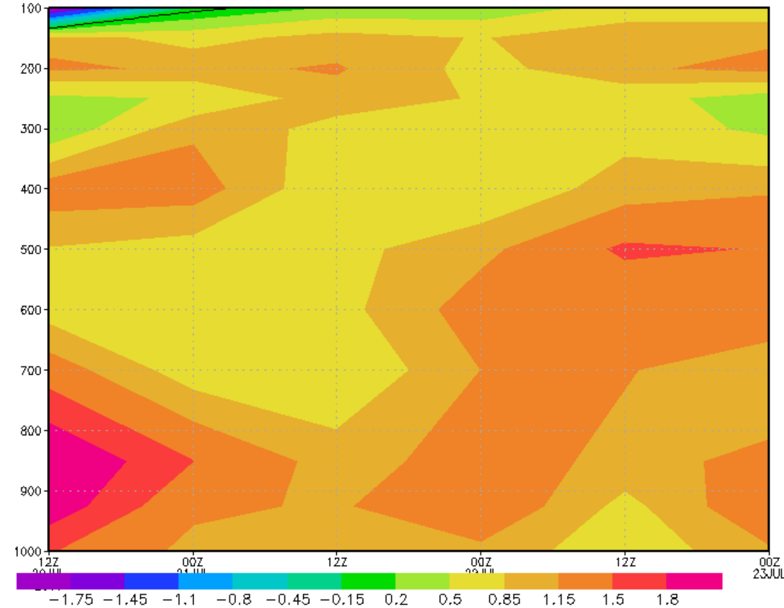
Impact of error covariance optimization on forecast accuracy



Improvement rate: Zonal wind



Improvement rate: Temperature



Results of a single case experiment of covariance optimization using the sensitivity method.

- TEST uses optimized **R**, CNTL uses original **R** (operational setting).
- The figure shows normalized forecast RMSE differences between TEST and CNTL: (CNTL – TEST)/CNTL.
- Warm (cold) color areas are forecast error decrease (increase) areas.

3. Analysis error estimation

Background

- We want to know analysis errors of a DAS because the analysis error information is useful to improve current DASs and to design future observational systems which can detect the analysis errors.
- Analysis error estimation is the same with construction of more accurate analysis than current DASs. Such analyses can be used as “pseudo truth”.

Previous studies

- “Key analysis error” (Rabier et al 1996, Klinker et al 1998, Isaksen et al 2005) can generate more accurate forecasts than current DASs.
- However, there are inconsistency between key analysis errors and observation information. SOSE (Marseille 2007) can reduce this problem.

Our approach

1. We construct the pseudo truth based on the data assimilation theory.
2. We construct the pseudo truth based on ADJ-based method.

Data assimilation theory based method

- Conditional PDF

$$P(\mathbf{x}|\mathbf{y},\mathbf{x}_b) \propto P(\mathbf{y}|\mathbf{x})P(\mathbf{x}_b|\mathbf{x})$$

Ordinary 4D-Var

$$P(\mathbf{x}|\mathbf{y},\mathbf{x}_b,\mathbf{x}_{ref}) \propto P(\mathbf{y}|\mathbf{x})P(\mathbf{x}_b|\mathbf{x})\boxed{P(\mathbf{x}_{ref}|\mathbf{x})}$$

Extended 4D-Var with reference analyses information

- Add reference analysis fields information

$$J = J_{org} + 1/2\left(\mathbf{x}_{ref} - M(\mathbf{x}_b + \delta\mathbf{x})\right)^T \mathbf{A}^{-1}\left(\mathbf{x}_{ref} - M(\mathbf{x}_b + \delta\mathbf{x})\right)$$

$$= J_{org} + 1/2\left(\mathbf{e}^t + \mathbf{M}\delta\mathbf{x}\right)^T \mathbf{A}^{-1}\left(\mathbf{e}^t + \mathbf{M}\delta\mathbf{x}\right)$$

$$J_{org} \equiv 1/2\,\delta\mathbf{x}^T\mathbf{B}^{-1}\delta\mathbf{x} + 1/2\left(\mathbf{d} - \mathbf{H}\delta\mathbf{x}\right)^T\mathbf{R}^{-1}\left(\mathbf{d} - \mathbf{H}\delta\mathbf{x}\right)$$

X: analysis,

Y: observations

X$_b$: background field

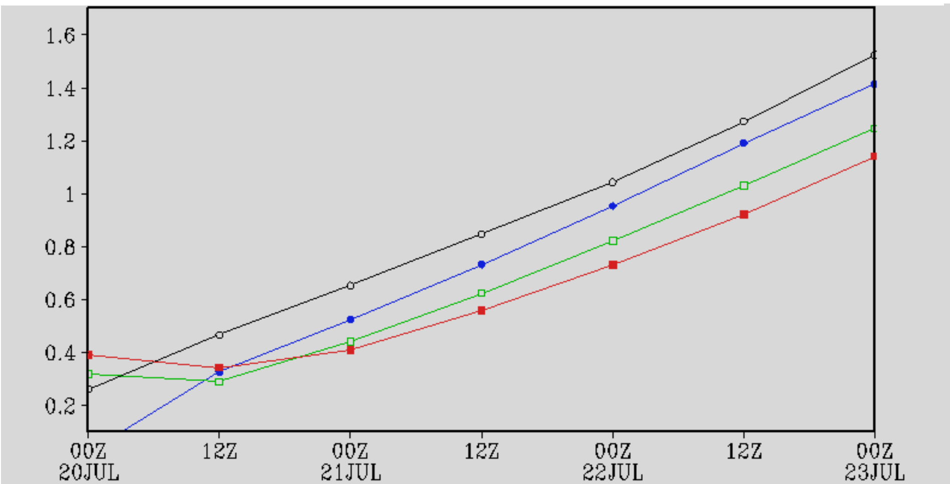
Xref: reference analyses

- Analytical solution has an error covariance matrix **A** of reference information in Kalman gain, and forecast error in input data, as follows,

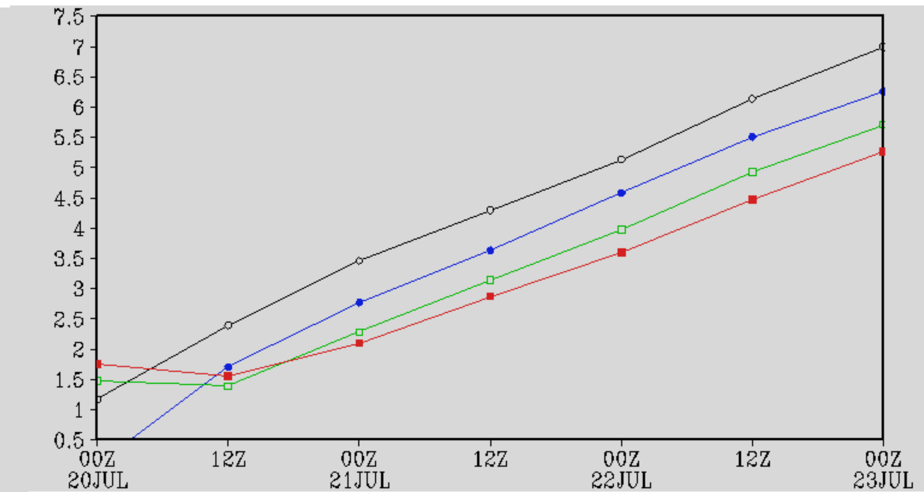
$$\delta\mathbf{x} = \left(\mathbf{B}^{-1} + \mathbf{H}^T\mathbf{R}^{-1}\mathbf{H} + \mathbf{M}^T\mathbf{A}^{-1}\mathbf{M}\right)^{-1}\left\{\mathbf{H}^T\mathbf{R}^{-1}\mathbf{d} - \mathbf{M}^T\mathbf{A}^{-1}\mathbf{e}^t\right\}$$

Accuracy of optimized forecasts

500hPa Temperature



250hPa Zonal wind



∗Inflation factor is one.

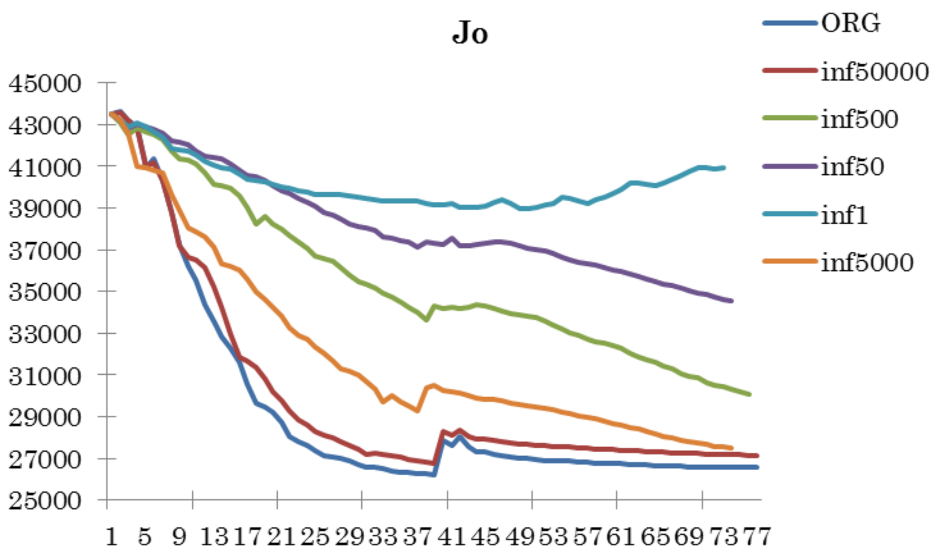
Optimized forecast with four reference analyses of every 6hours.

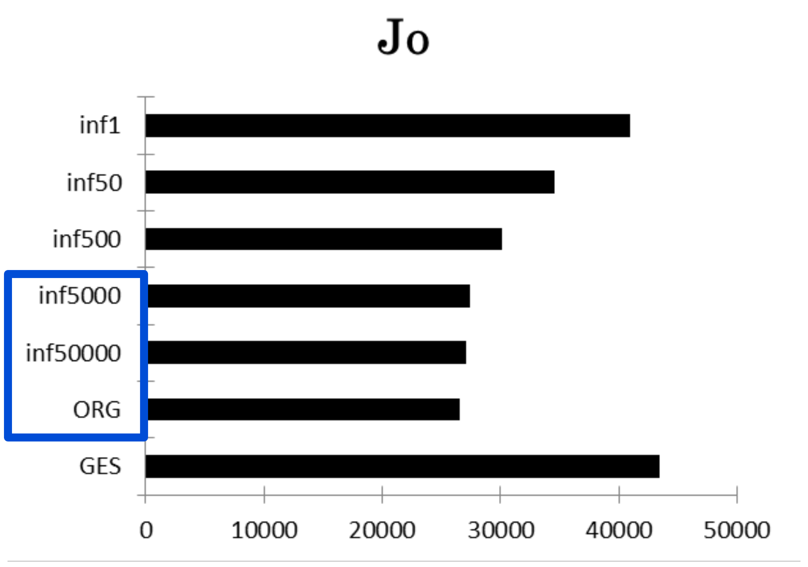
Optimized forecast with only two reference analyses

Original forecast

Original forecast from 6 hours after initial.

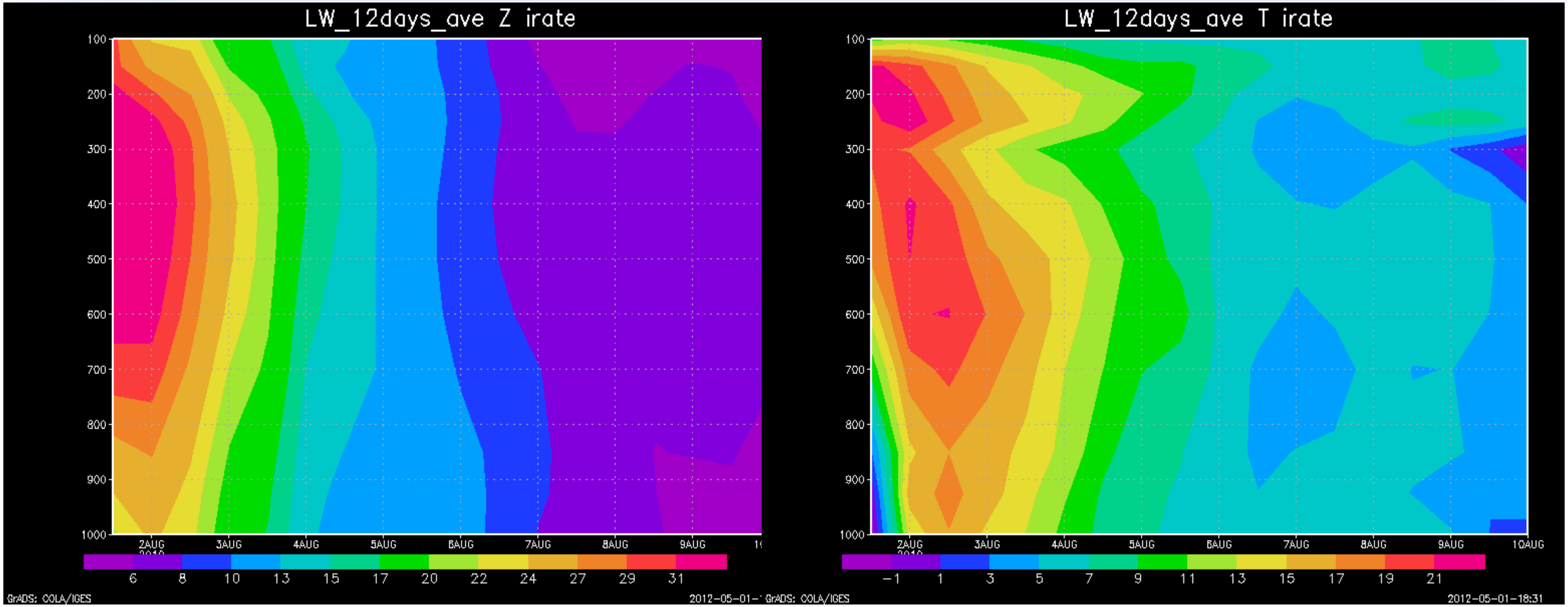
Assimilation of reference analyses





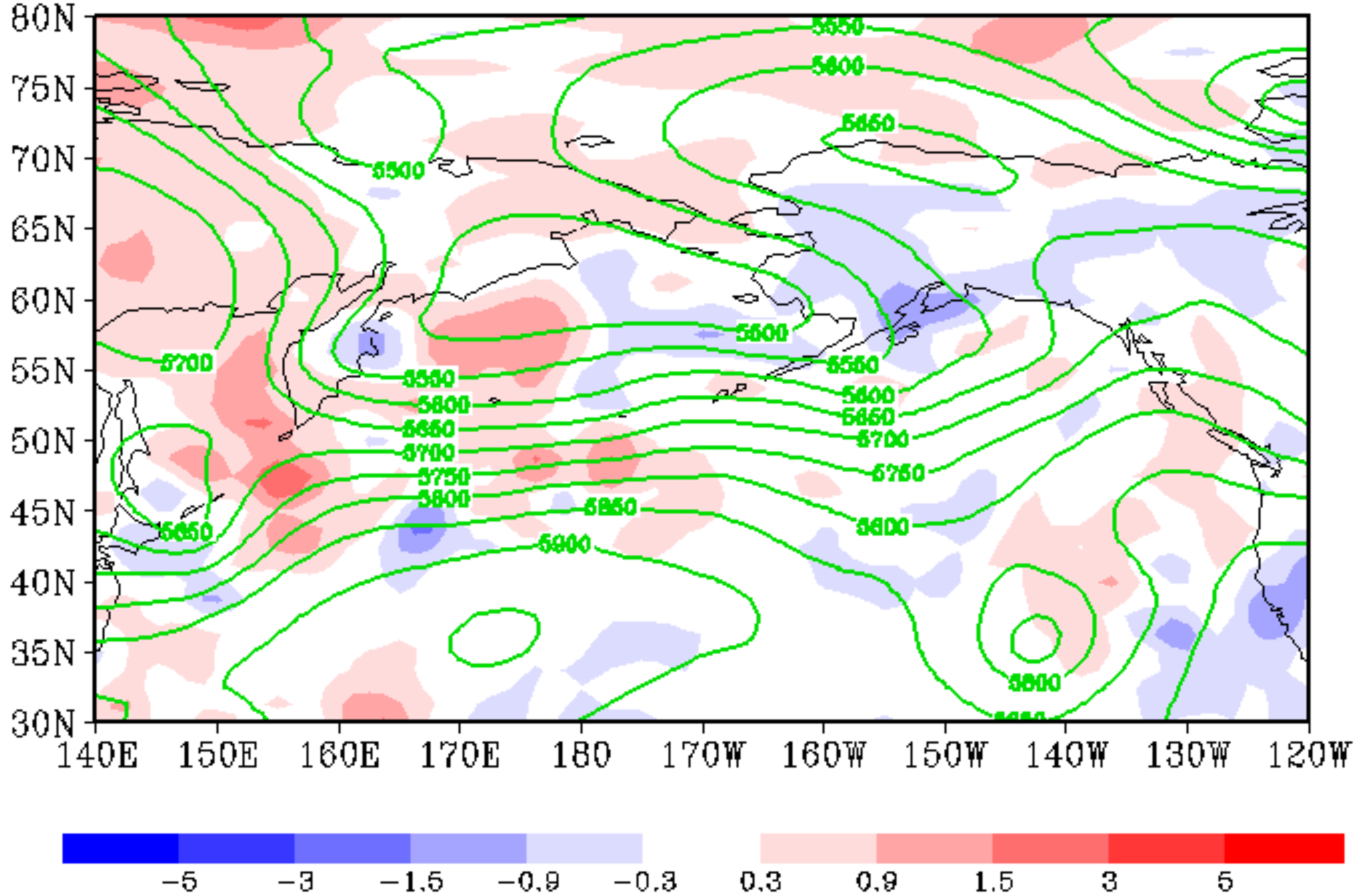
- The inflation factor dominates the fittings of analysis to observations.
- The inflation factors larger than 500 achieve good fitting nearly the original 4D-Var.

Two weeks statistics



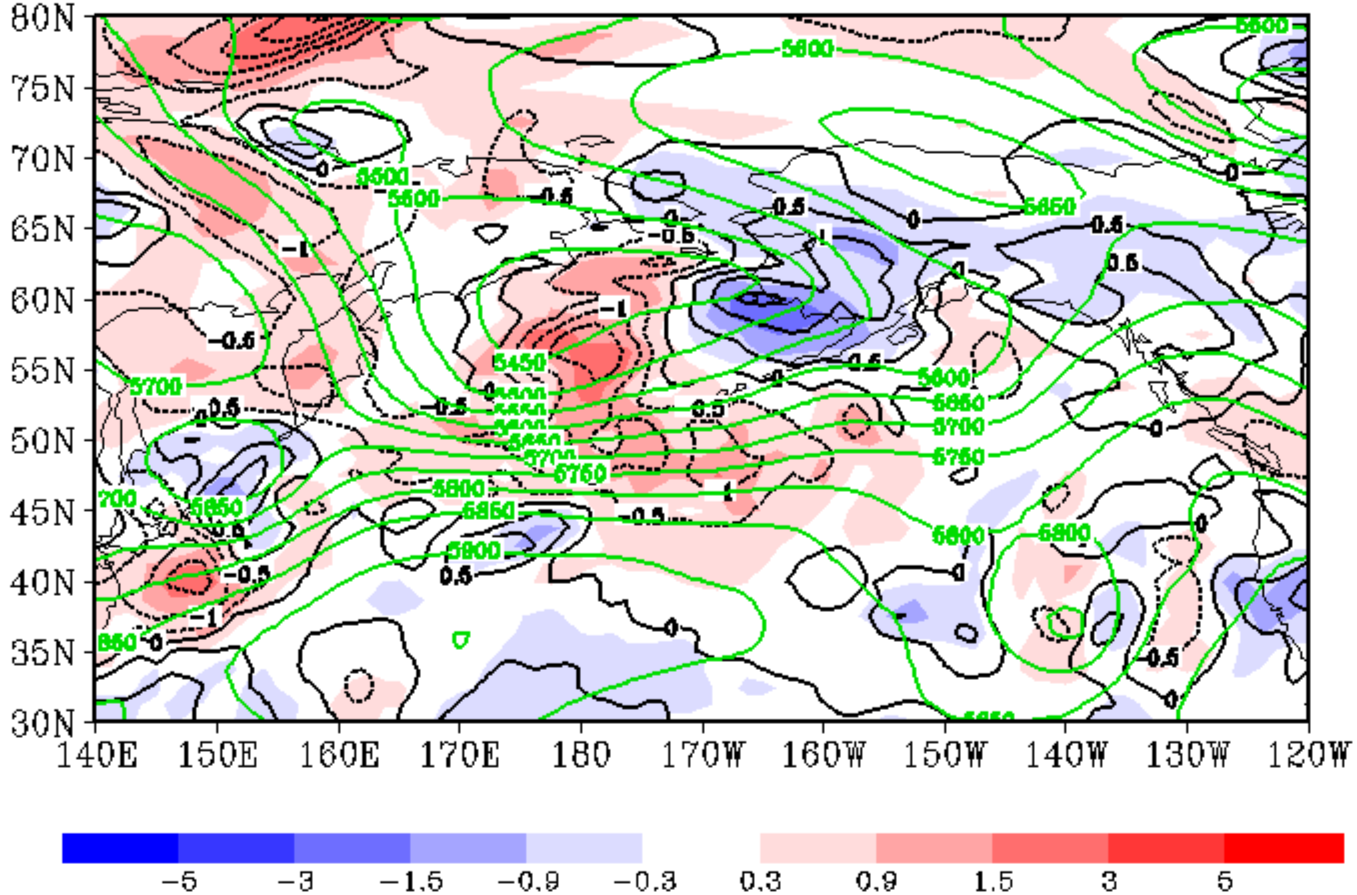
- Forecast accuracy keeps until 9days with 95% statistical significance until 6 or 7days.

Optimized analysis increments and background error 072000



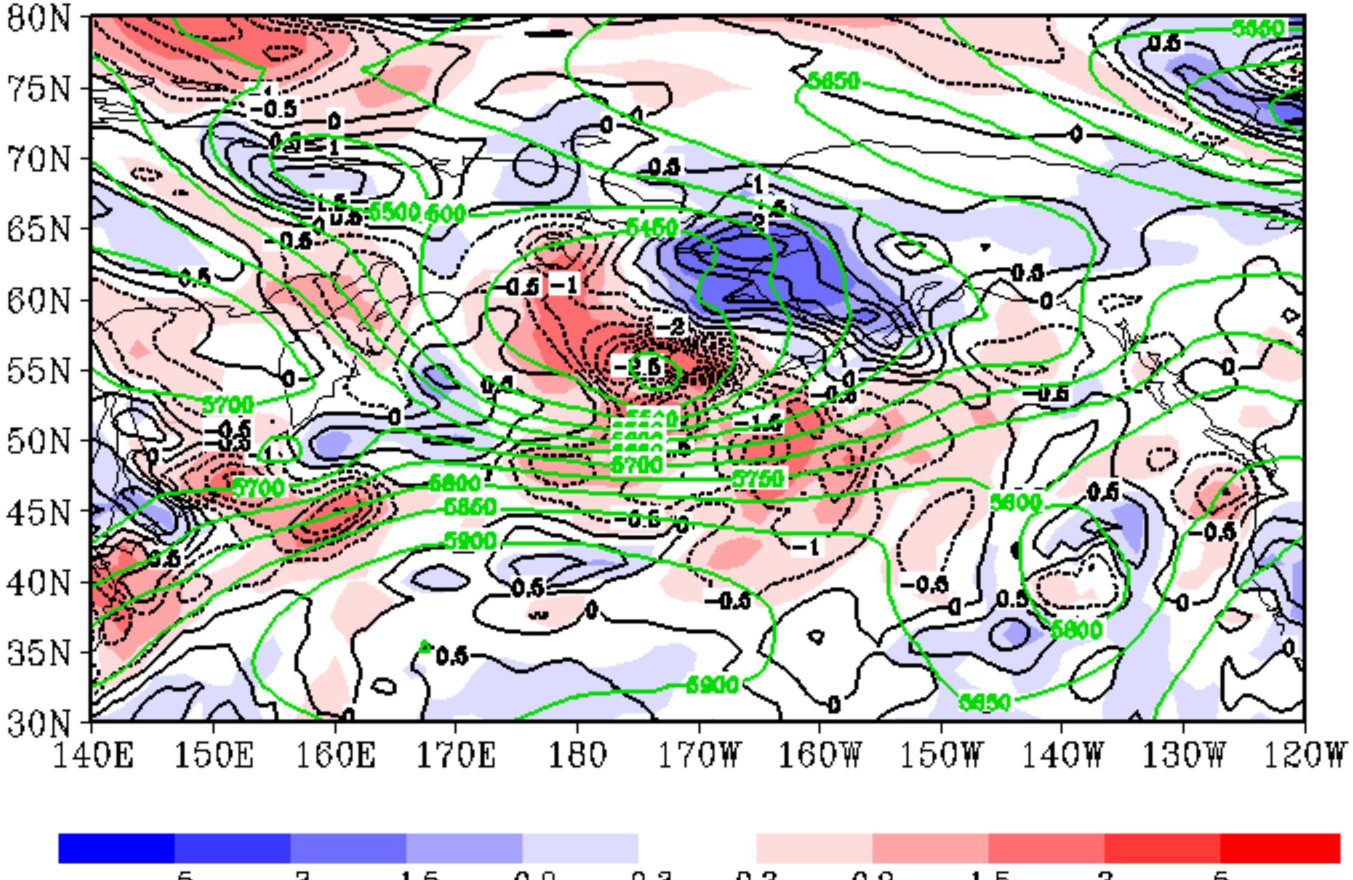
- Color shade: Optimized increments, red=plus, blue=minus.
- Green contour: 500hPa height.
- Black contour: Integrated background error, solid lines=plus, dotted lines=minus.

Optimized analysis increments and background error 072012



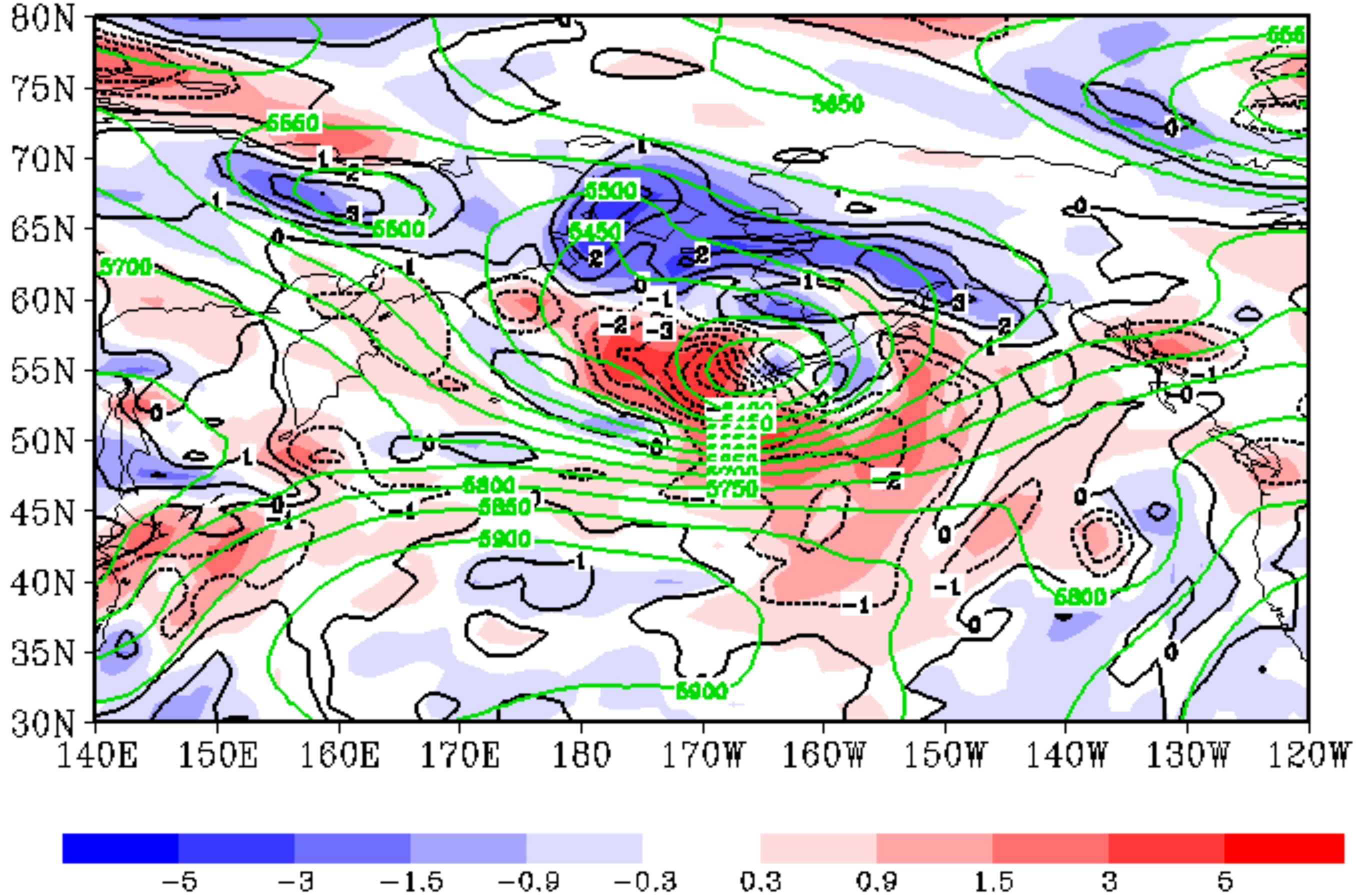
- Color shade: Optimized increments, red=plus, blue=minus.
- Green contour: 500hPa height.
- Black contour: Integrated background error, solid lines=plus, dotted lines=minus.

Optimized analysis increments and background error 072100



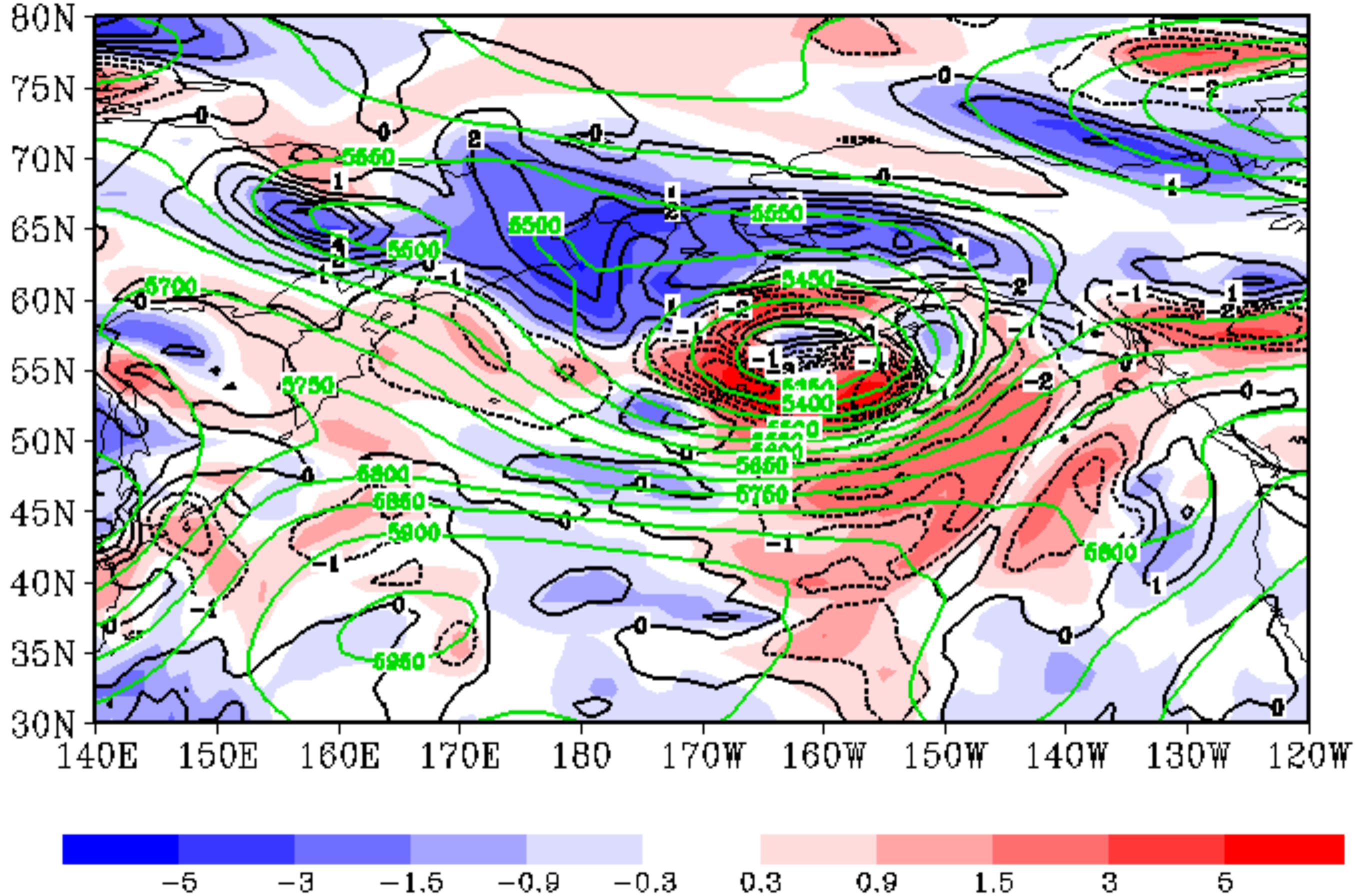
- Color shade: Optimized increments, red=plus, blue=minus.
- Green contour: 500hPa height.
- Black contour: Integrated background error, solid lines=plus, dotted lines=minus.

Optimized analysis increments and background error 072112



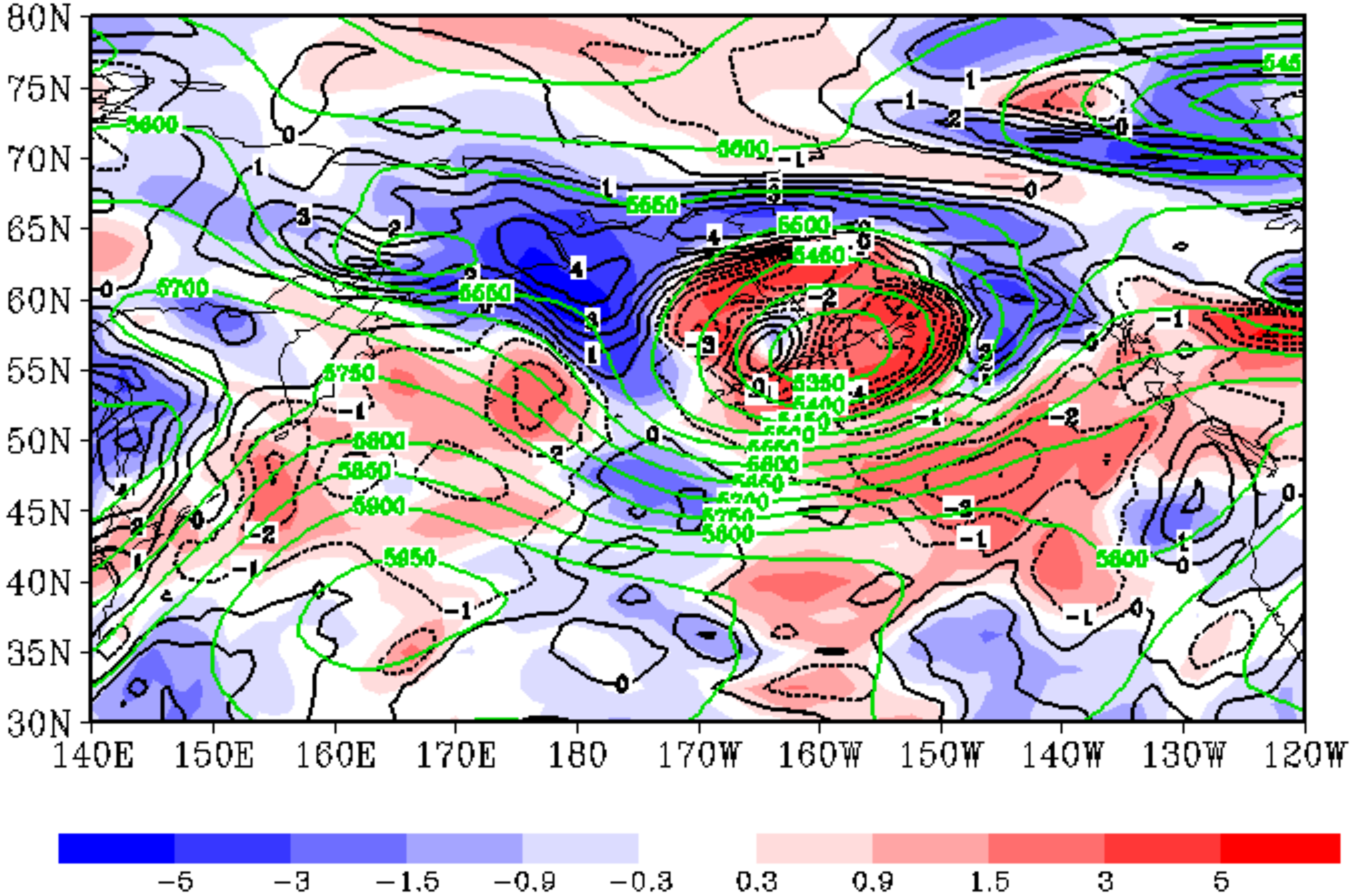
- Color shade: Optimized increments, red=plus, blue=minus.
- Green contour: 500hPa height.
- Black contour: Integrated background error, solid lines=plus, dotted lines=minus.

Optimized analysis increments and background error 072200



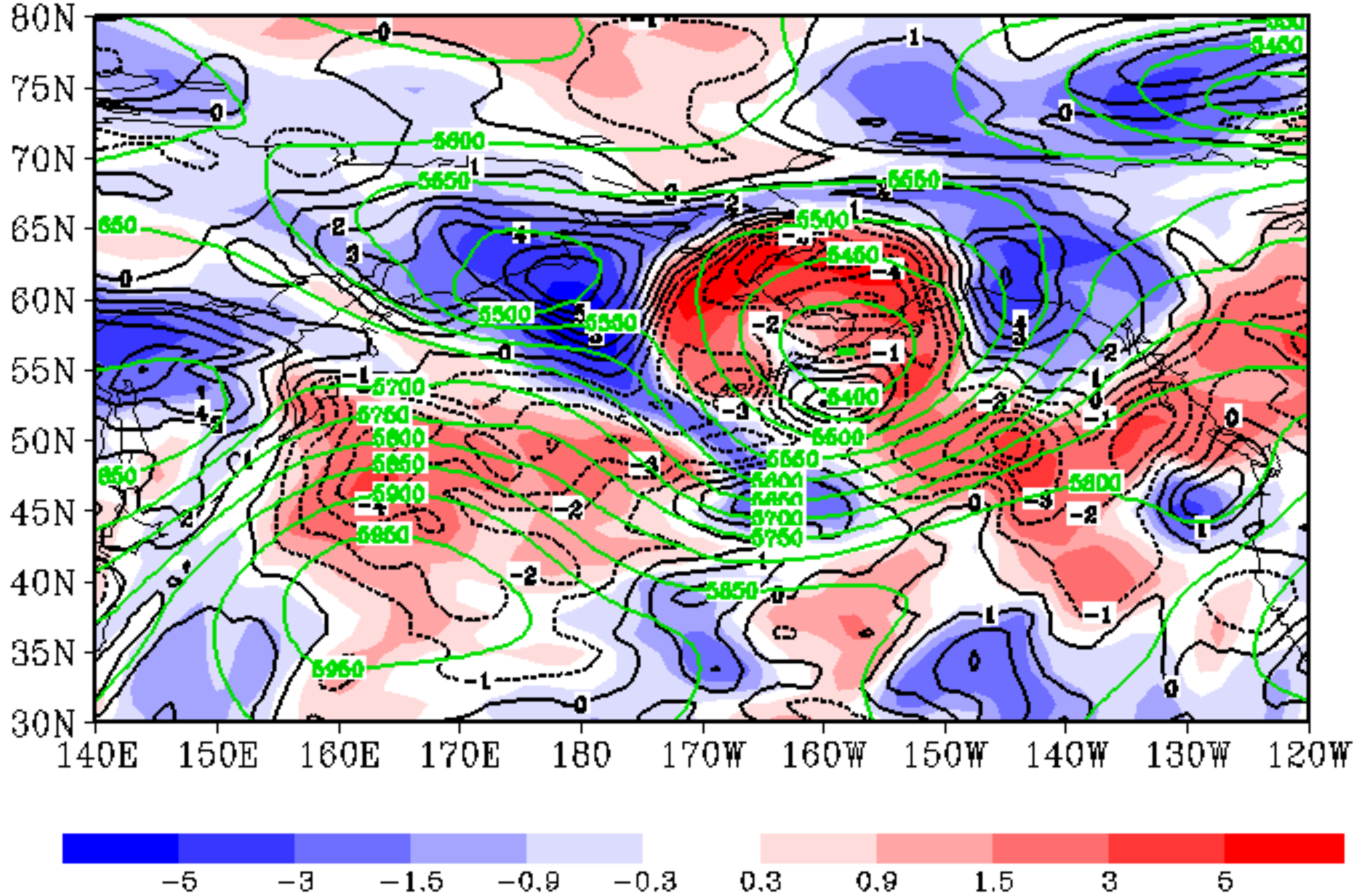
- Color shade: Optimized increments, red=plus, blue=minus.
- Green contour: 500hPa height.
- Black contour: Integrated background error, solid lines=plus, dotted lines=minus.

Optimized analysis increments and background error 072212



- Color shade: Optimized increments, red=plus, blue=minus.
- Green contour: 500hPa height.
- Black contour: Integrated background error, solid lines=plus, dotted lines=minus.

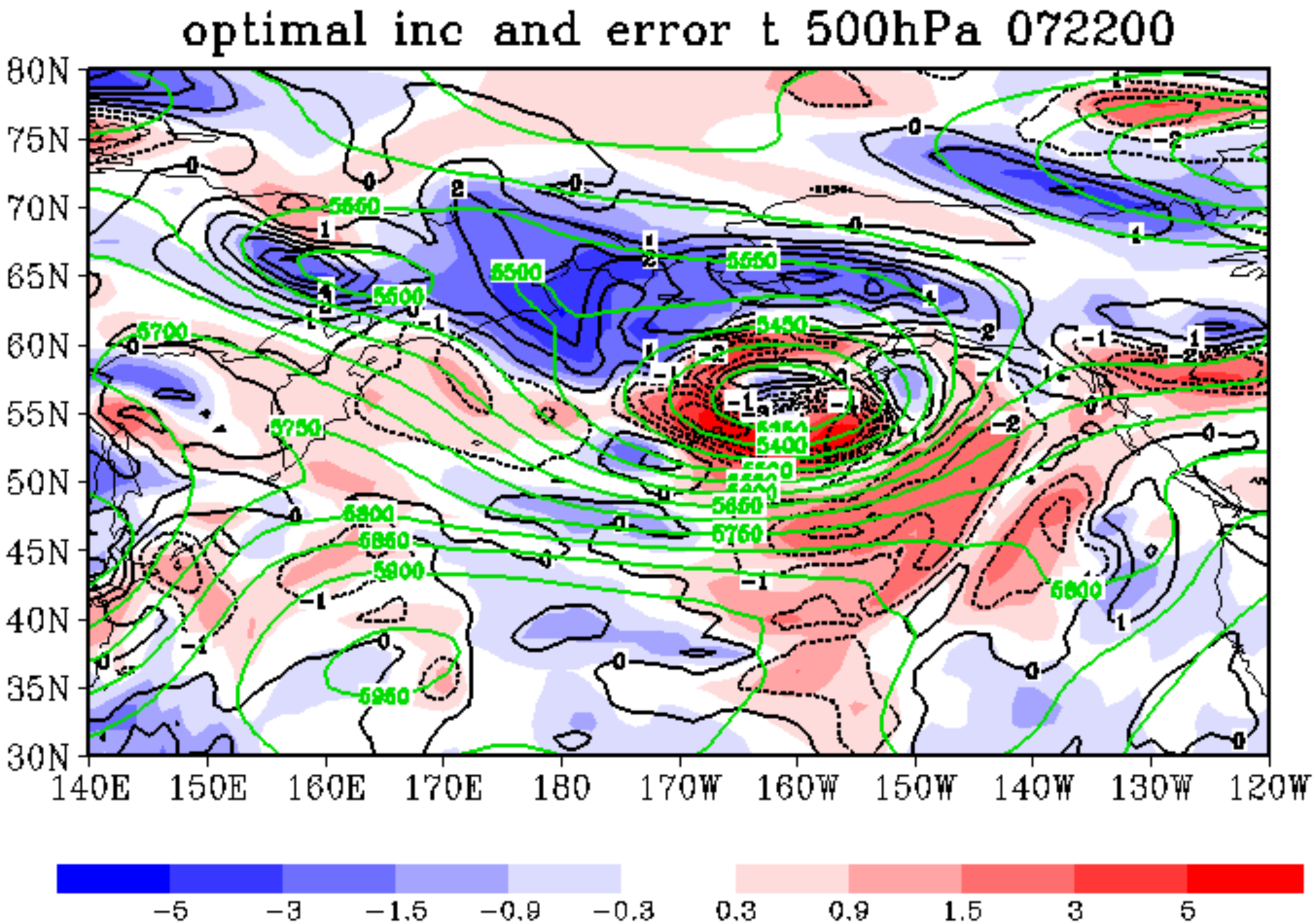
Optimized analysis increments and background error 072300



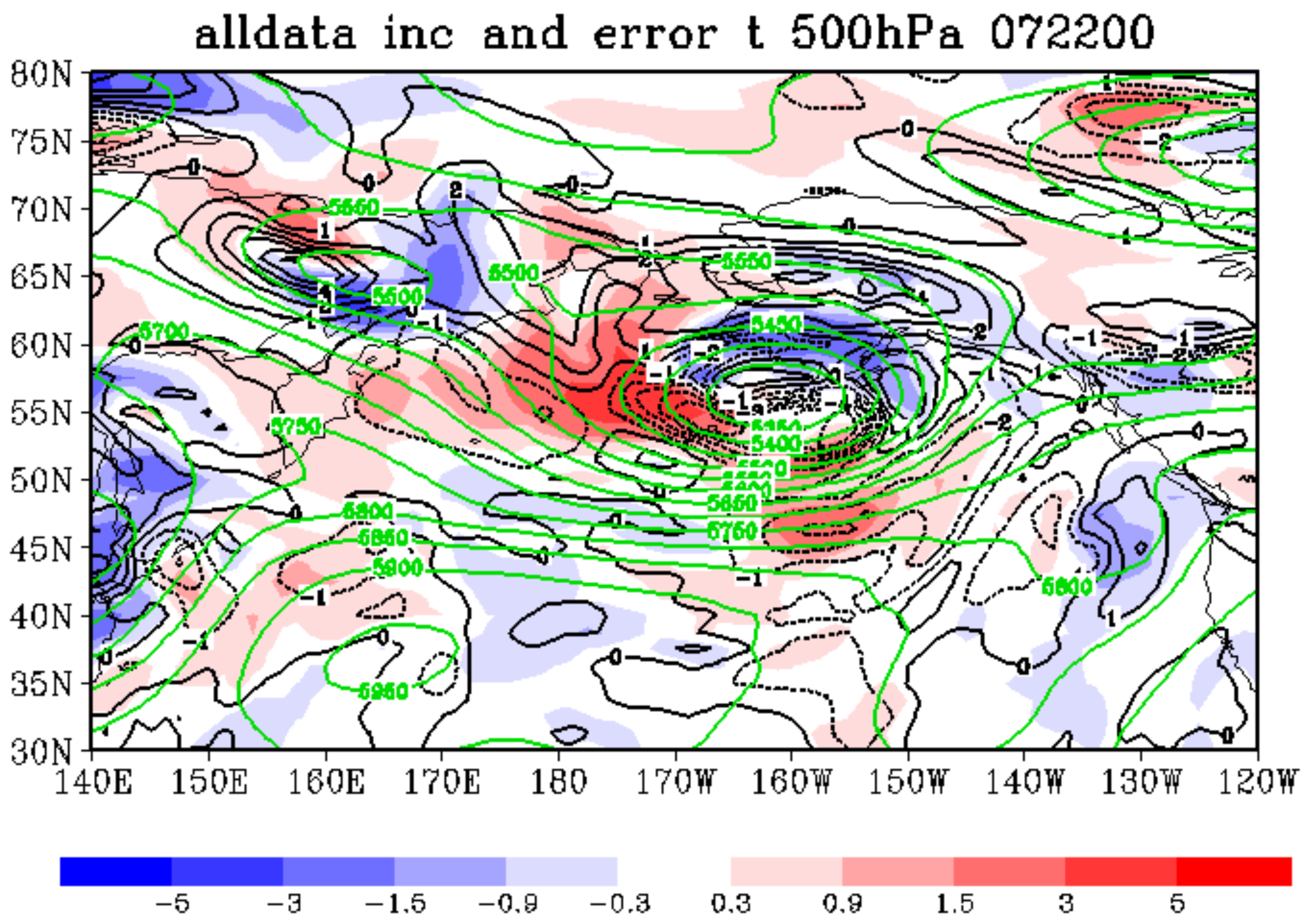
- Color shade: Optimized increments, red=plus, blue=minus.
- Green contour: 500hPa height.
- Black contour: Integrated background error, solid lines=plus, dotted lines=minus.

Comparison between original analysis and optimized analysis

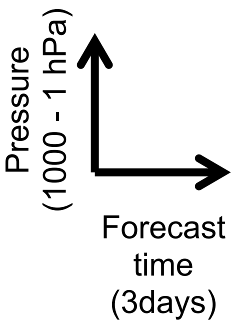
Optimized :FT48 500hPa T



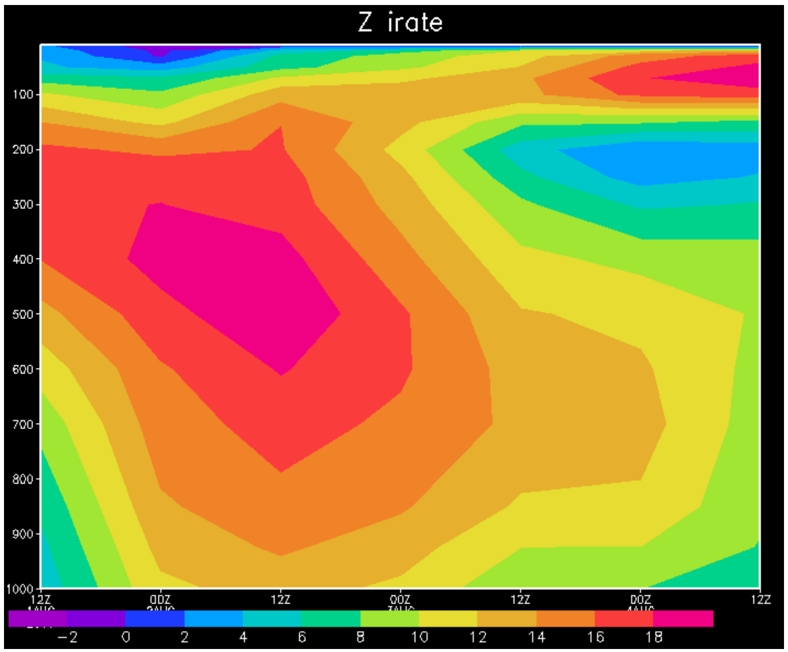
Original: FT48 500hPa T



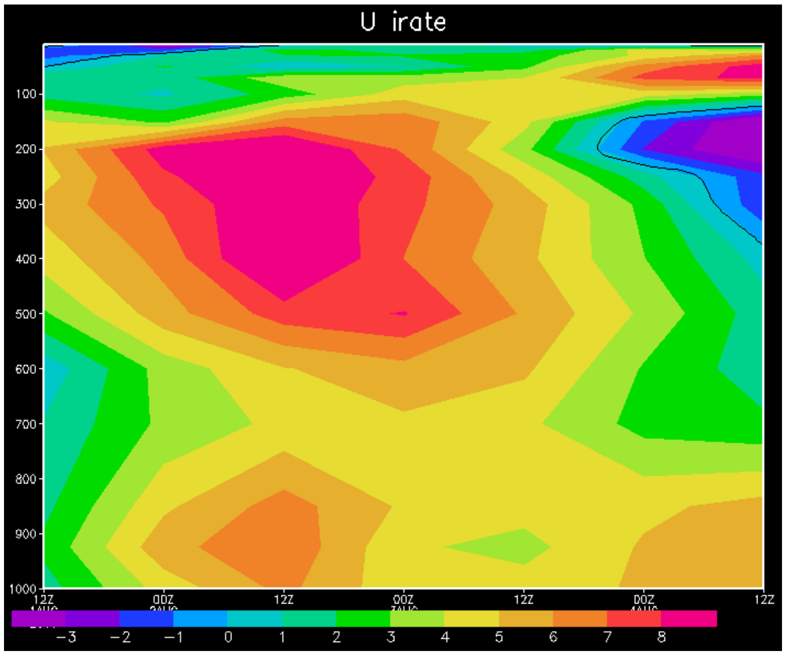
Pseudo truth with ADJ-based method

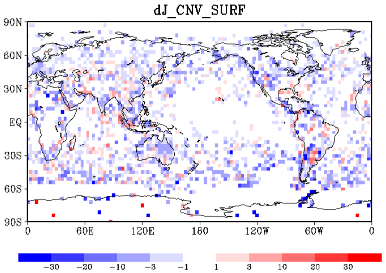


Improvement rate: Height

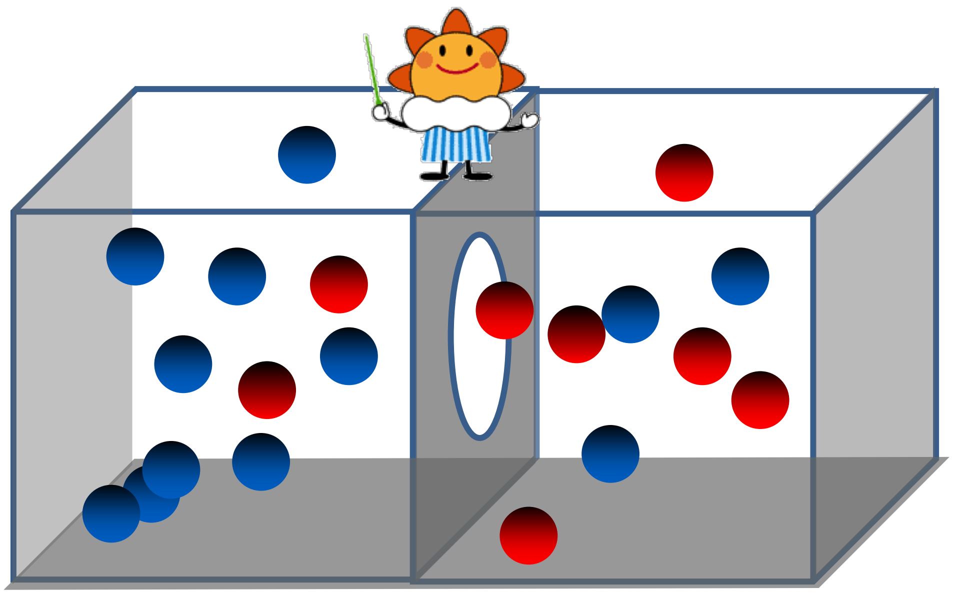


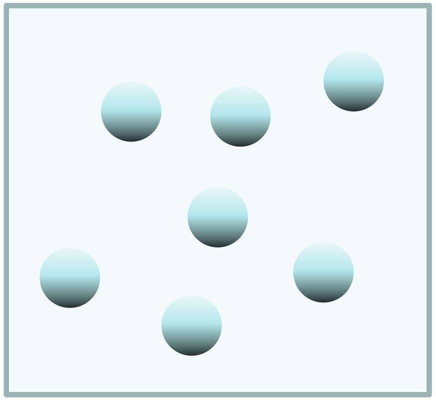
Improvement rate: Zonal wind



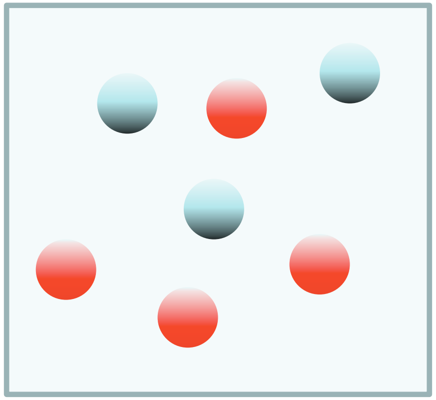


Maxwell demon ?





We know only statistical property of data, R and B.



We know property of each observation and can use this information.

Lets think about a system on thermal equilibrium at temperature T. We know only statistical property of the system, temperature T. While, if we can know velocity of each particle, we can get usable energy from this max entropy state, This is the Maxwell demon.

Summary

Observation impact

- We defined two types of observation impact; the linear impact and the non-linear impact.
- Diagnoses of the JMA global 4D-Var shows almost all observational dataset types contribute forecast error reduction in monthly averages.
- The diagnoses imply that it is possible to derive more information from radiance data by improving usage of these data.
- The TL-based method has been introduced.
- We can see time evolutions and space distribution of each dataset impact, and evaluate them by comparison with integrated background errors.

Covariance matrix optimization

- Optimization methods include observation impact estimations.
- Sensitivity based method diagnosed the JMA GDAS has too large (small) **R** (**B**).
- The single case experiment of optimization showed the explicit forecast error reductions.

Analysis error estimation

- We constructed new method based on data assimilation theory. The method assimilate reference analysis fields.
- The method reduce forecast error explicitly and also consistent with observations when inflation factor is used.

Thank you very much for kind attention.

Comments or Questions ?

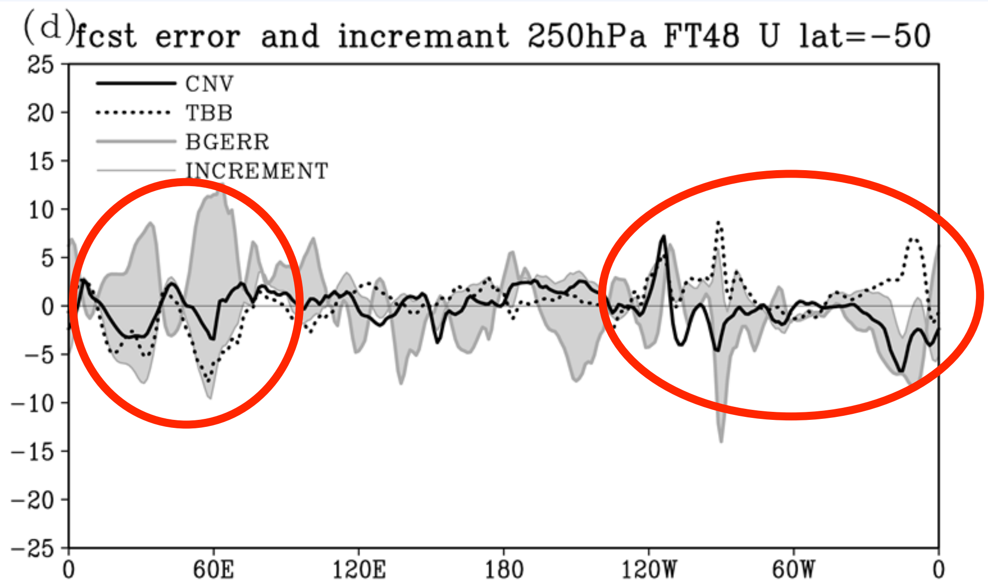
Please speak very slowly!

<u>One sentence question welcome!!</u>

More long complicated questions also welcome,
but answer will be after this session or by email.
I'm so sorry for my insufficient English.

BACKUPSLIDES

TL-based method



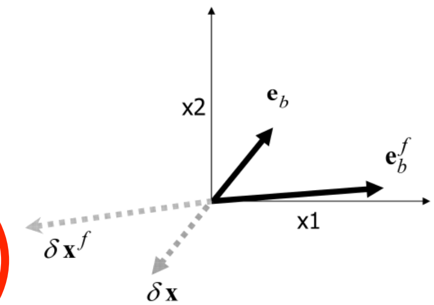


Figure 14. The comparison of the spatial structures of 48 hour integrated PIVs with that of the background error vector. The structure of the CNV-PIV (black solid line), the TBB-PIV (black dotted line), the background error (thick grey solid line), and the analysis increment of ALLDATA (thin grey solid line) are shown at specified latitudes. The areas between the background error and the analysis increment of ALLDATA are shaded with grey. The panels (a), (b), and (c) are for 500 hPa temperature (K) at 50° N, 50° S, and 0°, respectively. The panel (d) is for 250 hPa zonal wind velocity (m s^{-1}) at 50° S. The horizontal axis of the each panel is the longitude.

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TL-based method

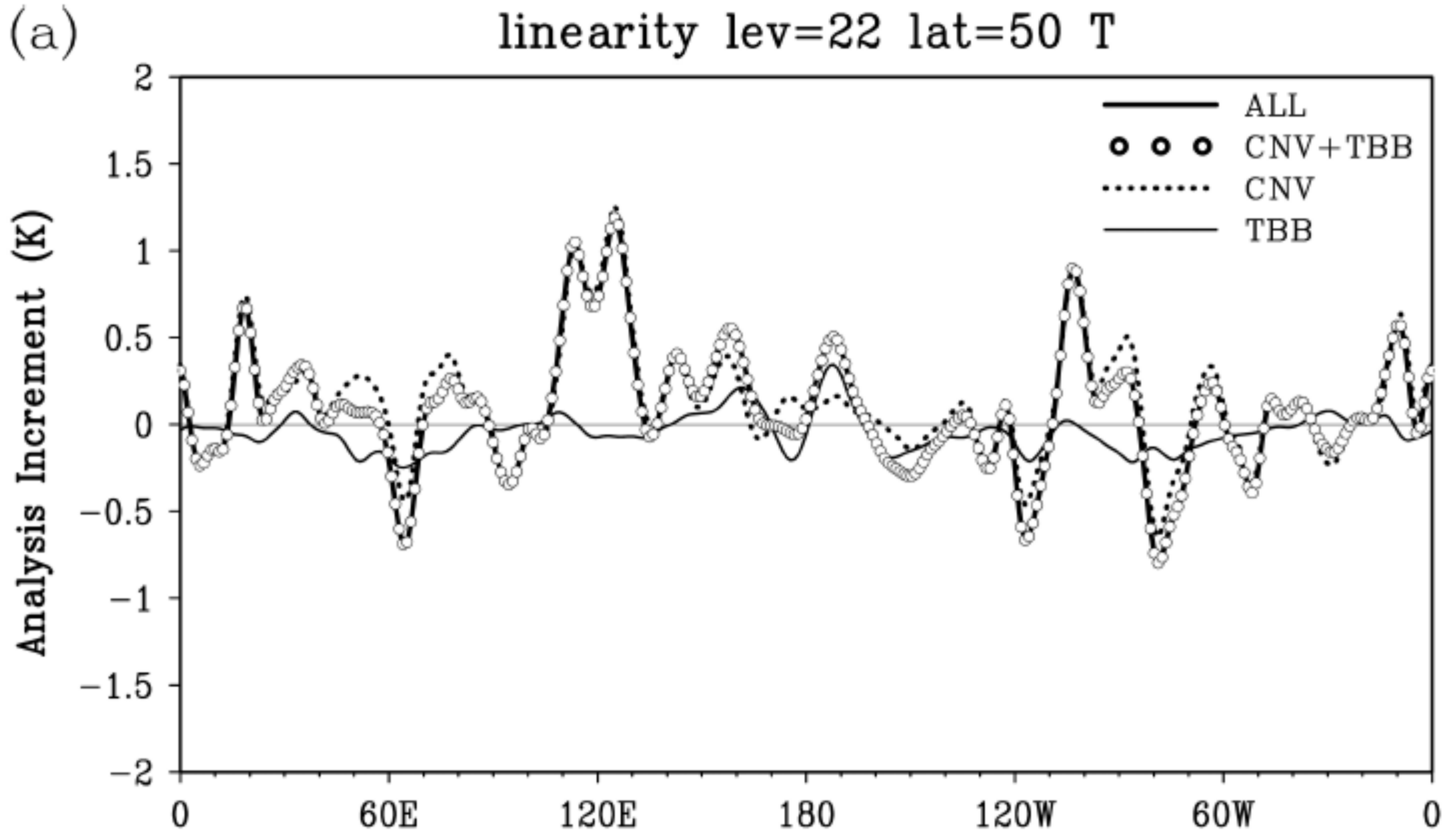

Figure 5. The linearity of the TL-based estimation for the analysis at 0000 UTC 20 July 2007. The analysis increment of the ALLDATA (thick solid line), the sum of the TBB-PIV and the CNV-PIV (open circles), the CNV-PIV (dotted line), and the TBB-PIV (thin solid line) are shown in each panel. Panels (a), (b) and (c) show these quantities for temperature at model level 22 (about 500 hPa) at 50°N, 0°N and 50°S, respectively. Panel (d) is specific humidity at the model level 12 (about 850 hPa) at 0°N. The vertical axis represents values of analysis increments and PIVs and the horizontal axis represents longitudes.

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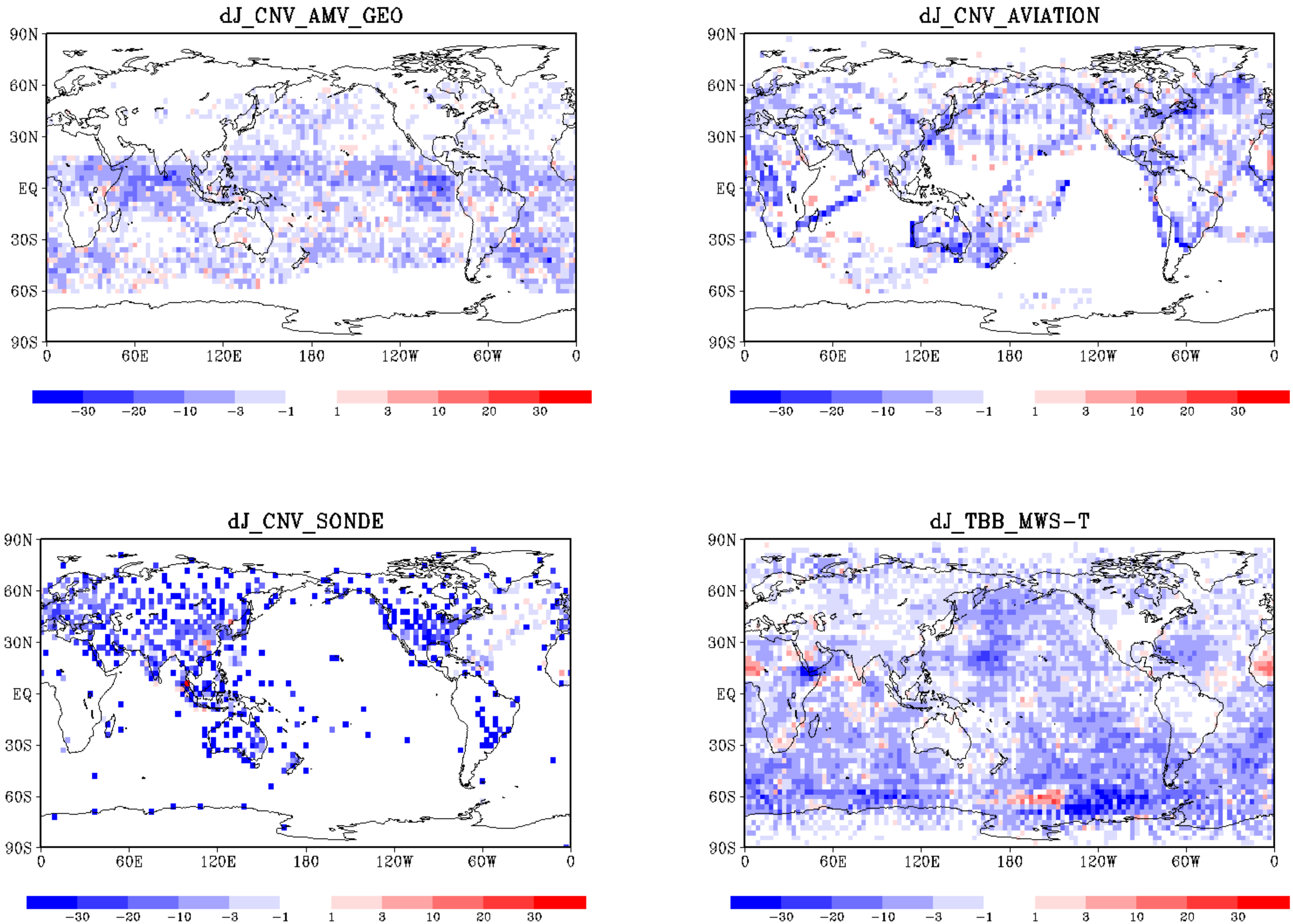
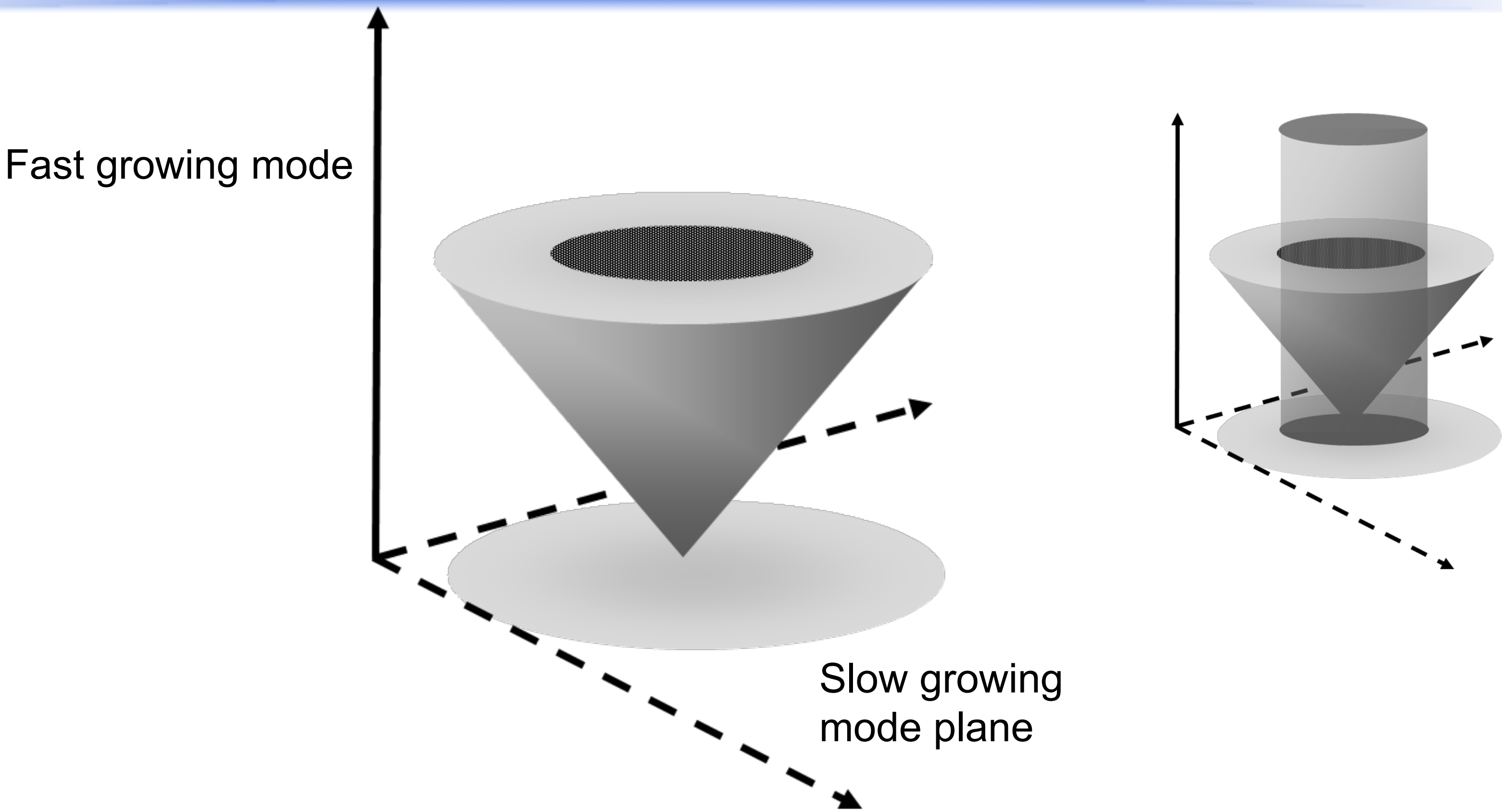
Vertically integrated contribution map



Fast and slow growing error modes



- Key analysis error based optimized analysis increment vector exist within grey corn. Precession motions are free with in the corn.
- Slow modes are constrained by observations and a background field, so constrain of these data is nearly cylinder shape.
- The both achieve narrow corn constrain.

3. Analysis error estimation

We want to know analysis errors of a DAS because of following reasons;

- If we know the analysis errors, we can start researches on what parts of the DAS causes those errors toward improvement of the DAS.
- We can start to design future observational systems, which can detect the analysis errors of current DASs.
- Since, analysis error estimation is the same with construction of more accurate analysis than current DASs. Such analyses may be able to use as "pseudo truth". The pseudo truth can be used validation of forecasts and operational analyses, and those of OSSEs. or directly use as in re-analyses.

- Our approach is

"***Collect more*** *information and integrate them correctly*"

Formulation of ADJ-based method using PIVs

$$
\begin{aligned}
\delta F &= 2\left(\delta\mathbf{x}^{\mathrm{f}*P}+\delta\mathbf{x}^{\mathrm{f}*Q}\right)^{\mathrm{T}}\mathbf{C}\mathbf{e}_{\mathrm{b}}^{\mathrm{f}} \\
&\quad+\left(\delta\mathbf{x}^{\mathrm{f}*P}+\delta\mathbf{x}^{\mathrm{f}*Q}\right)^{\mathrm{T}}\mathbf{C}\left(\delta\mathbf{x}^{\mathrm{f}*P}+\delta\mathbf{x}^{\mathrm{f}*Q}\right) \qquad (18)\\
&\equiv \delta F^{*P}+\delta F^{*Q}+\delta F^{*PQ},
\end{aligned}
$$

where we introduce the following quantities:

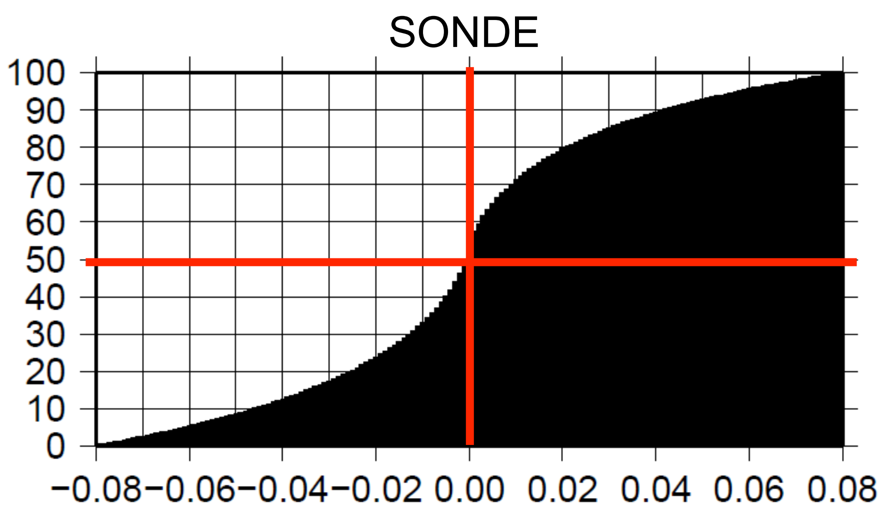
$$
\begin{aligned}
\delta F^{*P} &\equiv 2\left(\delta\mathbf{x}^{\mathrm{f}*P}\right)^{\mathrm{T}}\mathbf{C}\mathbf{e}_{\mathrm{b}}^{\mathrm{f}}+\left(\delta\mathbf{x}^{\mathrm{f}*P}\right)^{\mathrm{T}}\mathbf{C}\delta\mathbf{x}^{\mathrm{f}*P},\\
\delta F^{*Q} &\equiv 2\left(\delta\mathbf{x}^{\mathrm{f}*Q}\right)^{\mathrm{T}}\mathbf{C}\mathbf{e}_{\mathrm{b}}^{\mathrm{f}}+\left(\delta\mathbf{x}^{\mathrm{f}*Q}\right)^{\mathrm{T}}\mathbf{C}\delta\mathbf{x}^{\mathrm{f}*Q},\\
\delta F^{*PQ} &\equiv \left(\delta\mathbf{x}^{\mathrm{f}*P}\right)^{\mathrm{T}}\mathbf{C}\delta\mathbf{x}^{\mathrm{f}*Q}+\left(\delta\mathbf{x}^{\mathrm{f}*Q}\right)^{\mathrm{T}}\mathbf{C}\delta\mathbf{x}^{\mathrm{f}*P}.
\end{aligned}
$$

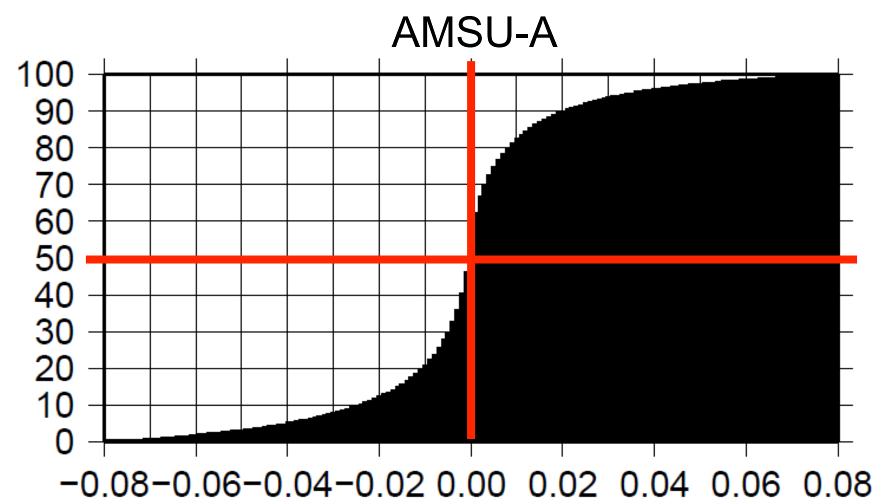
$$\mathbf{B}-\mathbf{A}=\mathbf{KHB}$$

$$
\mathbf{KHB}=-E\left\{\begin{array}{l}\mathbf{e}_{\mathrm{b}}\left(\delta\mathbf{x}^{*P}+\delta\mathbf{x}^{*Q}\right)^{\mathrm{T}}+\left(\delta\mathbf{x}^{*P}+\delta\mathbf{x}^{*Q}\right)\mathbf{e}_{\mathrm{b}}^{\mathrm{T}}\\ \quad+\left(\delta\mathbf{x}^{*P}+\delta\mathbf{x}^{*Q}\right)\left(\delta\mathbf{x}^{*P}+\delta\mathbf{x}^{*Q}\right)^{\mathrm{T}}\end{array}\right\}, \qquad (21)
$$

Ishibashi (2011) QJRMS

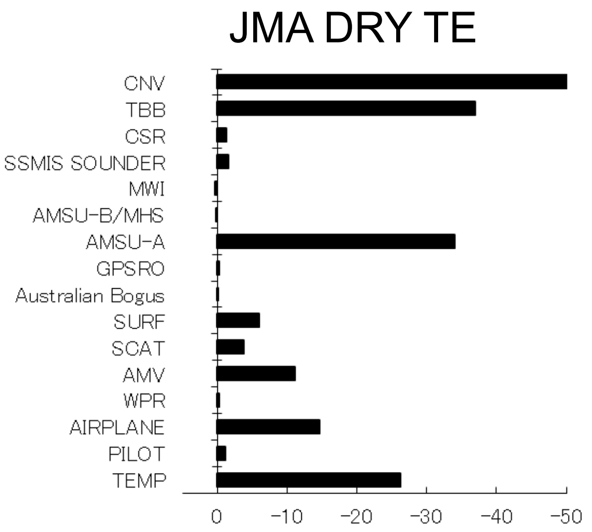
PDF

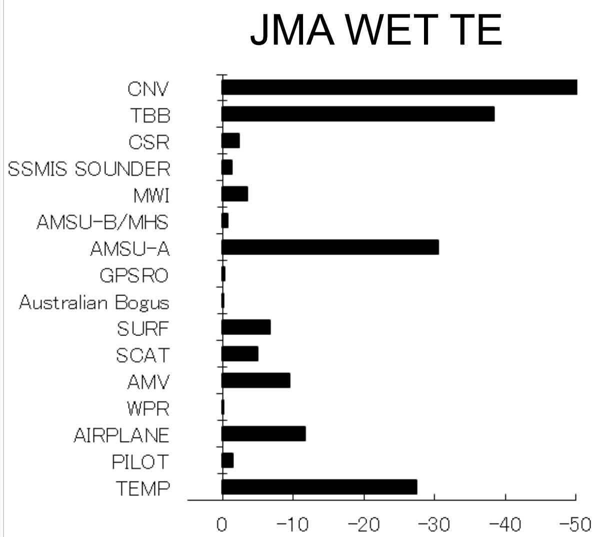


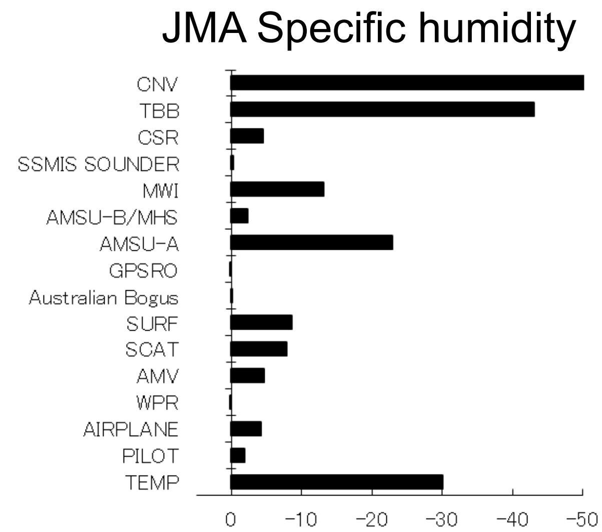


- About a half observations decrease forecast errors, and the other increase them.
- The slope of PDF is different between these data. SONDE is shallower than AMSU-A.
- We can guess that the gradient of the slope depends on **R** and **B** settings and real information amount of data.

Norm dependency





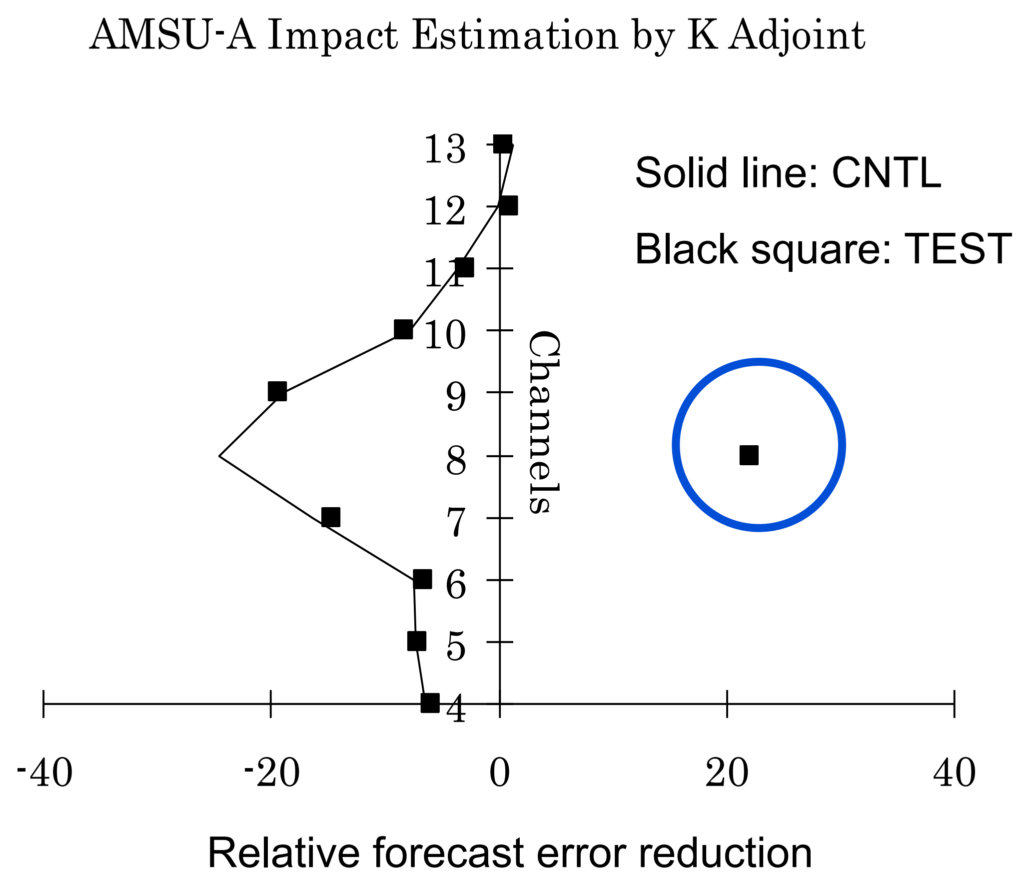


- Impacts of humidity sensor (MWI, AMSUB/MHS, and CSR) become clear when we use WET TE or Q norm than DRY TE norm.
- Impacts of humidity non sensitive data (AMSU-A, AMV, etc) are still large in the case of WET and Q norm.

Wrong data detection

- This figures shows impacts of AMSU-A/METOP2 in two NWP systems, TEST and CNTL.
  - CNTL uses adequate observation error settings (operational settings).
  - TEST uses inadequately small observation error setting for channel eight.

- The AD-based method can detect wrong observations which degenerate forecast accuracy.



TL-based method

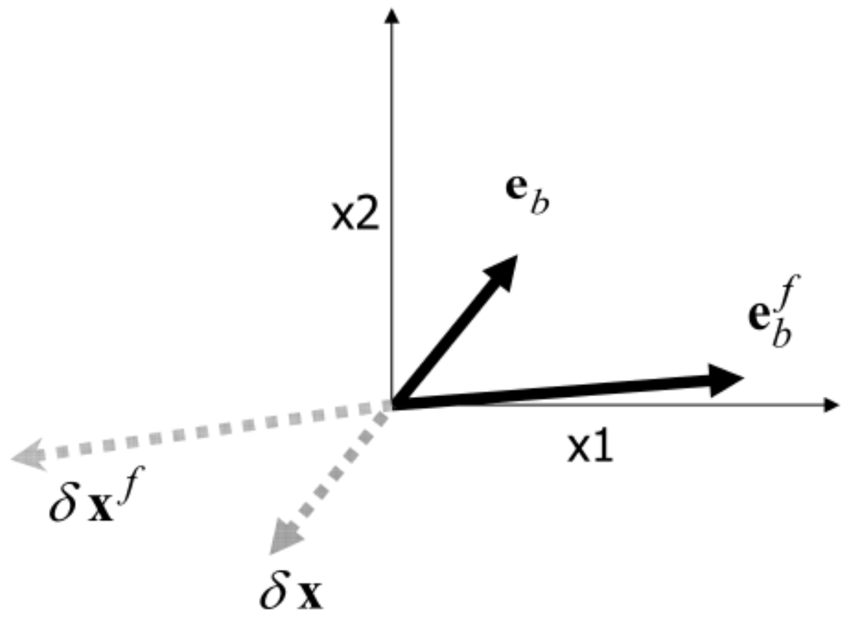
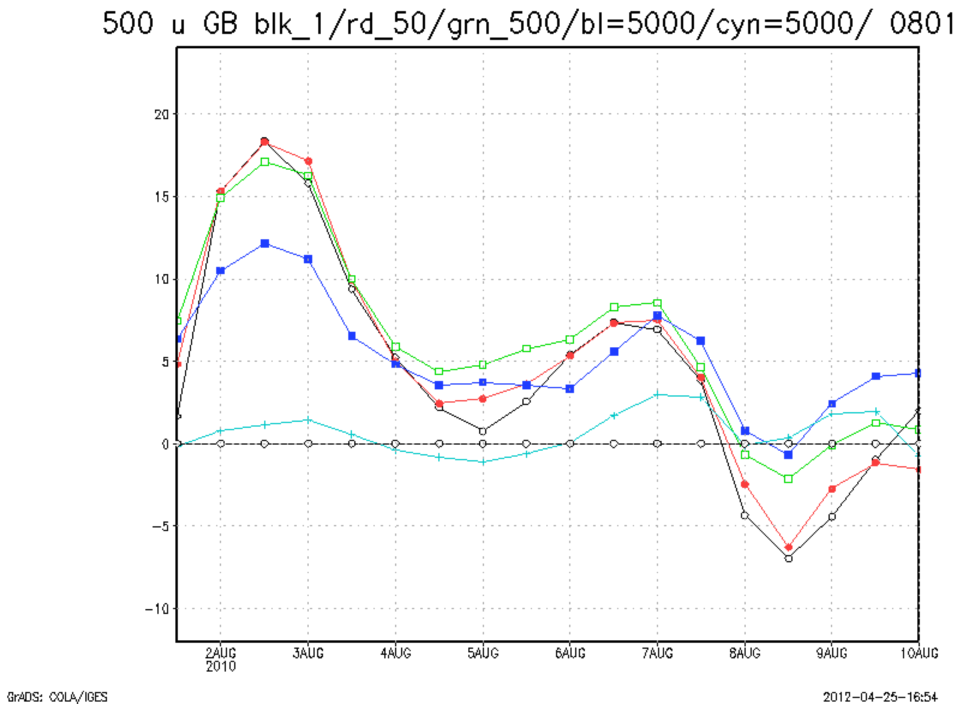
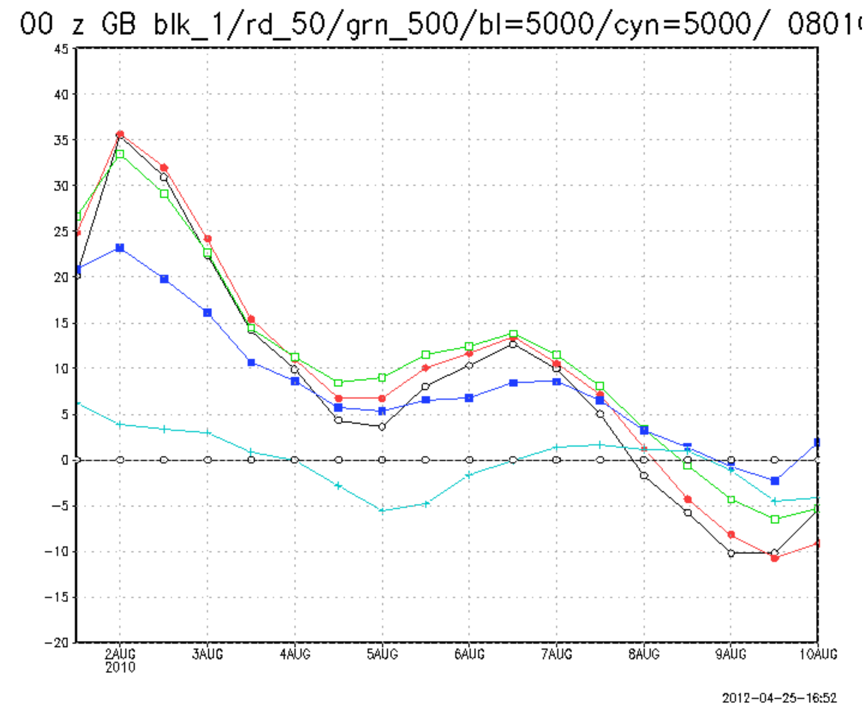


Figure 3. Schematic plot of an analysis increment vector $\delta\mathbf{x}$, the background error $\mathbf{e}_\mathrm{b}$, and the corresponding vectors $\delta\mathbf{x}^\mathrm{f}$ and $\mathbf{e}_\mathrm{b}^\mathrm{f}$, integrated using the forecast model. The plot is shown in a two-dimensional phase space.

Inflation and forecast accuracy





- Inflation factors smaller than 500 have almost same forecast accuracy during fast 4days, and after 5days, larger inflation factors have smaller forecast errors,
- Inflation 5000 has smaller improvement during 4days, however, smaller errors after 5days.

Variances of functions of analyses

$$\delta x_a(\mathbf{x}_b,\ \mathbf{y},\ \mathbf{B},\ \mathbf{R})$$
$$= (\partial x_a/\partial \mathbf{x}_b)\delta\mathbf{x}_b + (\partial x_a/\partial \mathbf{y})\delta\mathbf{y} + (\partial x_a/\partial \mathbf{B})\delta\mathbf{B} + (\partial x_a/\partial \mathbf{R})\delta\mathbf{R}$$

$$\delta F(x_a) = \partial F/\partial x_a\ \delta x_a$$
$$= \partial F/\partial x_a \left\{(\partial x_a/\partial \mathbf{x}_b)\delta\mathbf{x}_b + (\partial x_a/\partial \mathbf{y})\delta\mathbf{y} + (\partial x_a/\partial \mathbf{B})\delta\mathbf{B} + (\partial x_a/\partial \mathbf{R})\delta\mathbf{R}\right\}$$
$$= (\partial F/\partial \mathbf{x}_b)\delta\mathbf{x}_b + (\partial F/\partial \mathbf{y})\delta\mathbf{y} + (\partial F/\partial \mathbf{B})\delta\mathbf{B} + (\partial F/\partial \mathbf{R})\delta\mathbf{R}$$

- In linear analyses, we consider four independent variables, a background field **x**b , observations **y**, a background error covariance matrix **B**, and an observation error covariance matrix **R**.
- We can evaluate impacts of each variable.

AD-based estimation

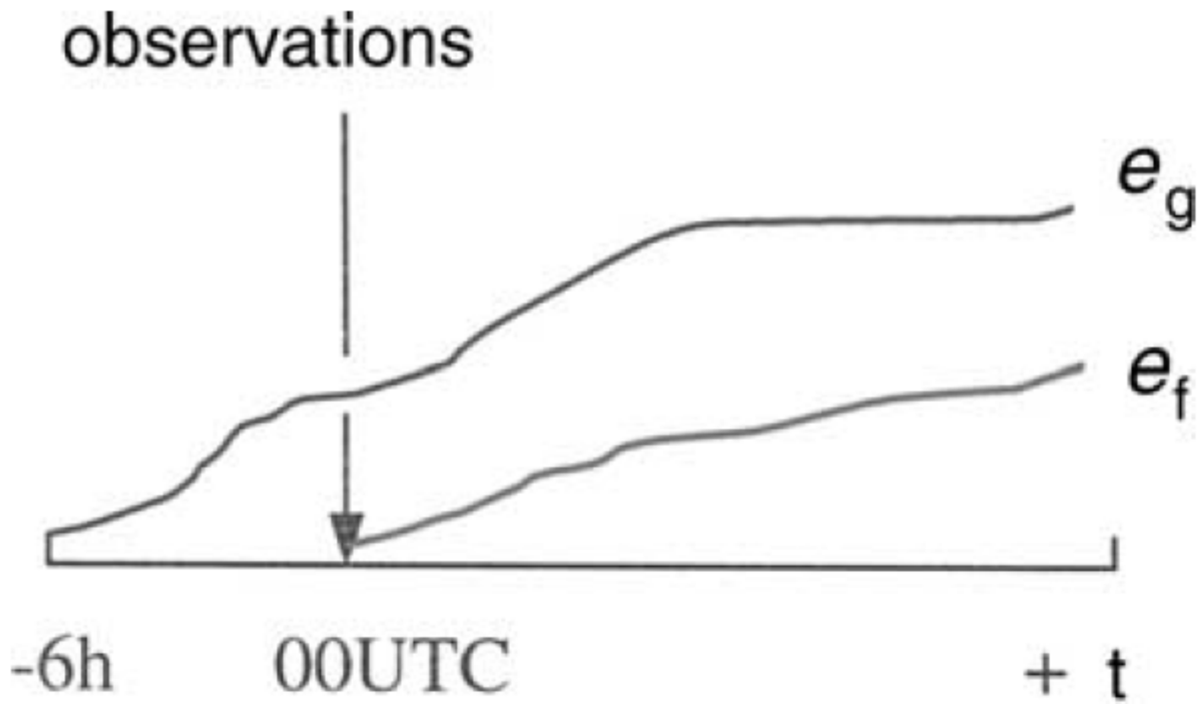


Fig 1. Observations are assimilated at 00UTC, creating initial conditions for a new trajectory, which has forecast error e_f. The old trajectory starts from 18UTC (−6 h), and has forecast error e_g. It also provides the background for the analysis at 00UTC. Both forecasts verify at time t, where t equals the length of forecast f. The difference between the errors $e_f - e_g$ is due solely to assimilation of observations.

Langland and Baker (2004) Tellus

AD-based estimation

The formulation of AD-based method under TL approximation:

- Definition of a scalar function *F* of a forecast error vector:

$$F \equiv \mathbf{e}^{f^T} \mathbf{C} \ \mathbf{e}^f$$

- Variation of *F* caused by an analysis

$$\delta F \cong 2(\mathbf{MKd})^T \mathbf{C}\,\mathbf{Me}_b + (\mathbf{M}\delta\mathbf{x})^T \mathbf{C}(\mathbf{M}\delta\mathbf{x})$$

$$= \mathbf{d}^T \left\{ 2\mathbf{K}^T\mathbf{M}^T\mathbf{C}\mathbf{M}\mathbf{e}_b + \mathbf{K}^T\mathbf{M}^T\mathbf{C}\mathbf{M}\delta\mathbf{x} \right\} \equiv \mathbf{d}^T\mathbf{g}$$

Each term is a inner product between the first-order quantities of TL approximation.

C: norm define matrix

M: tangent linear model

e^f**:** forecast error vector

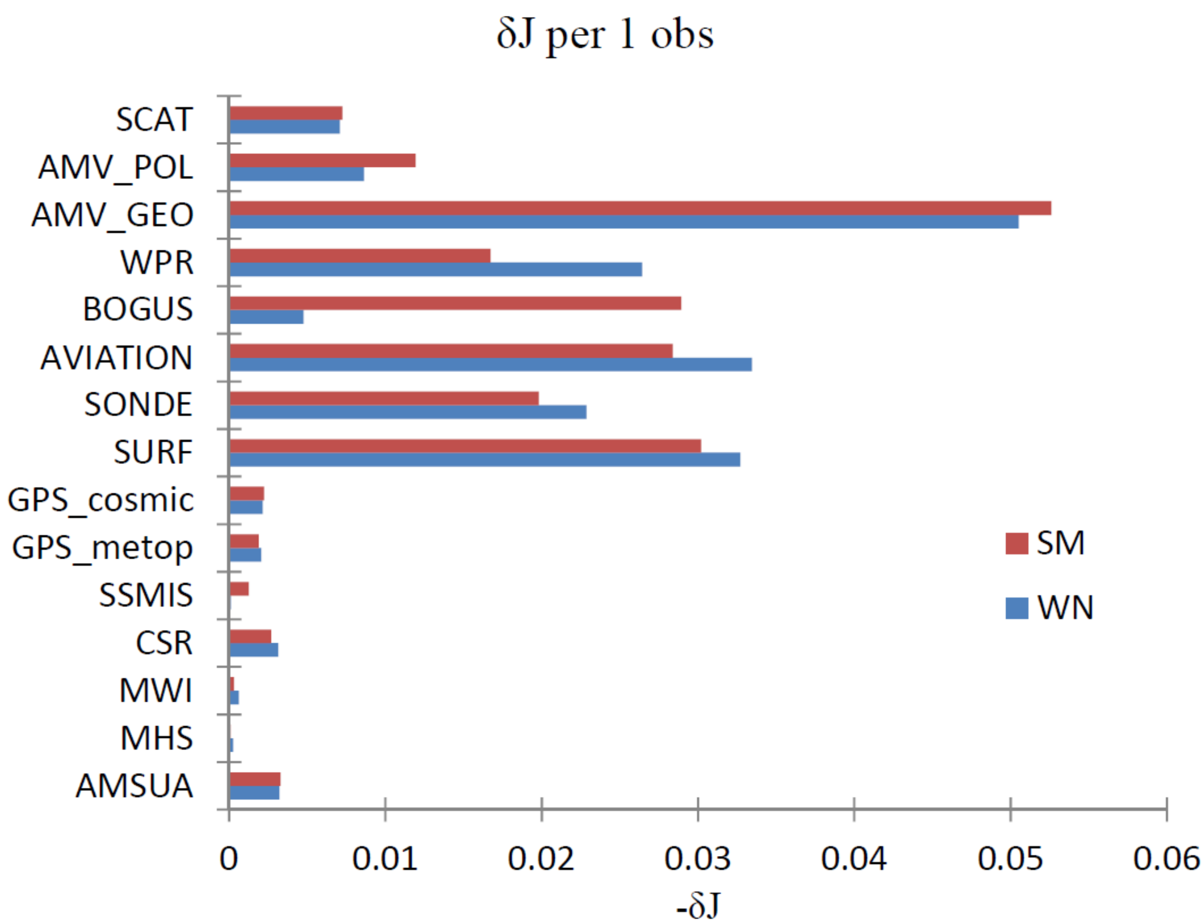
e$_b$**:** background error

δ**x:** analysis increment

- For example, a linear observation impact of a dataset P is calculated as follows;

$$\delta F_P \equiv \sum_{r \in P} g_r \ d_r$$

AD-based estimation in JMA global 4D-Var



TL-based method

$$
\begin{aligned}
\delta x_i &= \sum_{j,r\in Z,s\in Z} \boxed{B_{i,j}H_{r,j}(D)^{-1}_{r,s}}\, d_s \\
&= \sum_{j,r\in P,s\in P} B_{i,j}H_{r,j}(D)^{-1}_{r,s} d_s \qquad \cdots termPP \\
&+ \sum_{j,r\in Q,s\in Q} \boxed{B_{i,j}H_{r,j}(D)^{-1}_{r,s}}\, d_s \qquad \cdots termQQ \\
&+ \sum_{j,r\in Q,s\in P} B_{i,j}H_{r,j}(D)^{-1}_{r,s} d_s \qquad \cdots termQP \\
&+ \sum_{j,r\in P,s\in Q} B_{i,j}H_{r,j}(D)^{-1}_{r,s} d_s \qquad \cdots termPQ, \qquad (14)
\end{aligned}
$$

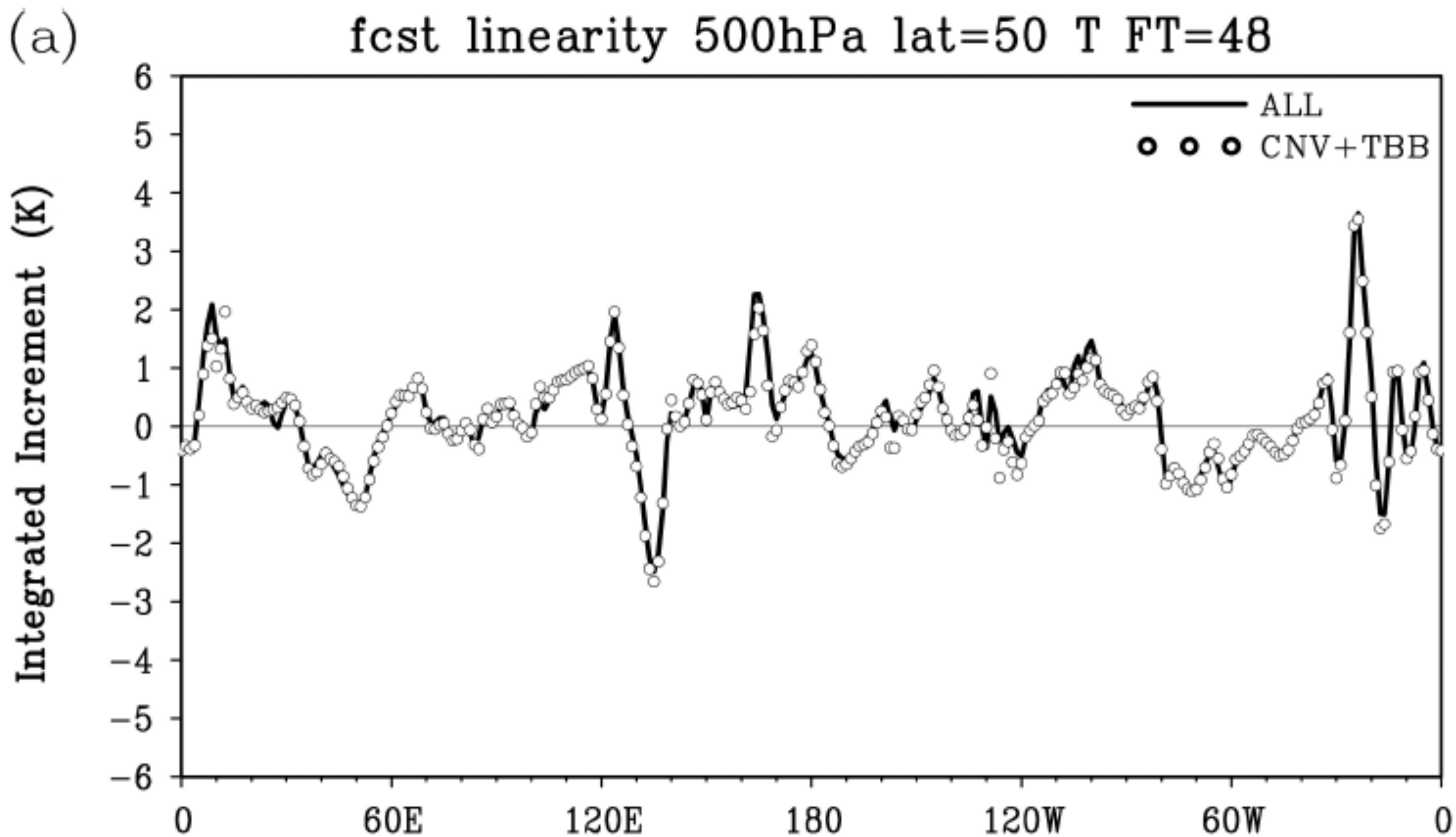
K

K'

Only the case that **D** is block diagonal.

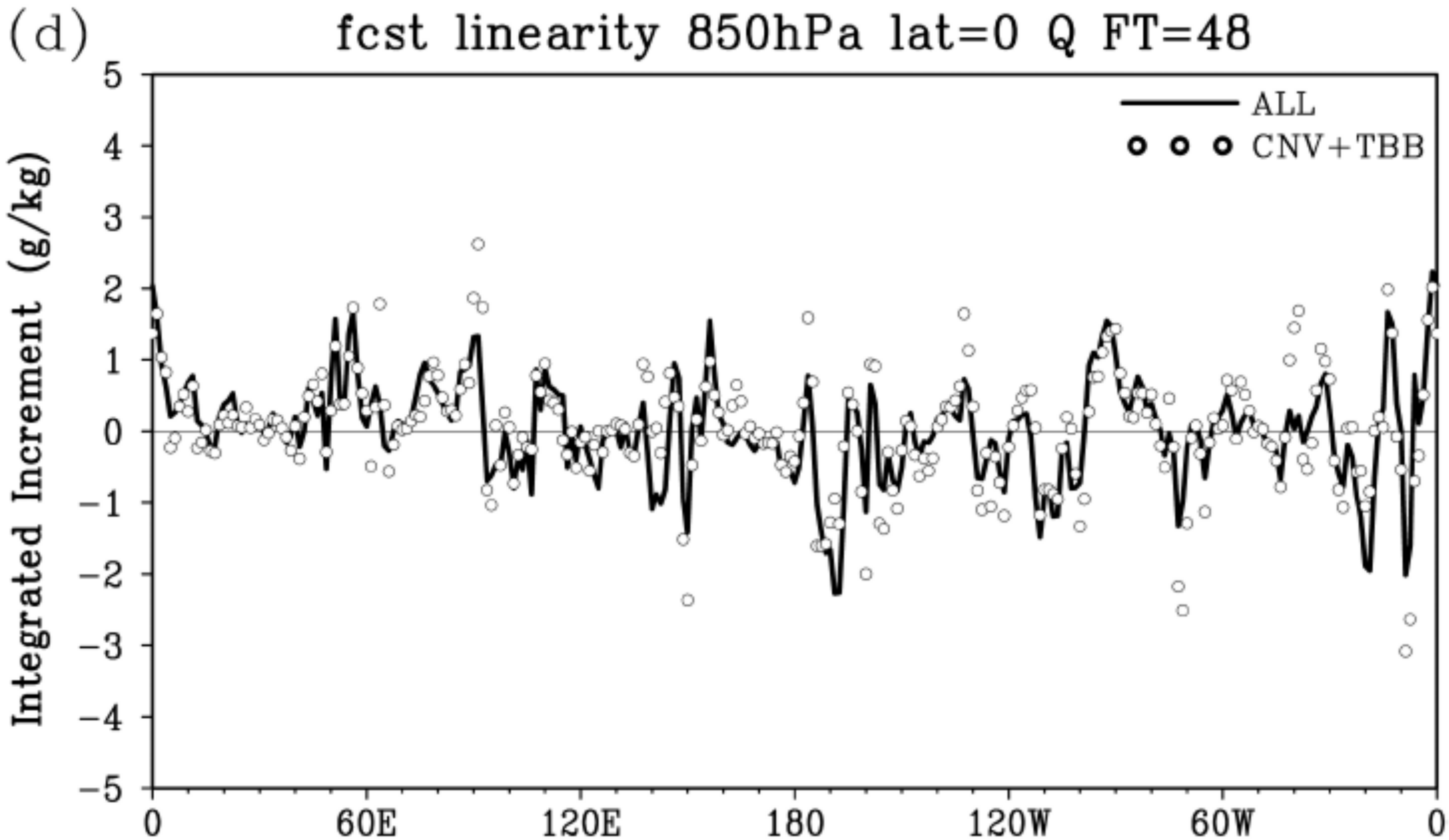
Ishibashi (2011) QJRMS

TL-based method



Ishibashi (2011) QJRMS

TL-based method



Ishibashi (2011) QJRMS

TL-based method

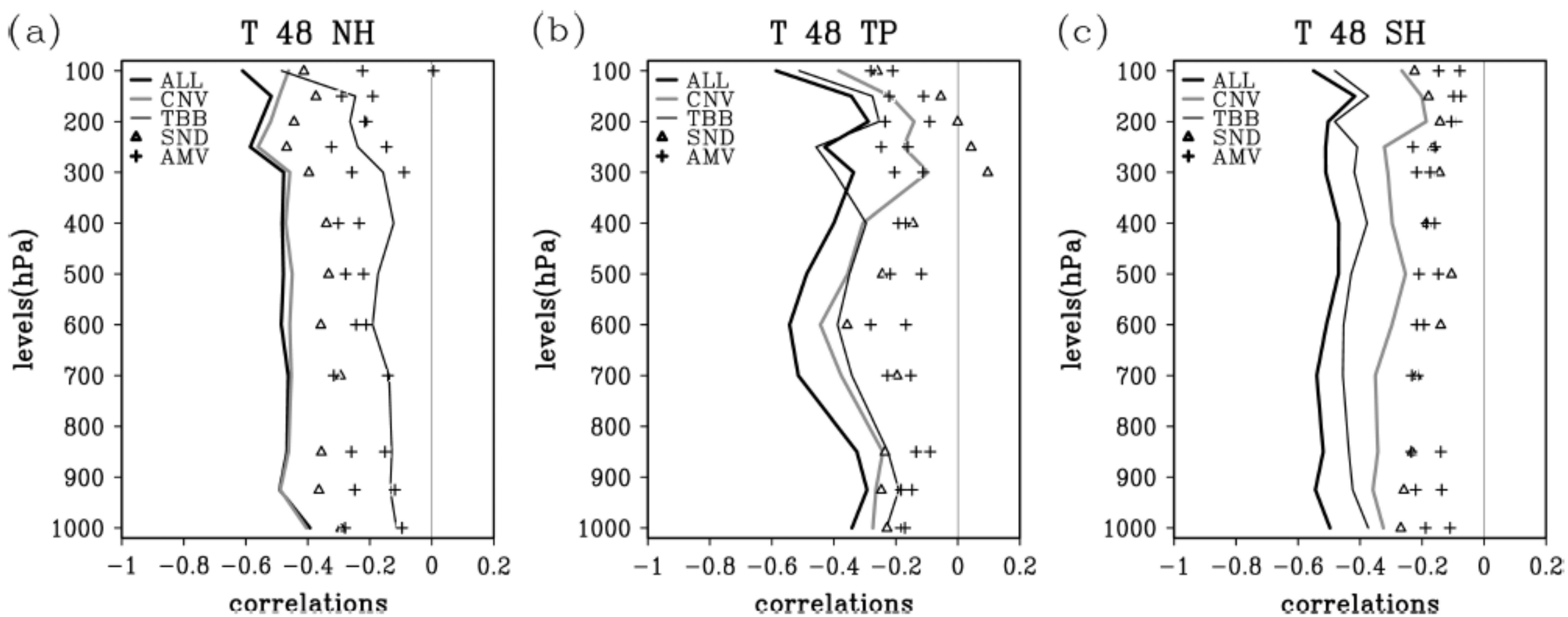
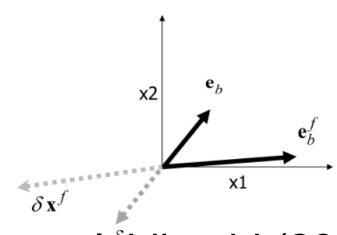


Figure 13. The trends of anti-parallel relationships between the 48-hour integrated PIVs and the background error vector. The values of $\cos\theta$ are shown for ALLDATA (black thick solid line), CNV (grey solid line), TBB (black thin solid lone), SONDE (triangles), and AMV (pluses). Here, θ is the angle between the integrated background error vector and each the integrated PIVs. Panels (a), (b) and (c) are the NH, the Tropics and the SH, respectively.



Ishibashi (2011) QJRMS

Formulation of AD-based method using Taylor series

$$\delta e = \sum_i \delta x_i^0 \left(\frac{\partial e}{\partial x_i^0} + \frac{1}{2} \sum_j \frac{\partial^2 e}{\partial x_i^0 \partial x_j^0} \, \delta x_j^0 \right.$$

$$\left. + \frac{1}{6} \sum_{j,k} \frac{\partial^3 e}{\partial x_i^0 \partial x_j^0 \partial x_k^0} \, \delta x_k^0 \, \delta x_j^0 + \cdots \right).$$

Errico (2007) Tellus

$$\delta e_1 = 2(\delta \mathbf{y})^{\mathrm{T}} \mathbf{K}^{\mathrm{T}} \mathbf{M}^{\mathrm{T}} \mathbf{C}(\mathbf{x}^f - \mathbf{x}^t)$$

$$\delta e_2 = (\delta \mathbf{y})^{\mathrm{T}} \mathbf{K}^{\mathrm{T}} \left[\mathbf{M}_b^{\mathrm{T}} \mathbf{C} \left(\mathbf{x}_a^f - \mathbf{x}^t \right) + \mathbf{M}_a^{\mathrm{T}} \mathbf{C} \left(\mathbf{x}_b^f - \mathbf{x}^t \right) \right]$$

$$\delta e_3 = (\delta \mathbf{y})^{\mathrm{T}} \mathbf{K}^{\mathrm{T}} \left[\mathbf{M}_b^{\mathrm{T}} \mathbf{C} \left(\mathbf{x}_b^f - \mathbf{x}^t \right) + \mathbf{M}_a^{\mathrm{T}} \mathbf{C} \left(\mathbf{x}_a^f - \mathbf{x}^t \right) \right]$$

$$+ \text{term involving } \frac{\partial^3 \mathbf{m}}{\partial \mathbf{x}^3},$$

Cross terms

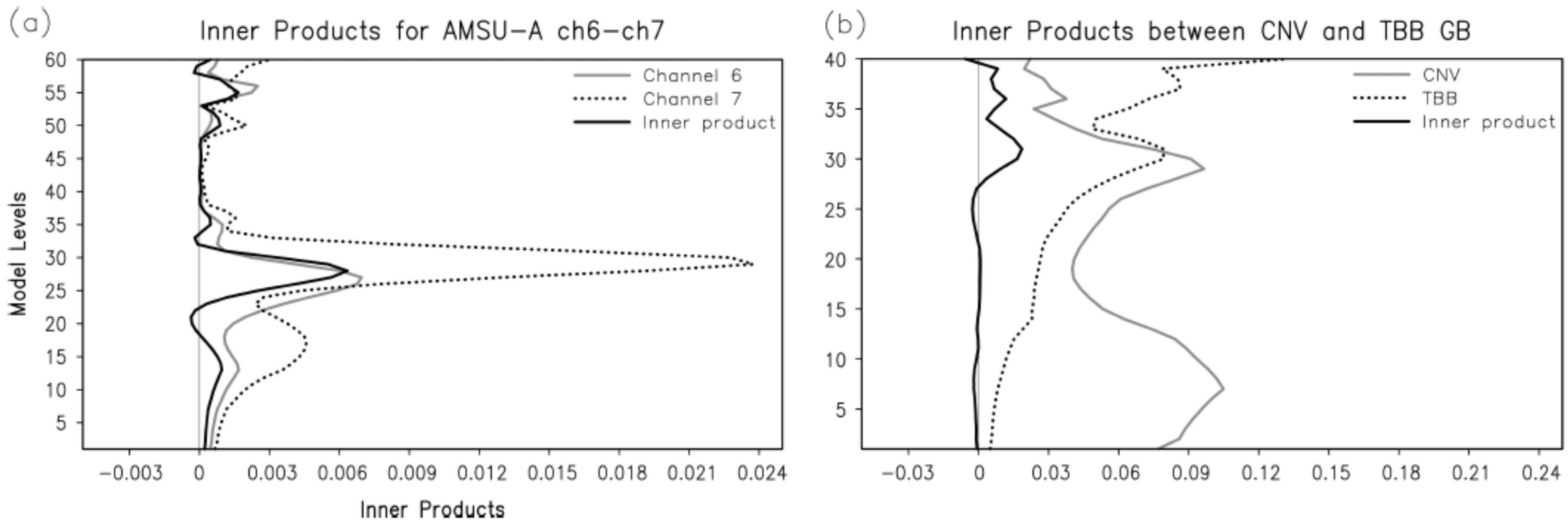


Figure 8. The inner products between PIVs in the global region. Panel (a) shows the inner product between the A6- PIV and itself (grey solid line), that of the A7-PIV (dotted line), and the inner product between the A6- PIV and the A7-PIV (black solid line). Panel (b) shows the inner product between the CNV- PIV and itself (grey solid line), that of the TBB-PIV (dotted line) and the inner product between the CNV- PIV and the TBB-PIV (black solid line). All PIVs are for temperature (K).

Ishibashi (2011) QJRMS

Sensitivity on covariance matrices

$$\partial J / \partial \mathbf{R}$$

$$\partial F/\partial R_{p,q} = \left(\partial F/\partial x_i\right)\left(\partial x_i/\partial R_{p,q}\right)$$

$$\begin{aligned}
\partial x_i/\partial R_{p,q} &= \partial K_{i,r} d_r/\partial R_{p,q} = d_r \partial K_{i,r}/\partial R_{p,q} \\
&= d_r \partial\left(B_{i,j} H_{s,j} D^{-1}_{s,r}\right)/\partial R_{p,q} \\
&= d_r B_{i,j} H_{s,j}\, \partial D^{-1}_{s,r}/\partial R_{p,q} \\
&= d_r B_{i,j} H_{s,j}\left(\partial D^{-1}_{s,r}/\partial D_{t,u}\right)\left(\partial D_{t,u}/\partial R_{p,q}\right) \\
&= -d_r B_{i,j} H_{s,j} D^{-1}_{s,t} D^{-1}_{u,r} \delta_{t,p} \delta_{u,q} \\
&= -d_r B_{i,j} H_{s,j} D^{-1}_{s,p} D^{-1}_{q,r} \\
&= -d_r K_{i,p} D^{-1}_{q,r}
\end{aligned}$$

$$\begin{aligned}
\partial F/\partial R_{p,q} &= -\left(\partial F/\partial x_i\right)\left(K_{i,p} D^{-1}_{q,r} d_r\right) \\
&= -\left(\partial F/\partial x_i\right)\left(\partial x_i/\partial d_p\right)\left(D^{-1}_{q,r} d_r\right) \\
&= -\left(\partial F/\partial d_p\right)\left(D^{-1}_{q,r} d_r\right) \\
&= \left(\partial F/\partial d_p\right) R^{-1}_{q,s}\left(H\delta x - d\right)_s
\end{aligned}$$

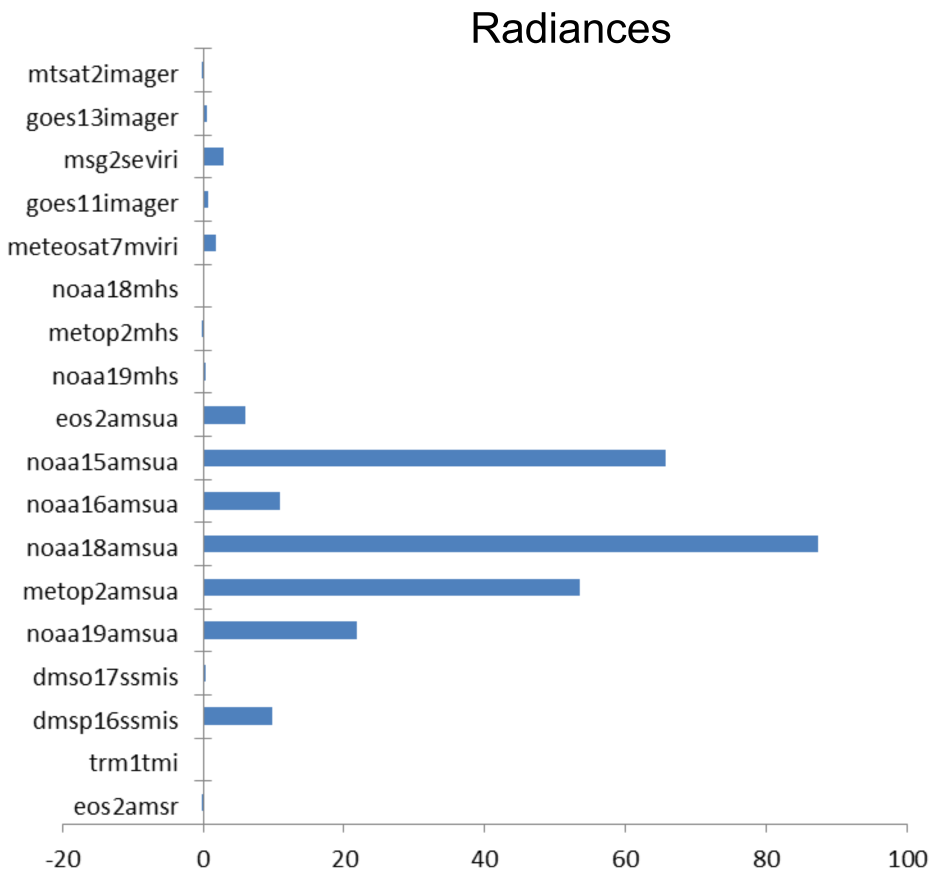
$$\partial J / \partial \mathbf{B}$$

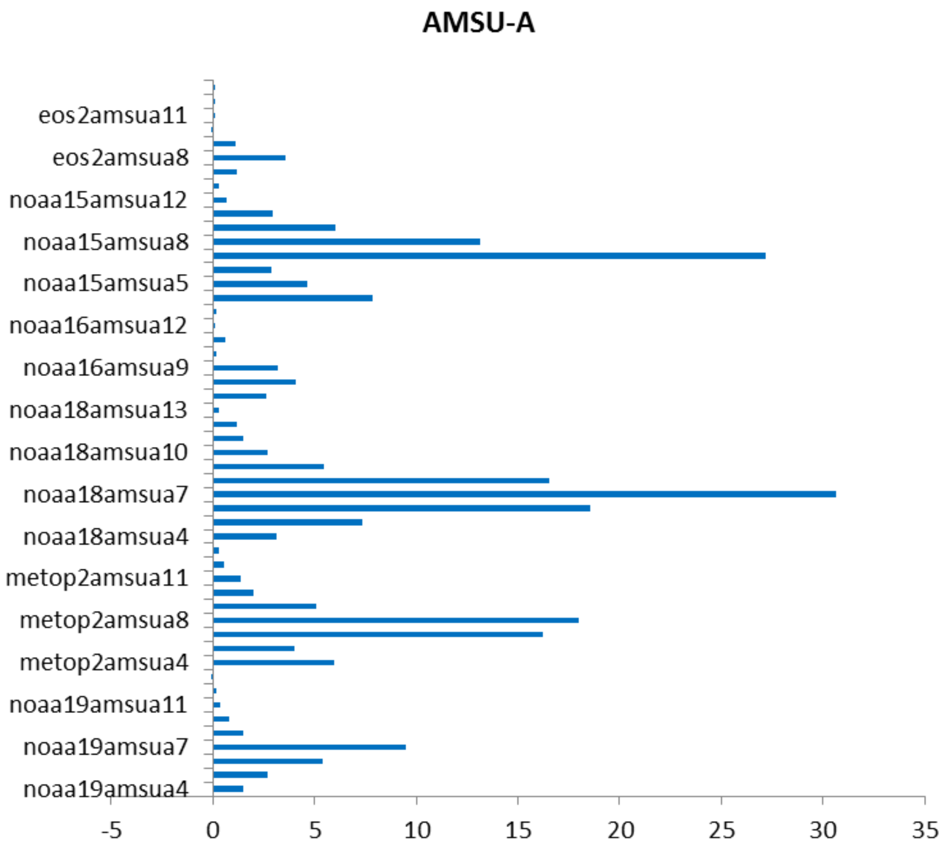
$$\partial J_t/\partial B_{k,l} = \left(\partial J_t/\partial x_i\right)\left(\partial x_i/\partial B_{k,l}\right)$$

$$\begin{aligned}
\partial x_i/\partial B_{k,l} &= \partial K_{i,r} d_r/\partial B_{k,l} = d_r \partial K_{i,r}/\partial B_{k,l} \\
&= d_r \partial\left(A^{-1}_{i,j} H_{p,j} R^{-1}_{p,r}\right)/\partial B_{k,l} \\
&= d_r H_{p,j} R^{-1}_{p,r}\, \partial A^{-1}_{i,j}/\partial B_{k,l} \\
&= d_r H_{p,j} R^{-1}_{p,r}\left(\partial A^{-1}_{i,j}/\partial A_{a,b}\right)\left(\partial A_{a,b}/\partial B_{k,l}\right) \\
&= d_r H_{p,j} R^{-1}_{p,r} A^{-1}_{i,a} A^{-1}_{b,j}\left(\partial B^{-1}_{a,b}/\partial B_{k,l}\right) \\
&= d_r H_{p,j} R^{-1}_{p,r} A^{-1}_{i,a} A^{-1}_{b,j} B^{-1}_{a,k} B^{-1}_{l,b} \\
&= d_r K_{b,r} A^{-1}_{i,a} B^{-1}_{a,k} B^{-1}_{l,b} \\
&= \left(B^{-1}_{l,b} K_{b,r} d_r\right)\left(A^{-1}_{i,a} B^{-1}_{a,k}\right)
\end{aligned}$$

$$\partial J_t/\partial B_{k,l} = \left(B^{-1}_{l,b} K_{b,r} d_r\right)\left(A^{-1}_{i,a} B^{-1}_{a,k}\right)\left(\partial J_t/\partial x_i\right)$$

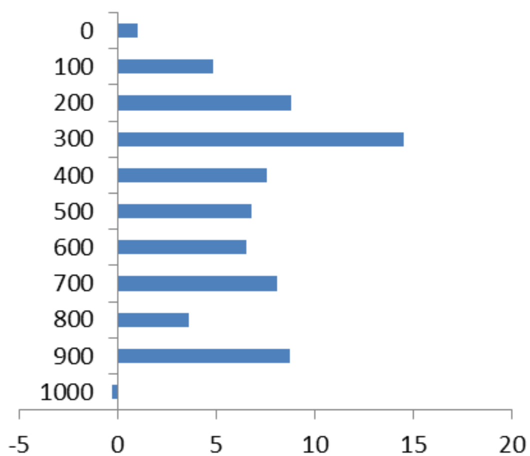
Diagnoses of the JMA global 4D-Var



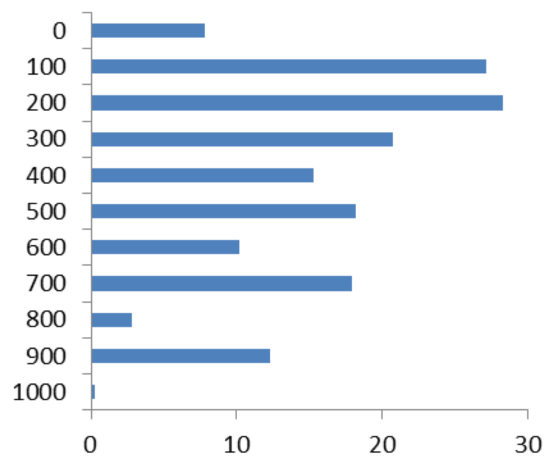


Diagnoses of the JMA global 4D-Var

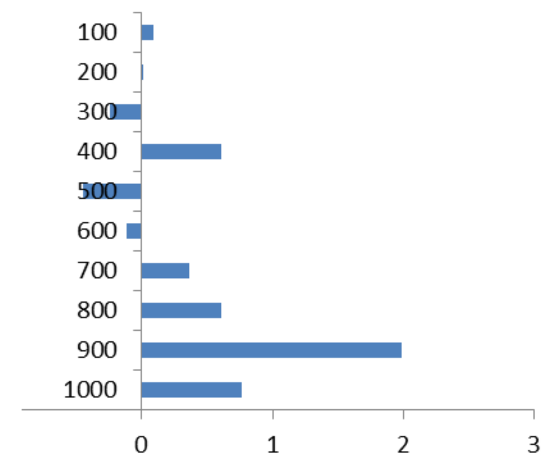
SONDE T



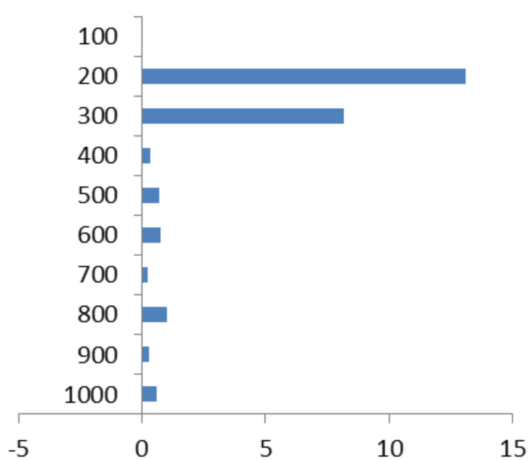
SONDE UV



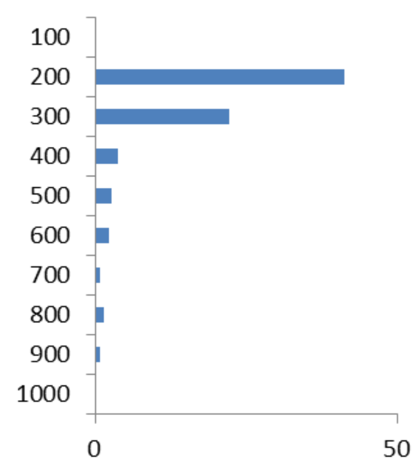
SOND RH



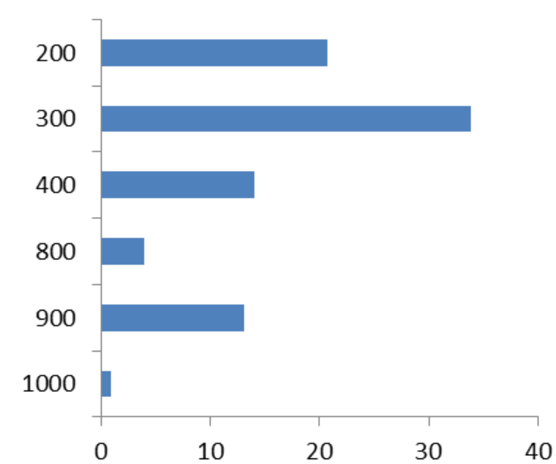
AVIATION T



AVIATION UV

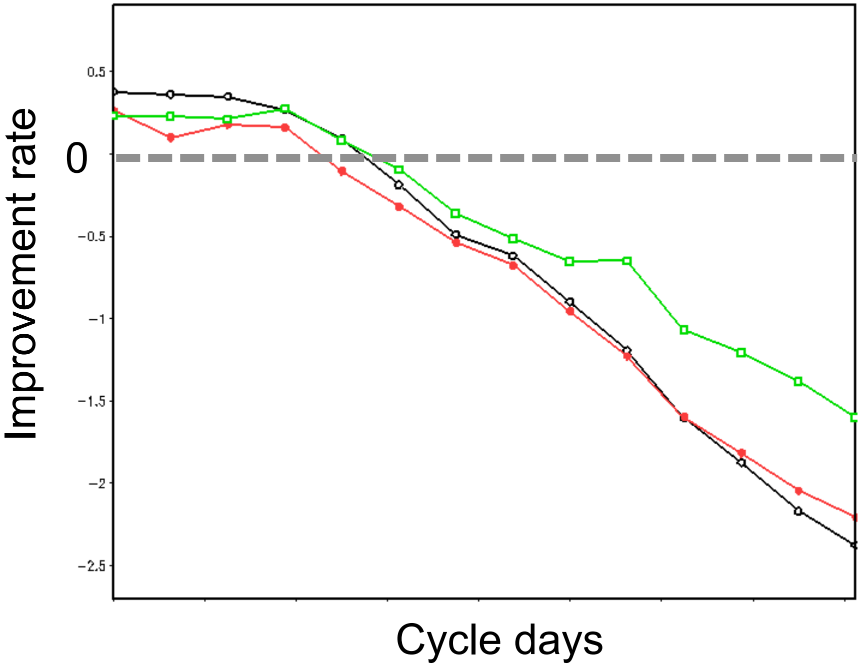


AMV_geo

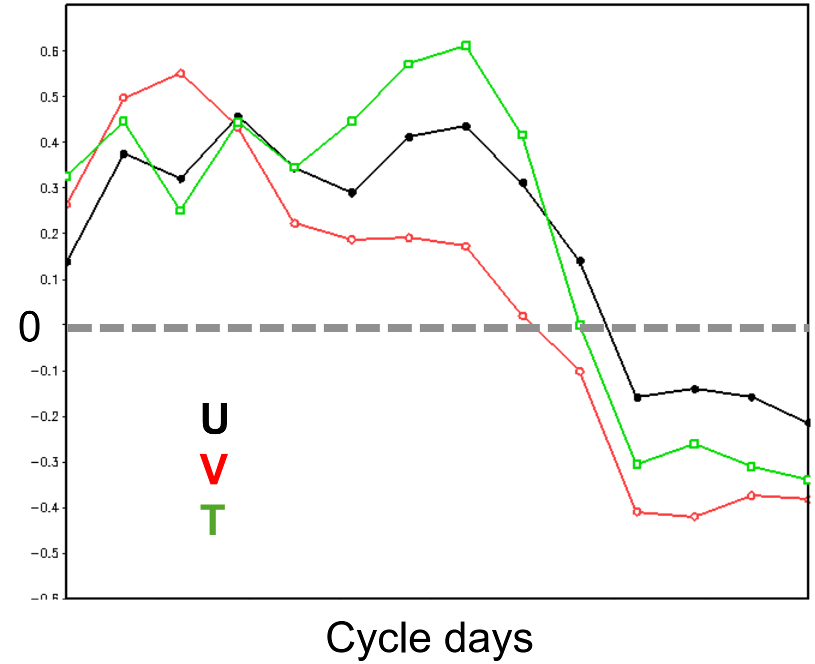


Cycle experiment with static optimization

FT24 500hPa



FT48 500hPa

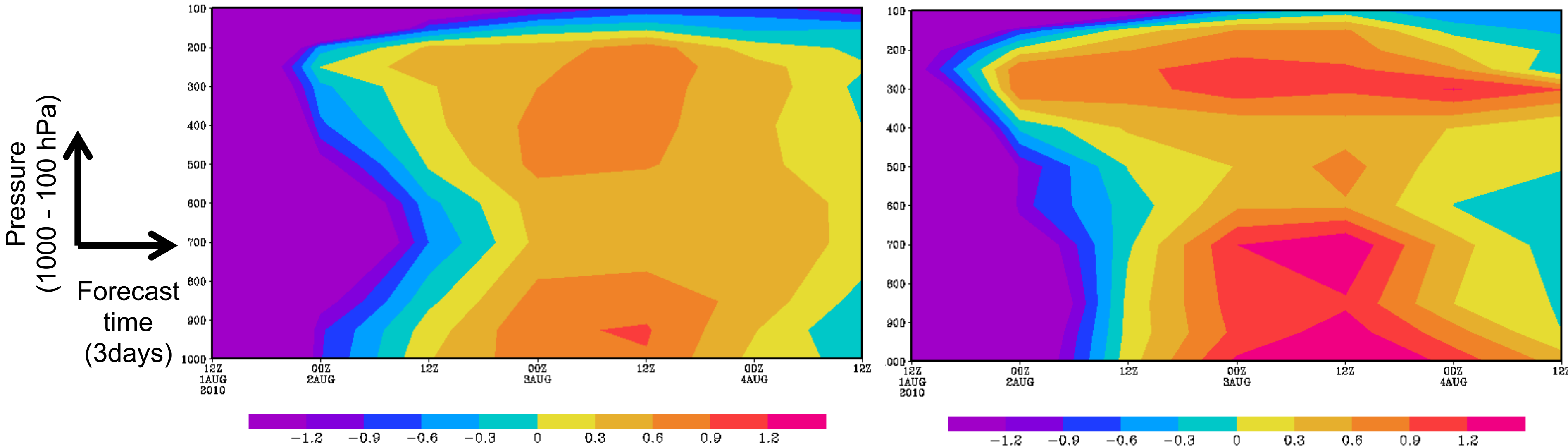


- Forecast errors increase as the cycle advances, within a week.
- This may bes because of this method does not take into account cycle (nonlinear) effect, variations of background fields caused by the static optimization.

Cycle experiment with online optimization

Improvement rate: Zonal wind

Improvement rate: Temperature



- One week averaged improvement rate.
- To treat nonlinear effects, here, tuning coefficients are calculated each 00UTC analysis and renewed.
- Some improvement can be fond but not enough.

Key analysis error

- The relationship between sensitivity vector and analysis error vector （Rabier et al, 1996）

C is metrics,

M is TL model,

a is a scalar coefficient.

$$J \equiv 1/2\, \mathbf{e}^{fT}\, \mathbf{C}\, \mathbf{e}^{f}$$

$$\mathbf{s} \equiv \mathbf{C}^{-1}\, \partial J / \partial \mathbf{e}^{0} = \mathbf{C}^{-1}\mathbf{M}^{T}\mathbf{C}\mathbf{M}\mathbf{e}^{0} \cong \frac{1}{a}\mathbf{e}^{0}$$

- Analytical determination of the coefficient a (Isaksen et al, 2005)

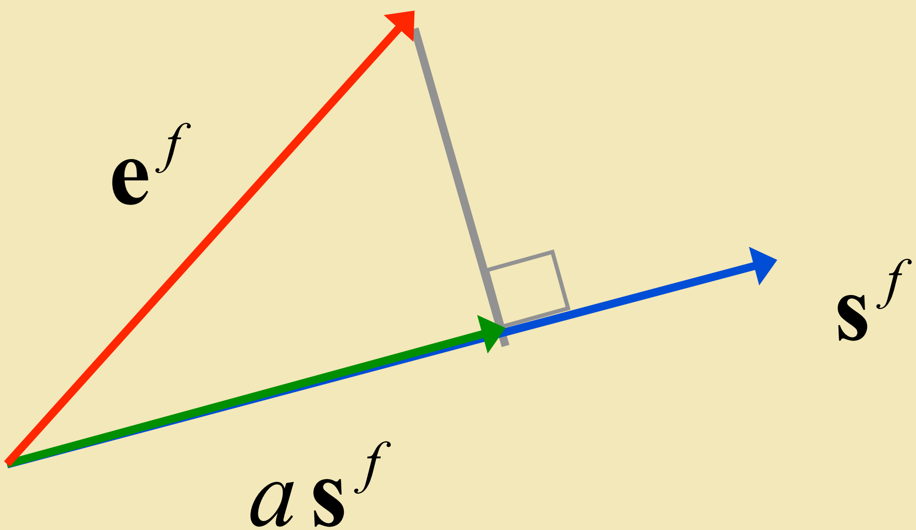
$$F \equiv \left(\mathbf{e}^{f} - a\mathbf{s}^{f}\right)^{T} \mathbf{C} \left(\mathbf{e}^{f} - a\mathbf{s}^{f}\right)$$

$$\partial F / \partial a = 2a\, \mathbf{s}^{fT}\, \mathbf{C}\, \mathbf{s}^{f} - 2\mathbf{s}^{fT}\, \mathbf{C}\, \mathbf{e}^{f} = 0$$

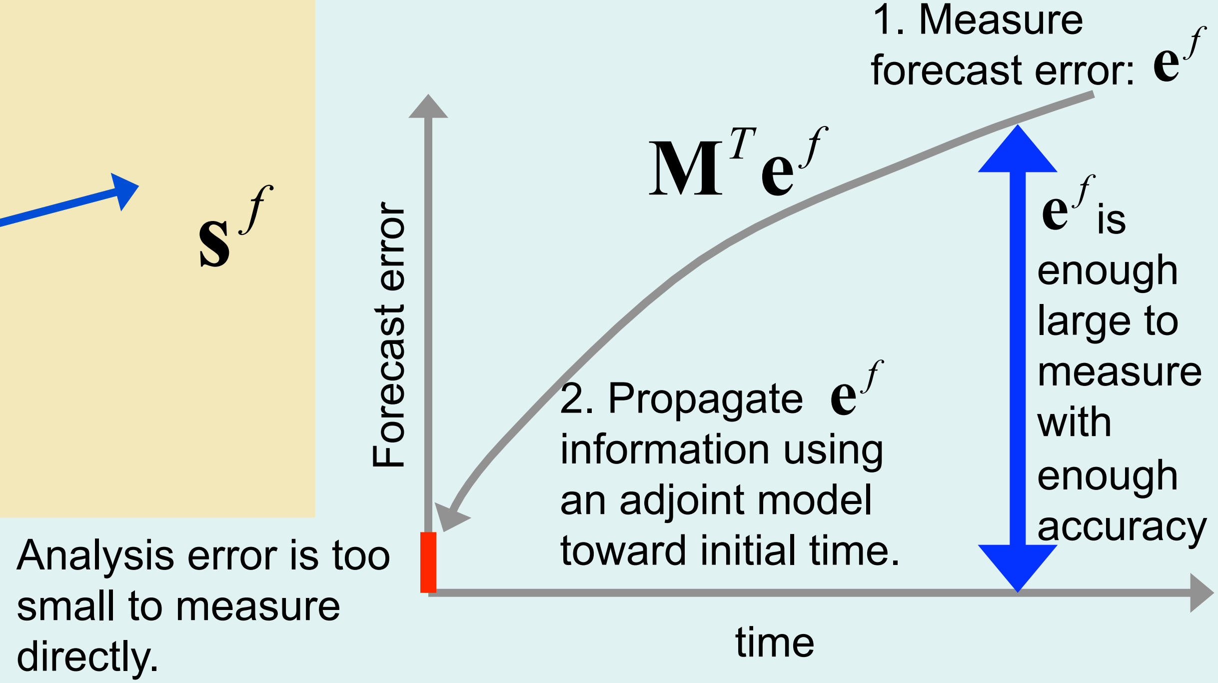
$$a = \mathbf{s}^{fT}\mathbf{C}\mathbf{e}^{f} / \mathbf{s}^{fT}\mathbf{C}\mathbf{s}^{f}$$

Key analysis error

- Geometric illustration of the coefficient ‘a’

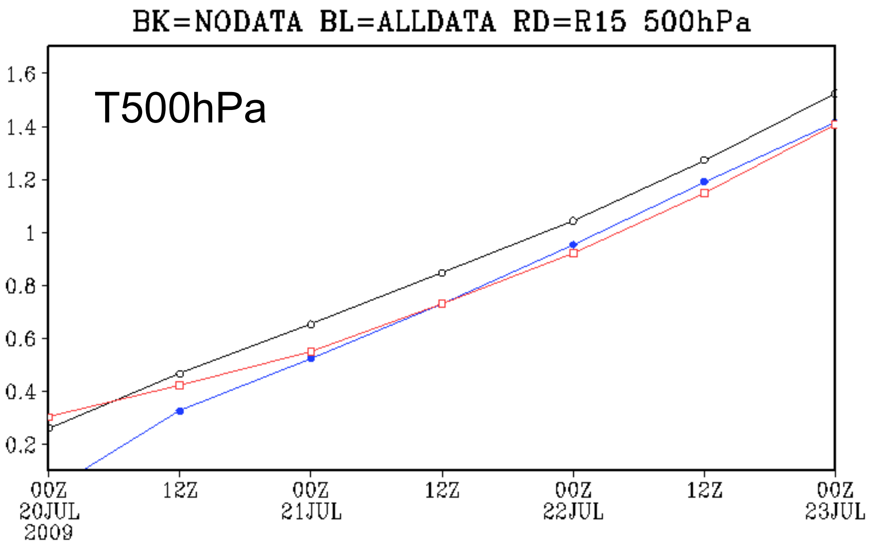


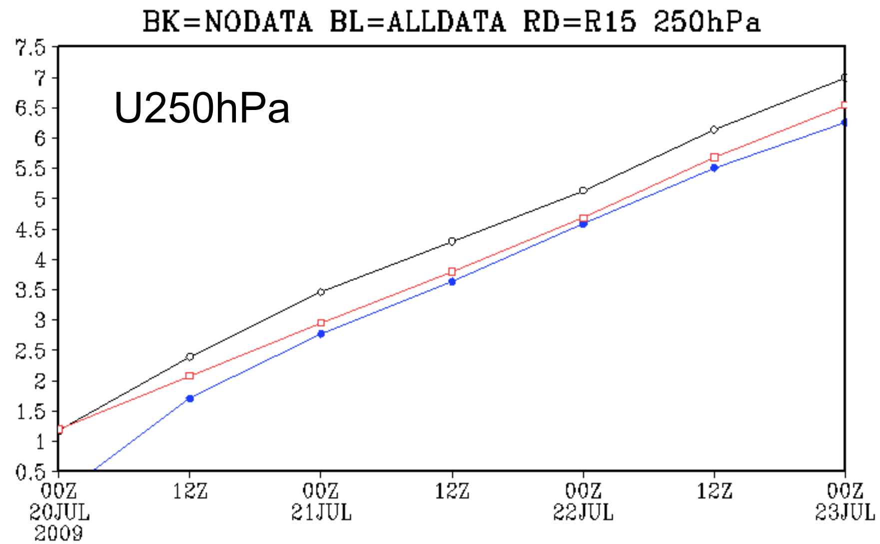
- Analysis error estimation using sensitivity vector



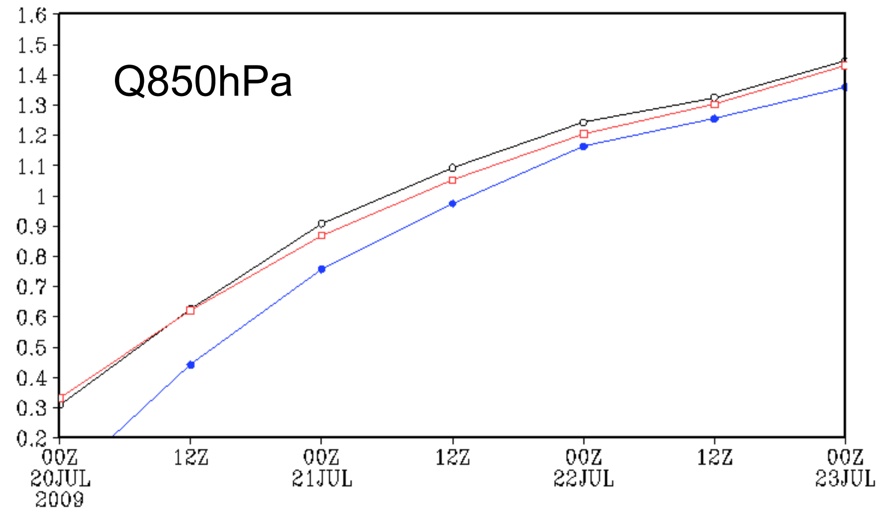
Optimization: To subtract an estimated analysis error from the original analysis.

Accuracy of optimized forecast





- Optimization time is FT 15hr.
- Vertical axis is RMSE, horizontal axis is forecast time, zero to three days.



Red: optimized forecast

Black: Original forecast

(Blue: forecast from an original next analysis time original)

Variational formulation of key analysis error

Cost function

$$J \equiv 1/2\left(\mathbf{e}^{\mathrm{t}} + \mathbf{M}\delta\mathbf{x}\right)^T \mathbf{A}^{-1}\left(\mathbf{e}^{\mathrm{t}} + \mathbf{M}\delta\mathbf{x}\right)$$

Klinker et al 1998

Minimize
J(δ**x**)

Here, $M\left(\mathbf{x}_b + \delta\mathbf{x}\right) - \mathbf{x}_{ref} \cong \mathbf{e}^{\mathrm{t}} + \mathbf{M}\delta\mathbf{x},\ \mathbf{e}^{\mathrm{t}} \equiv M\left(\mathbf{x}_b\right) - \mathbf{x}_{ref}$

M: TL model, δ**x**: analysis increment vector, **A**: norm define matrix.

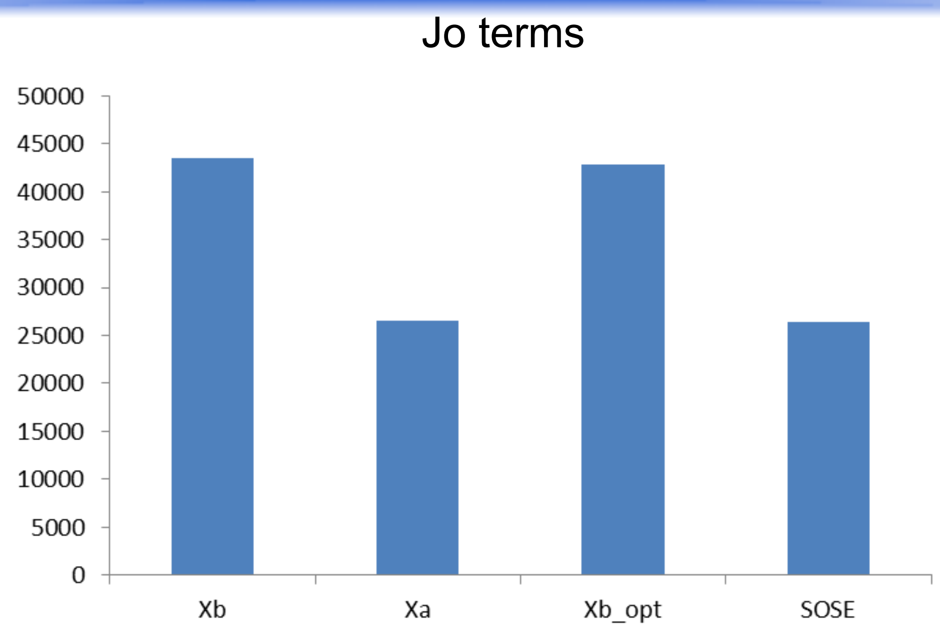
Analysis
increment

$$\delta\mathbf{x} = -\left(\mathbf{M}^T \mathbf{A}^{-1} \mathbf{M}\right)^{-1} \mathbf{M}^T \mathbf{A}^{-1} \mathbf{e}^t \cong -\mathbf{e}^0$$

$$\left(\partial J/\partial \delta\mathbf{x}\right)_{first} = \mathbf{M}^T \mathbf{A}^{-1} \mathbf{e}^t$$

- This method put 100% confidence on reference analysis information, since any compatible information are used.
- This property causes inconsistency between observations and a background field (Klinker et al 1998, Isaksen et al 2005, Pu, Carron).
- So, previous studies stop iteration at once or dozen times to avoid over fitting.

Problem of key analysis error

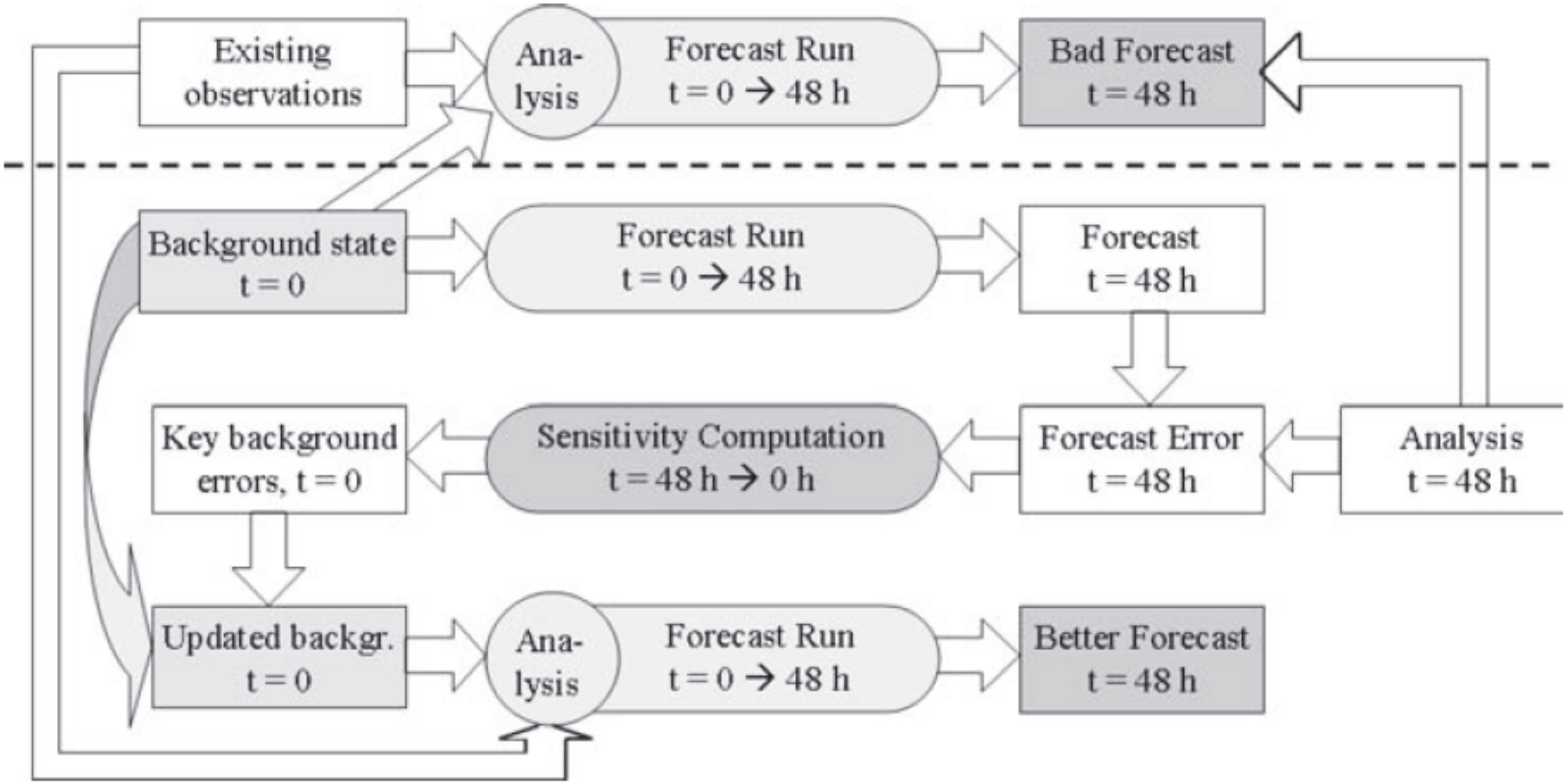


SOSE: Sensitivity observing system experiment (Marseille et al 2008)

- Although the optimized analysis based on the key analysis error can generate more accurate forecasts, there is problems that the optimized analyses are inconsistent with observations (Isaksen et al 2005) .

Next, we consider to achieve analysis fields that are able to generate more accurate forecasts and consistent with observations, simultaneously.

SOSE



Adapted from Marseille et al (2008 Tellus A)

Data assimilation theory based method

- We ignore cross terms. Validity of this assumption is checked by forecast errors and observation fitting.

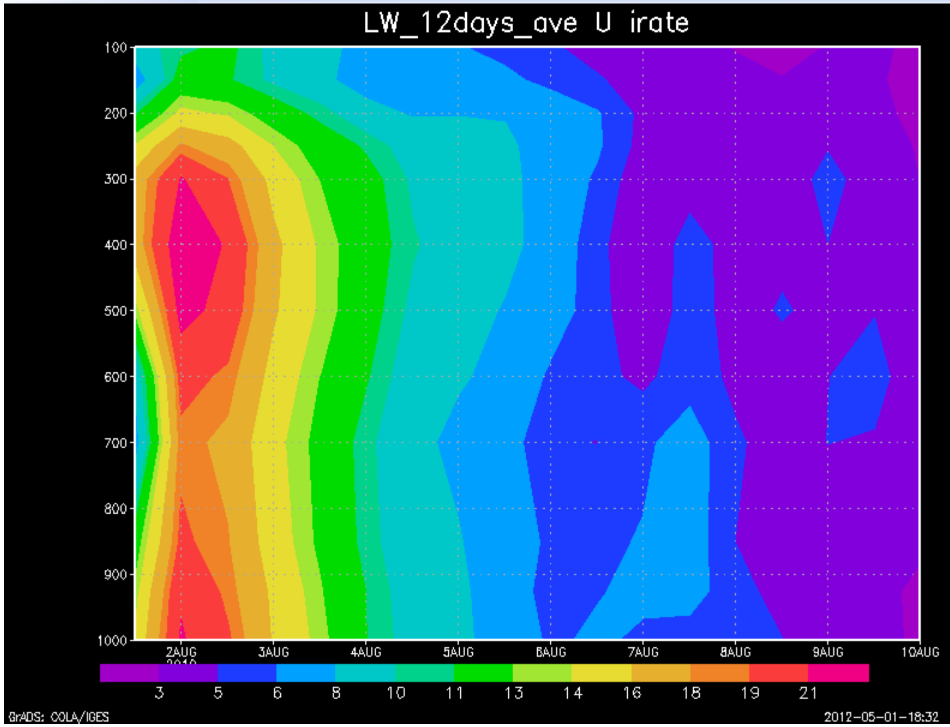
$$\mathbf{T} = \begin{pmatrix} \mathbf{B} & \mathbf{X}_{BR} & \mathbf{X}_{BA} \\ \mathbf{X}_{BR} & \mathbf{R} & \mathbf{X}_{RA} \\ \mathbf{X}_{BA} & \mathbf{X}_{RA} & \mathbf{A} \end{pmatrix} \cong \begin{pmatrix} \mathbf{B} & 0 & \mathbf{X}_{BA} \\ 0 & \mathbf{R} & \mathbf{X}_{RA} \\ \mathbf{X}_{BA} & \mathbf{X}_{RA} & \mathbf{A} \end{pmatrix} \approx \begin{pmatrix} \mathbf{B} & 0 & 0 \\ 0 & \mathbf{R} & 0 \\ 0 & 0 & \mathbf{A} \end{pmatrix}$$

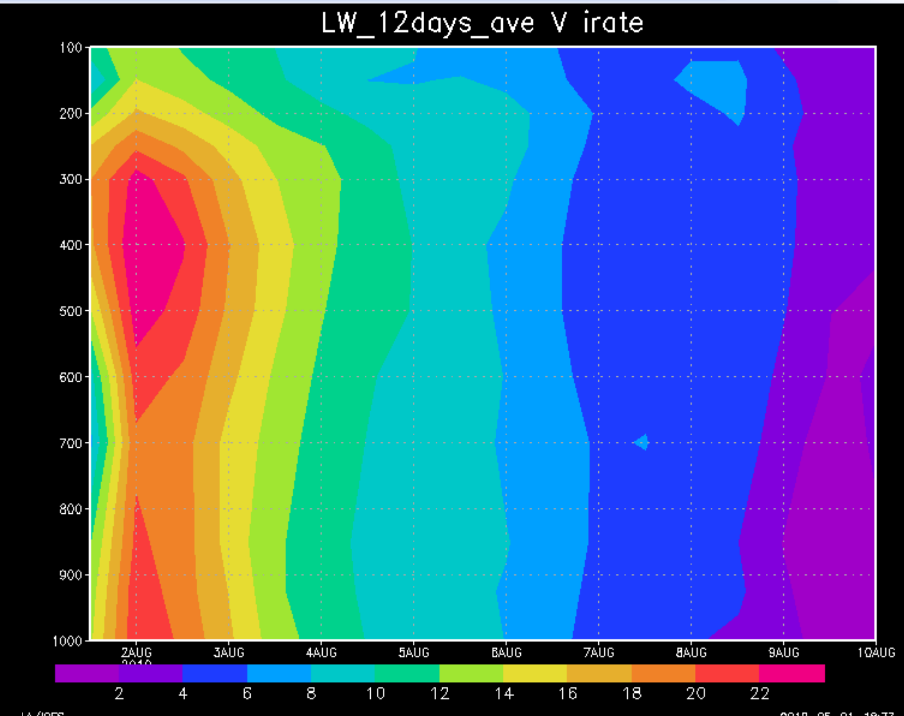
- We give **A** as diagonal matric, and variances of **A** are give as 30% of forecast error variance of background field multiplied by inflation factor *f*.

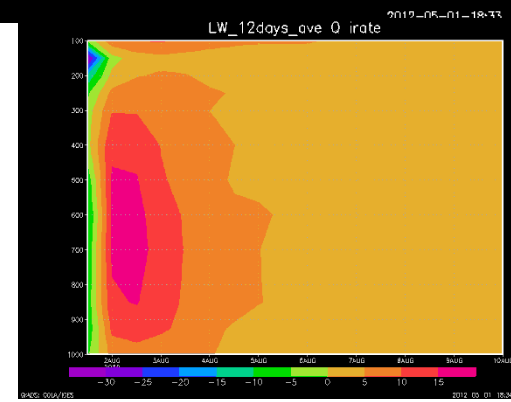
$$\begin{aligned} \mathbf{e}_x^f &\equiv \mathbf{x}_{ref} - M(\mathbf{x}_{true}) = \mathbf{x}_{ref} - \mathbf{x}_{true} - \left(M(\mathbf{x}_{true}) - \mathbf{x}_{true}\right) \\ &= \mathbf{e}_x^m - \left(M(\mathbf{x}_{true}) - M_{true}(\mathbf{x}_{true}) + M_{true}(\mathbf{x}_{true}) - \mathbf{x}_{true}\right) \\ &= \mathbf{e}_x^m - \mathbf{e}_x^{func} - \mathbf{e}_x^{rep} \end{aligned}$$

$$\begin{aligned} \mathbf{e}^o &\equiv \mathbf{y} - H(\mathbf{x}_{true}) = \mathbf{y} - \mathbf{y}_{true} - \left(H(\mathbf{x}_{true}) - \mathbf{y}_{true}\right) \\ &= \mathbf{e}^m - \left(H(\mathbf{x}_{true}) - H_{true}(\mathbf{x}_{true}) + H_{true}(\mathbf{x}_{true}) - \mathbf{y}_{true}\right) \\ &= \mathbf{e}^m - \mathbf{e}^{func} - \mathbf{e}^{rep} \end{aligned}$$

Two weeks statistics







- Forecast accuracy keeps until 9days with 95% statistical significance until 6 or 7days.

Thank you for listening

Comments or Questions ?

Please speak very slowly!

One sentence question welcome!!

Mathematical language also welcome!

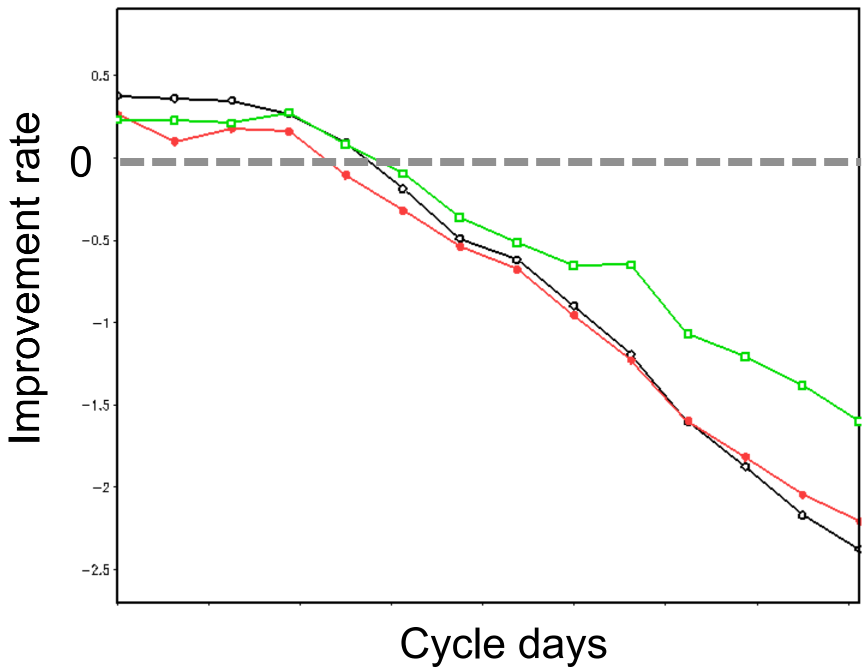
More long complicate questions also welcome,

but answer will be after this session or by email.

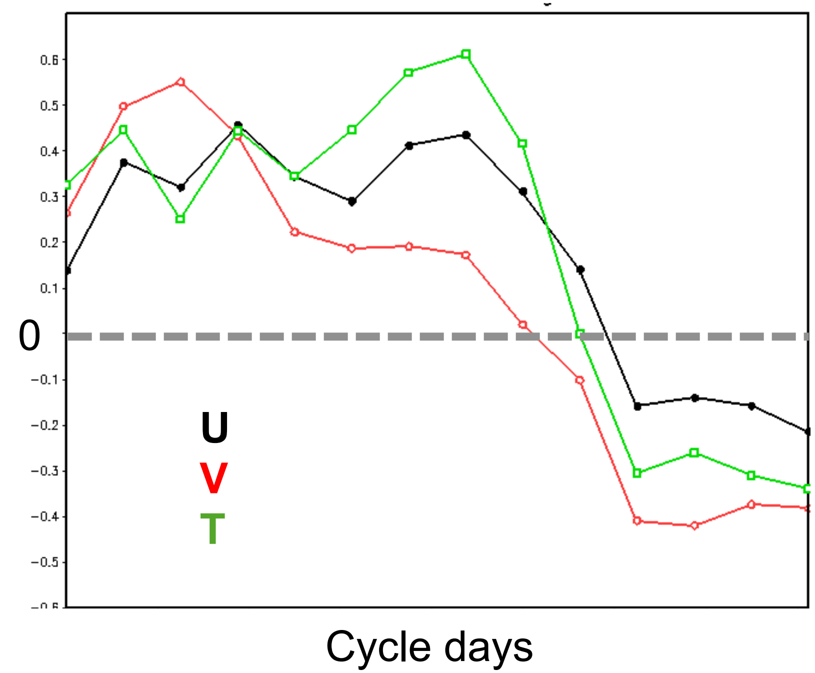
So sorry my very developing stage English.

Cycle experiment with static optimization

FT24 500hPa



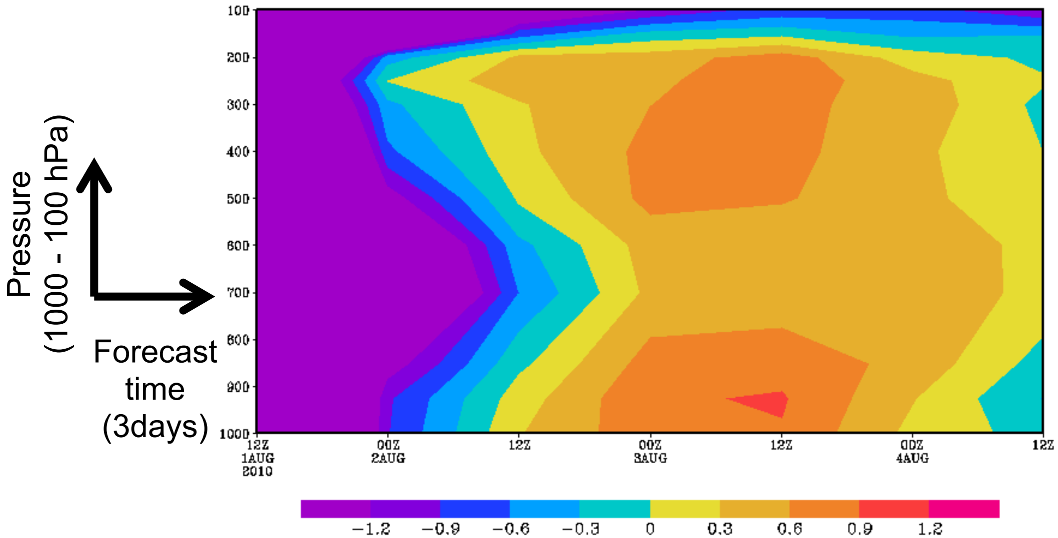
FT48 500hPa



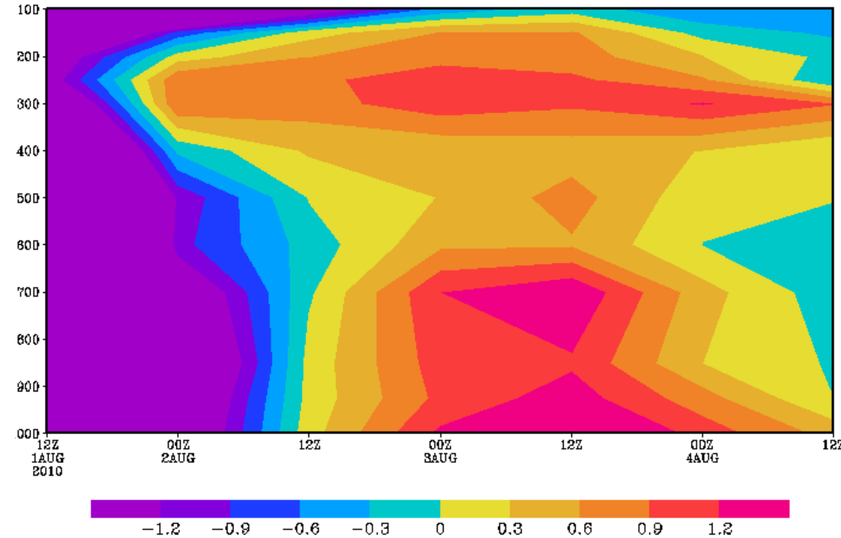
- Forecast errors increase as the cycle advances, within a week.
- This may bes because of this method does not take into account cycle (nonlinear) effect, variations of background fields caused by the static optimization.

Cycle experiment with online optimization

Improvement rate: Zonal wind



Improvement rate: Temperature



- One week averaged improvement rate.
- To treat nonlinear effects, here, tuning coefficients are calculated each 00UTC analysis and renewed.
- Some improvement can be fond but not enough. Because convergence of coefficients is bad. Because large STD.

$$F \equiv 1/2\, \mathbf{e}^{fT}\, \mathbf{C}\, \mathbf{e}^{f} = \tilde{\mathbf{e}}^{fT}\tilde{\mathbf{e}}^{f},$$

$$\tilde{\mathbf{s}}_j \equiv \partial F/\partial \tilde{\mathbf{e}} = \tilde{\mathbf{M}}_{i,j}\tilde{\mathbf{M}}_{i,k}\tilde{\mathbf{e}}_k = \tilde{\mathbf{M}}_{i,j}\tilde{\mathbf{M}}_{i,k}\tilde{\mathbf{U}}_{k,n}b_n = \sum_n \tilde{\mathbf{U}}_{j,n}b_n\sigma_n^2$$

where, E is the vector constructed from sin *eigen values.*

$$\mathbf{s} \equiv \partial F/\partial \mathbf{e} = \mathbf{M}^T\mathbf{CMe}$$

$$\tilde{\mathbf{s}} = \tilde{\mathbf{M}}^T\tilde{\mathbf{M}}\tilde{\mathbf{e}} = \mathbf{C}^{-1/2}\mathbf{MC}^{1/2}\mathbf{C}^{1/2}\mathbf{MC}^{-1/2}\mathbf{C}^{1/2}\mathbf{e} = \mathbf{C}^{-1/2}\mathbf{s}$$

$$\tilde{\mathbf{U}} = \mathbf{C}^{1/2}\mathbf{U}$$

$$\mathbf{s} = \mathbf{C}^{1/2}\tilde{\mathbf{s}} = \mathbf{C}^{1/2}\sum_n \tilde{\mathbf{U}}_{j,n}b_n\sigma_n^2 = \mathbf{C}^{1/2}\sum_n \left(\mathbf{C}^{1/2}\mathbf{U}\right)_{j,n}b_n\sigma_n^2 = \mathbf{C}\sum_n \mathbf{U}_{j,n}b_n\sigma_n^2$$

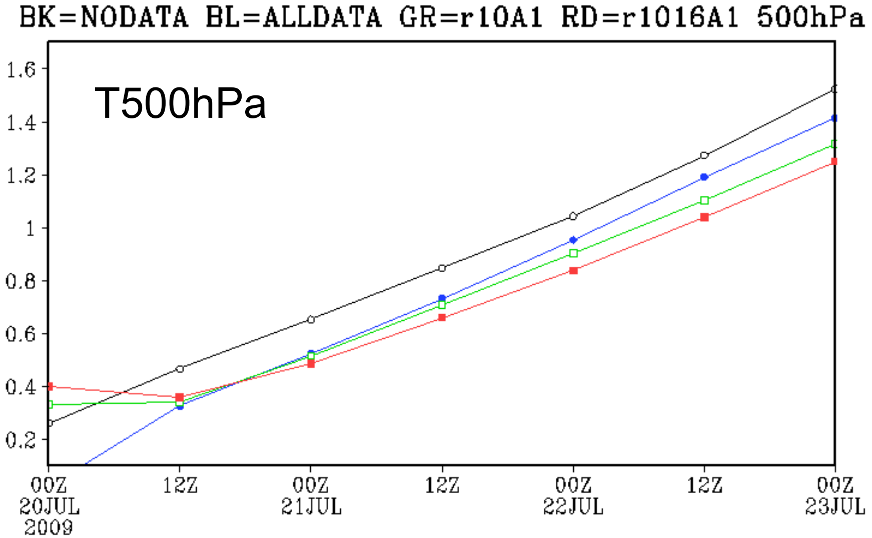
If $b_n = b_1\delta_{1n}$ *then*

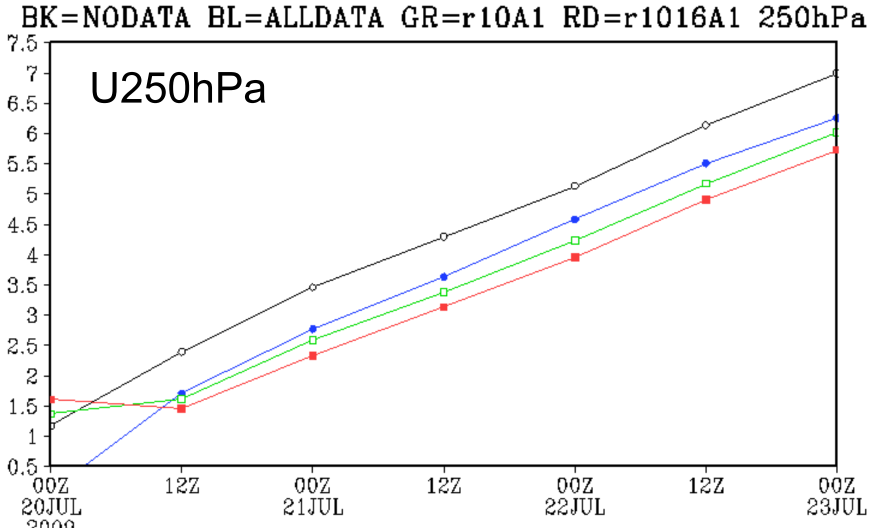
$$\mathbf{e}_j = \sum_n \mathbf{U}_{j,n}b_n = \mathbf{U}_{j,1}b_1$$

$$\mathbf{s} = \mathbf{CU}_{j,1}b_1\sigma_1^2 = \mathbf{C}b_1\sigma_1^2\mathbf{e} \equiv \mathbf{Ce}\frac{1}{a}$$

$$\mathbf{e} = a\mathbf{C}^{-1}\mathbf{s} \equiv a\mathbf{s}_{\dim}$$

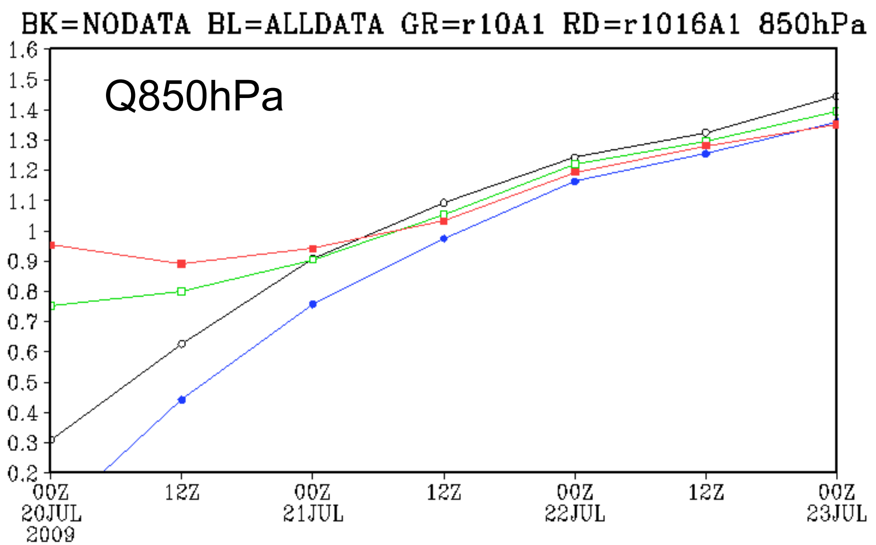
“One opt” vs “two opt”



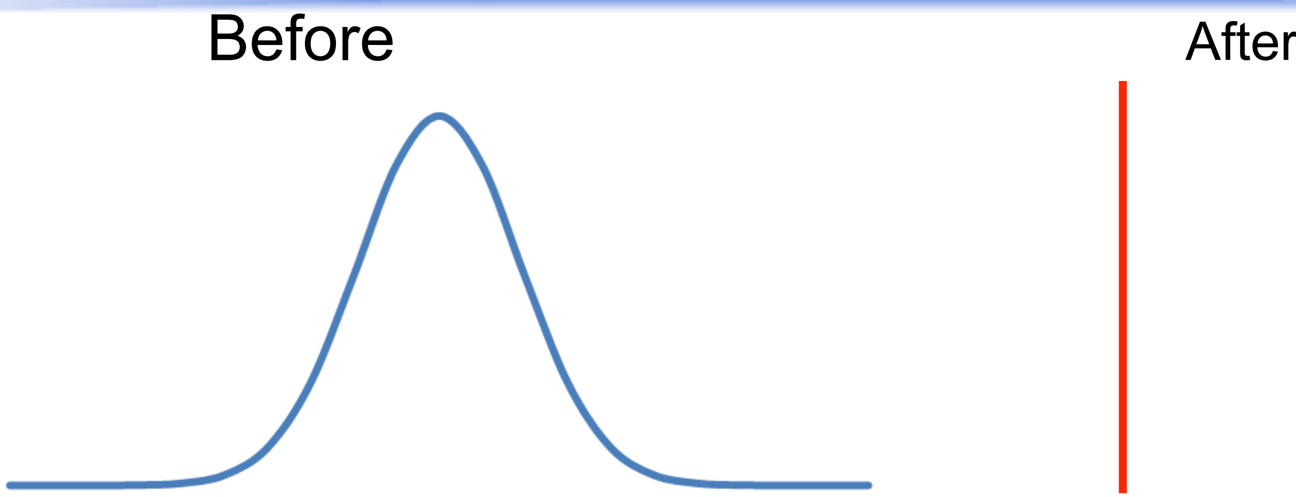


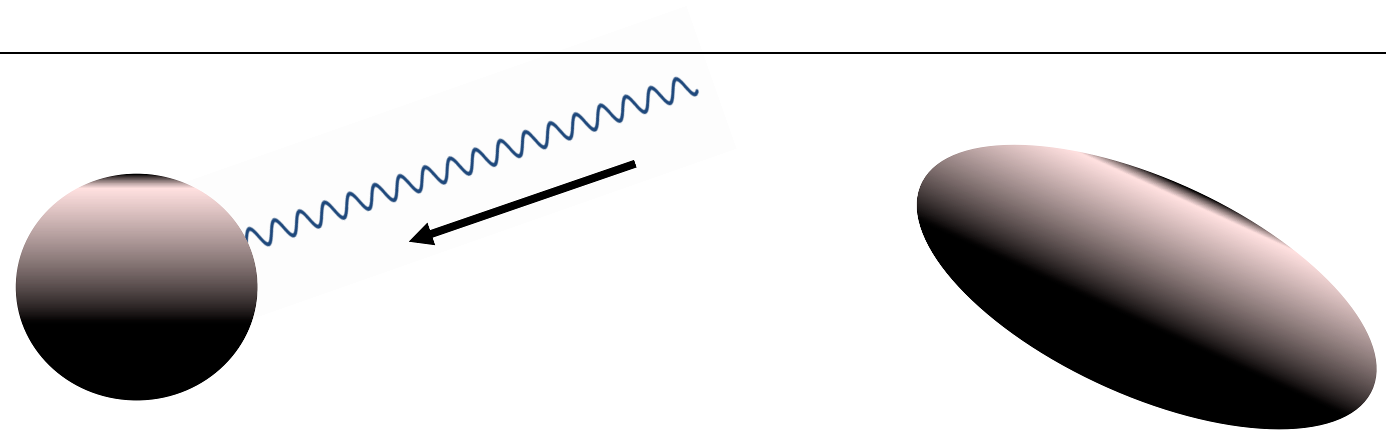
- **A = I** で与えている。

赤 : FT9FT15で最適化
緑 : FT9でのみ最適化
黒 : 最適化なし
青 : 6時間後の初期値からの予報



TL-based method





Sensitivity on covariance matrices

$$X_{i,j}X^{-1}_{j,k} = \delta_{i,k}$$

$$\partial X_{i,j}X^{-1}_{j,k} / \partial X_{l,m} = \left(\partial X_{i,j}/\partial X_{l,m}\right)X^{-1}_{j,k} + X_{i,j}\left(\partial X^{-1}_{j,k}/\partial X_{l,m}\right)$$

$$= \delta_{i,l}\delta_{j,m}X^{-1}_{j,k} + X_{i,j}\left(\partial X^{-1}_{j,k}/\partial X_{l,m}\right)$$

$$= \delta_{i,l}X^{-1}_{m,k} + X_{i,j}\left(\partial X^{-1}_{j,k}/\partial X_{l,m}\right) = 0$$

$$X^{-1}_{n,i}\left\{\delta_{i,l}X^{-1}_{m,k} + X_{i,j}\left(\partial X^{-1}_{j,k}/\partial X_{l,m}\right)\right\}$$

$$= X^{-1}_{n,l}X^{-1}_{m,k} + \partial X^{-1}_{n,k}/\partial X_{l,m} = 0$$

$$\partial X^{-1}_{n,k}/\partial X_{l,m} = -X^{-1}_{n,l}X^{-1}_{m,k}$$

$$\partial J_t/\partial B_{k,l} = \left(\partial J_t/\partial x_i\right)\left(\partial x_i/\partial B_{k,l}\right)$$

$$\partial x_i/\partial B_{k,l} = \partial K_{i,r}d_r/\partial B_{k,l} = d_r\partial K_{i,r}/\partial B_{k,l}$$

$$= d_r\partial\left(A^{-1}_{i,j}H_{p,j}R^{-1}_{p,r}\right)/\partial B_{k,l}$$

$$= d_rH_{p,j}R^{-1}_{p,r}\,\partial A^{-1}_{i,j}/\partial B_{k,l}$$

$$= d_rH_{p,j}R^{-1}_{p,r}\left(\partial A^{-1}_{i,j}/\partial A_{a,b}\right)\left(\partial A_{a,b}/\partial B_{k,l}\right)$$

$$= d_rH_{p,j}R^{-1}_{p,r}A^{-1}_{i,a}A^{-1}_{b,j}\left(\partial B^{-1}_{a,b}/\partial B_{k,l}\right)$$

$$= d_rH_{p,j}R^{-1}_{p,r}A^{-1}_{i,a}A^{-1}_{b,j}B^{-1}_{a,k}B^{-1}_{l,b}$$

$$= d_rK_{b,r}A^{-1}_{i,a}B^{-1}_{a,k}B^{-1}_{l,b}$$

$$= \left(B^{-1}_{l,b}K_{b,r}d_r\right)\left(A^{-1}_{i,a}B^{-1}_{a,k}\right)$$

$$\partial J_t/\partial B_{k,l} = \left(B^{-1}_{l,b}K_{b,r}d_r\right)\left(A^{-1}_{i,a}B^{-1}_{a,k}\right)\left(\partial J_t/\partial x_i\right)$$

$$\partial F/\partial R_{p,q} = \left(\partial F/\partial x_i\right)\left(\partial x_i/\partial R_{p,q}\right)$$

$$\partial x_i/\partial R_{p,q} = \partial K_{i,r}d_r/\partial R_{p,q} = d_r\partial K_{i,r}/\partial R_{p,q}$$

$$= d_r\partial\left(B_{i,j}H_{s,j}D^{-1}_{s,r}\right)/\partial R_{p,q}$$

$$= d_rB_{i,j}H_{s,j}\,\partial D^{-1}_{s,r}/\partial R_{p,q}$$

$$= d_rB_{i,j}H_{s,j}\left(\partial D^{-1}_{s,r}/\partial D_{t,u}\right)\left(\partial D_{t,u}/\partial R_{p,q}\right)$$

$$= -d_rB_{i,j}H_{s,j}D^{-1}_{s,t}D^{-1}_{u,r}\delta_{t,p}\delta_{u,q}$$

$$= -d_rB_{i,j}H_{s,j}D^{-1}_{s,p}D^{-1}_{q,r}$$

$$= -d_rK_{i,p}D^{-1}_{q,r}$$

$$\partial F/\partial R_{p,q} = -\left(\partial F/\partial x_i\right)\left(K_{i,p}D^{-1}_{q,r}d_r\right)$$

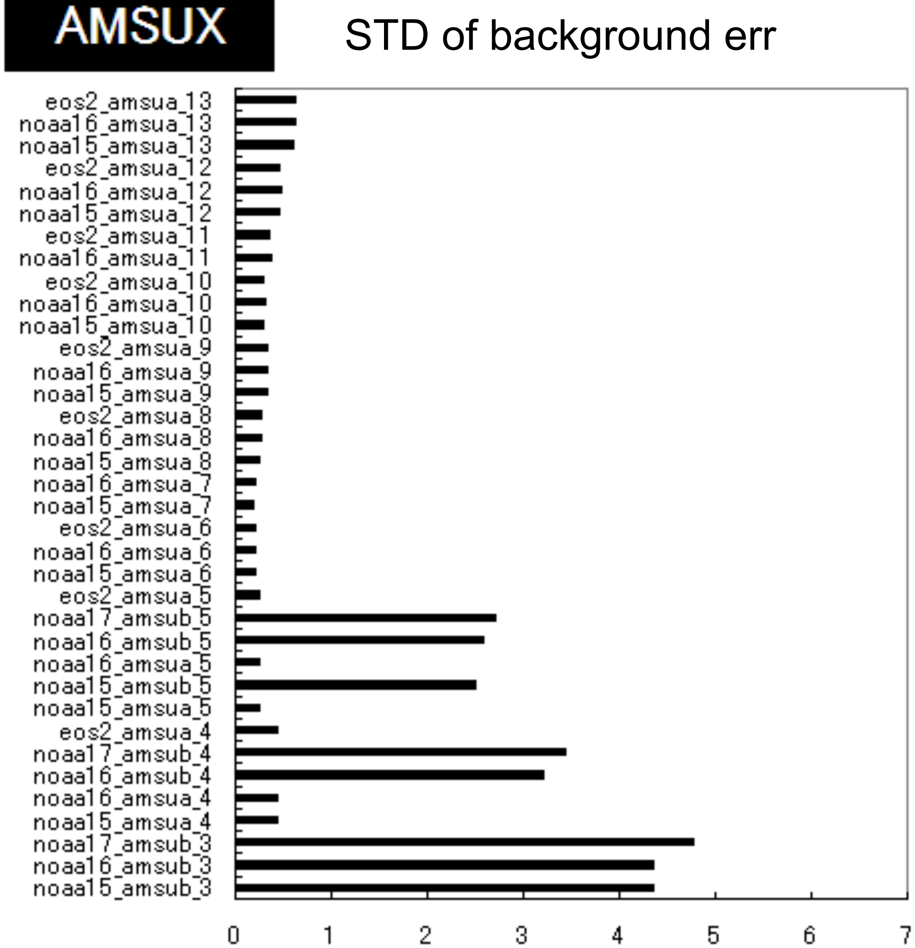
$$= -\left(\partial F/\partial x_i\right)\left(\partial x_i/\partial d_p\right)\left(D^{-1}_{q,r}d_r\right)$$

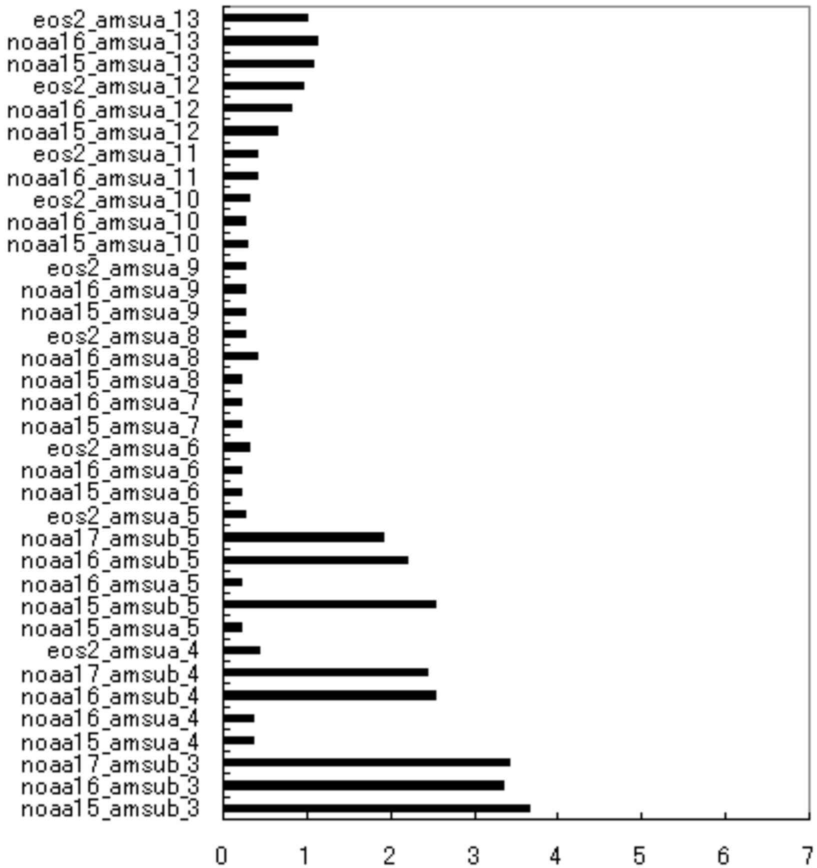
$$= -\left(\partial F/\partial d_p\right)\left(D^{-1}_{q,r}d_r\right)$$

$$= \left(\partial F/\partial d_p\right)R^{-1}_{q,s}\left(H\delta x - d\right)_s$$

2. Covariance matrix optimization

AMSUX

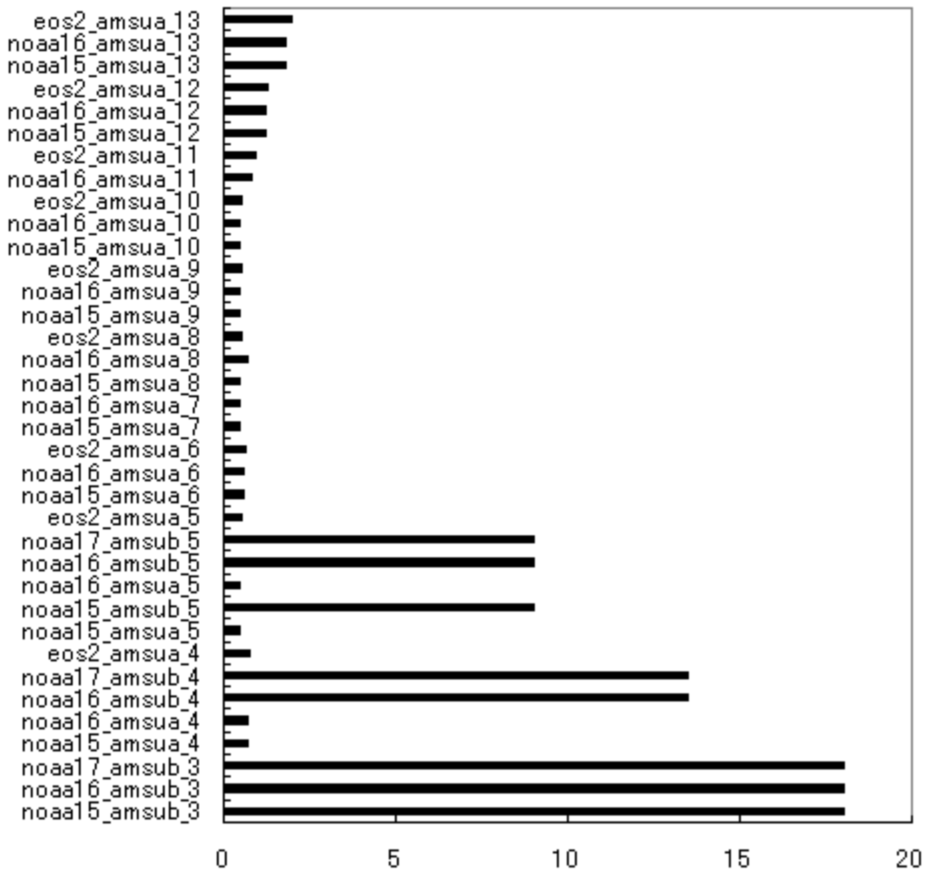




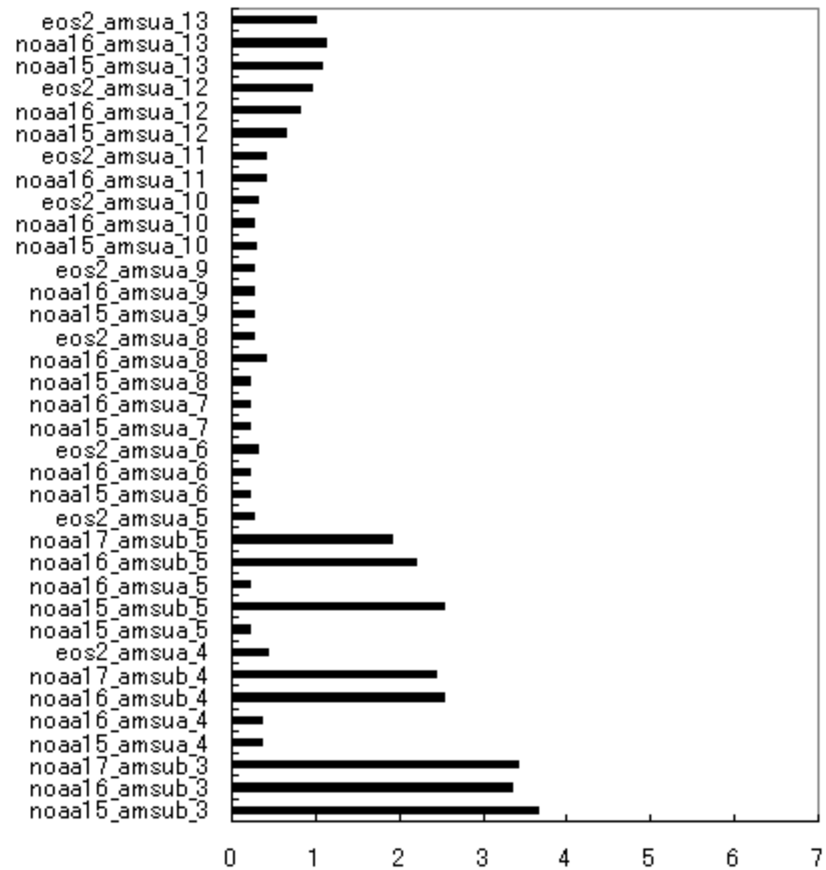
In general, B is too large especially for humidity sensitive sensors.

2. Covariance matrix optimization

STD of observation err



STD of departure



In general, R is too large especially for humidity sensitive sensors.

Variational formulation of key analysis error

Cost function $J \equiv 1/2\left(\mathbf{e}^{\mathrm{t}} + \mathbf{M}\delta\mathbf{x}\right)^T \mathbf{A}^{-1}\left(\mathbf{e}^{\mathrm{t}} + \mathbf{M}\delta\mathbf{x}\right)$ Klinker et al 1998

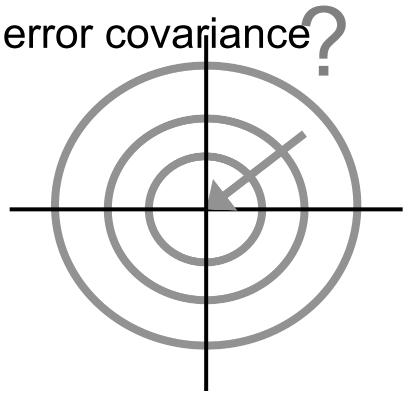
Minimize J(δ**x**)

Here, $M\left(\mathbf{x}_b + \delta\mathbf{x}\right) - \mathbf{x}_{ref} \cong \mathbf{e}^{\mathrm{t}} + \mathbf{M}\delta\mathbf{x},\ \mathbf{e}^{\mathrm{t}} \equiv M\left(\mathbf{x}_b\right) - \mathbf{x}_{ref}$

M: TL model, δ**x**: analysis increment vector, **A**: error covariance matrix of these information

Analysis increment

$$\delta\mathbf{x} = -\left(\mathbf{M}^T\mathbf{A}^{-1}\mathbf{M}\right)^{-1}\mathbf{M}^T\mathbf{A}^{-1}\mathbf{e}^t \cong -\mathbf{e}^0$$



We can see the sensitivity analysis is an approximate of this optimization wit only once iteration.

$$\left(\partial J/\partial\delta\mathbf{x}\right)_{first} = \mathbf{M}^T\mathbf{A}^{-1}\mathbf{e}^t$$

感度場による解析が解析解の良い近似となる条件

$$\left(\partial J/\partial\delta\mathbf{x}\right)_{first} \cong \left(-1/a\right)\delta\mathbf{x}$$

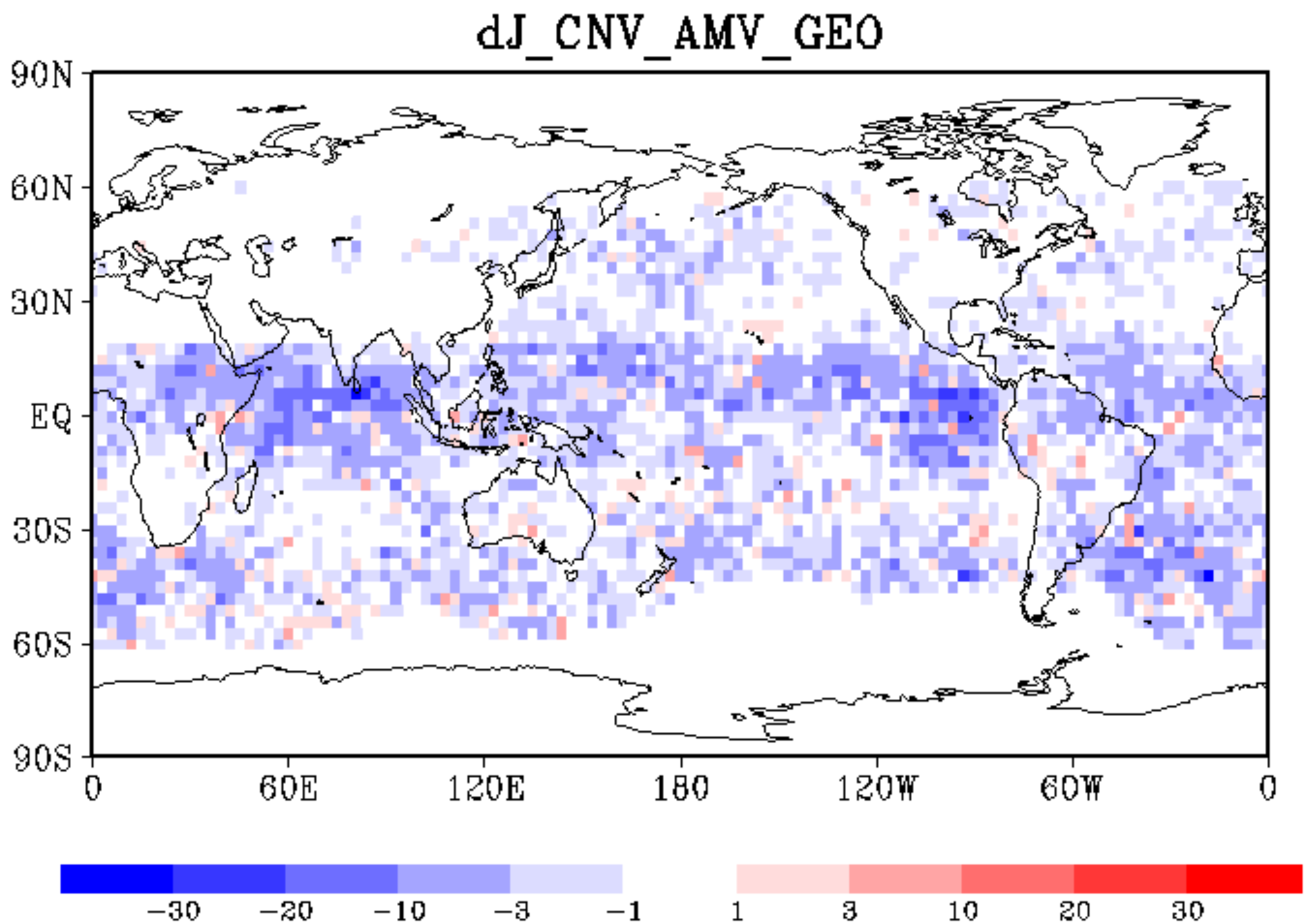
$$\mathbf{M}^T\mathbf{A}^{-1}\mathbf{M}\mathbf{e}^0 \cong \frac{1}{a}\mathbf{e}^0$$

これを満たす場合は2とおりある。

① $\mathbf{e}^0$ が**M**の特異値1/aに属する特異ベクトルである。

② $\mathbf{M}^T\mathbf{A}^{-1}\mathbf{M} = -a\mathbf{I}$

①は任意の摂動が特異値-aで発展することを意味する。少なくとも，変数変換をしない場合は物理的にありえない。



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60N
30N
EQ
30S
60S
90S
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120W
60W
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−3
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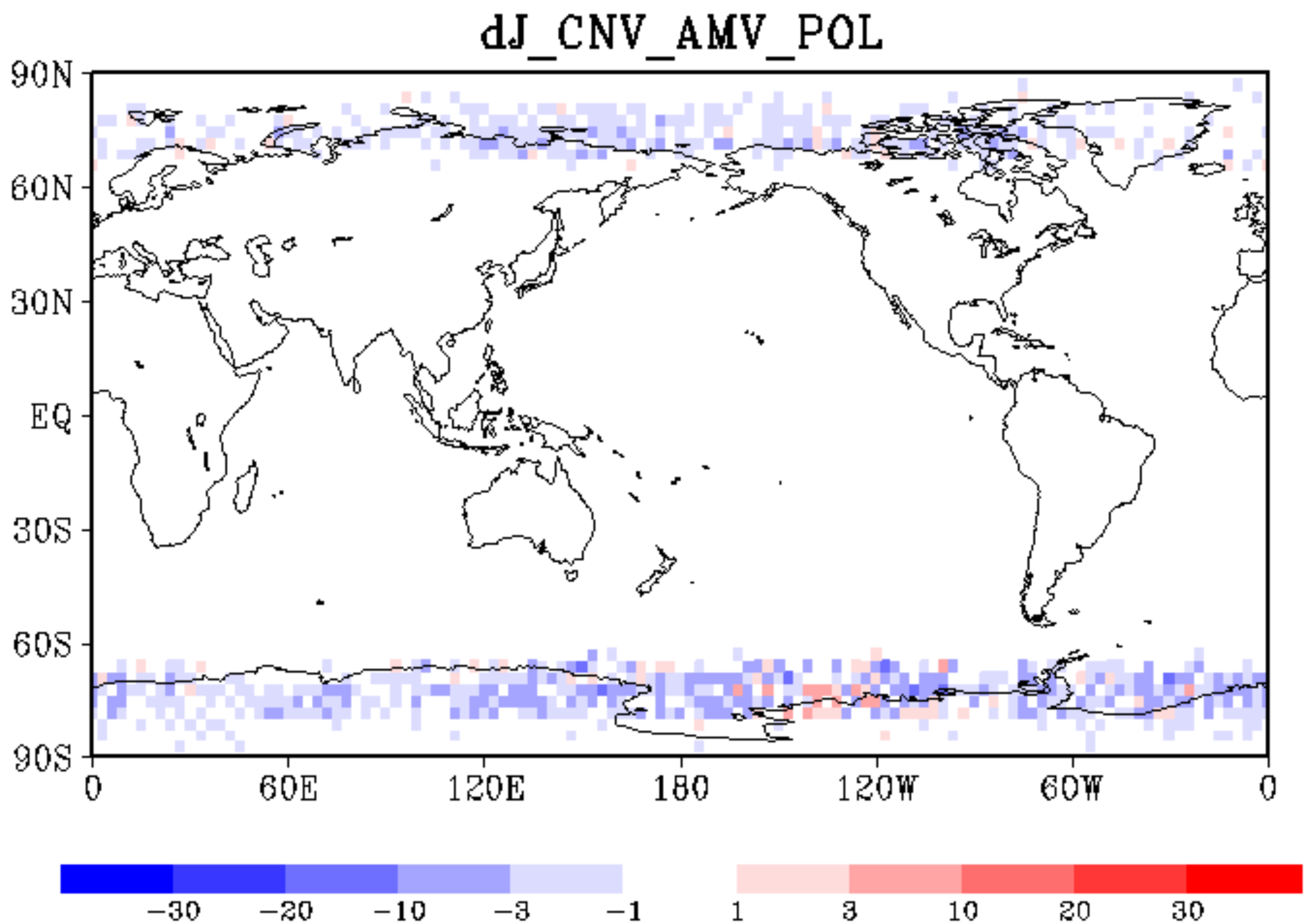
1
3
10
20
30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-3
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3
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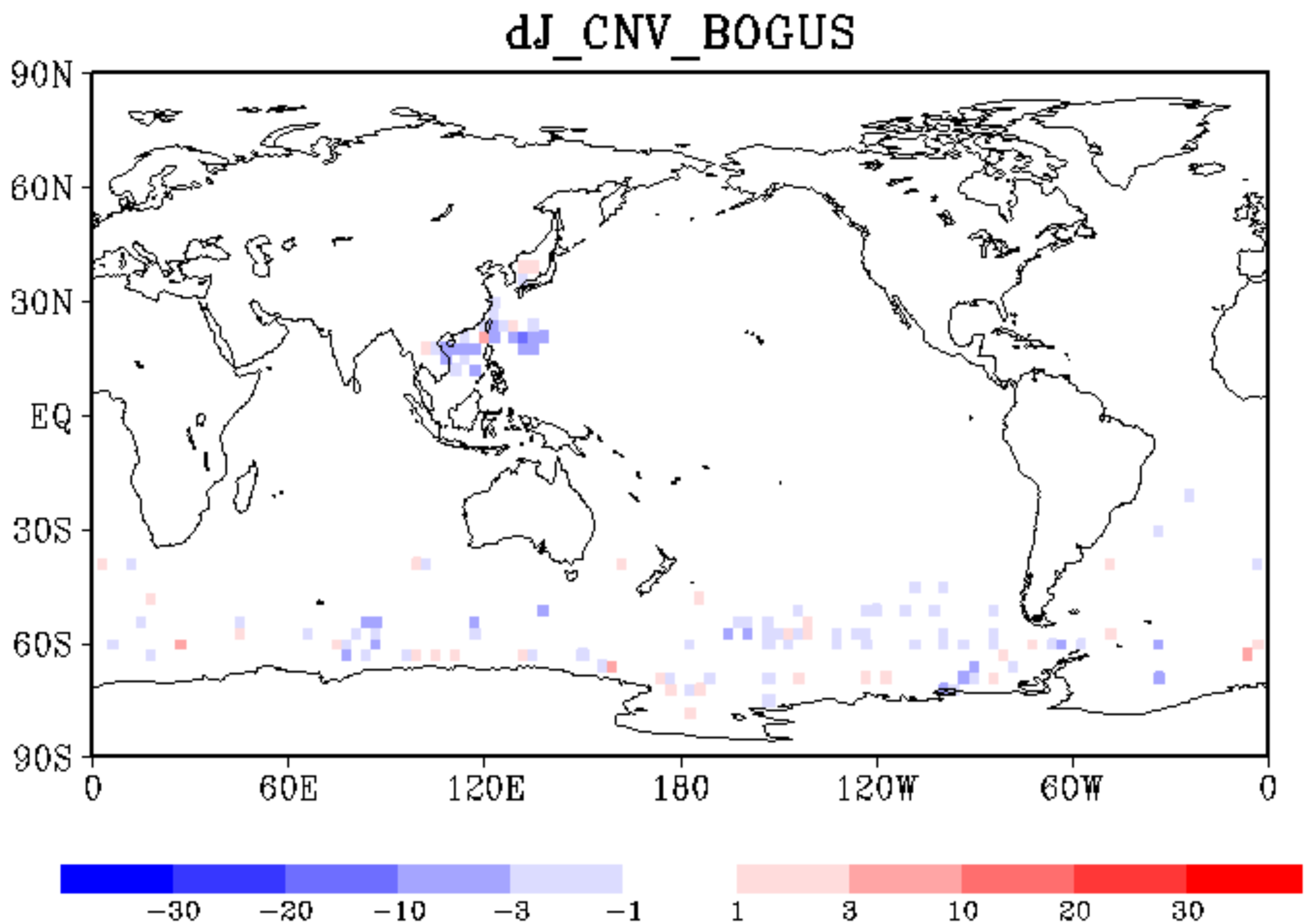
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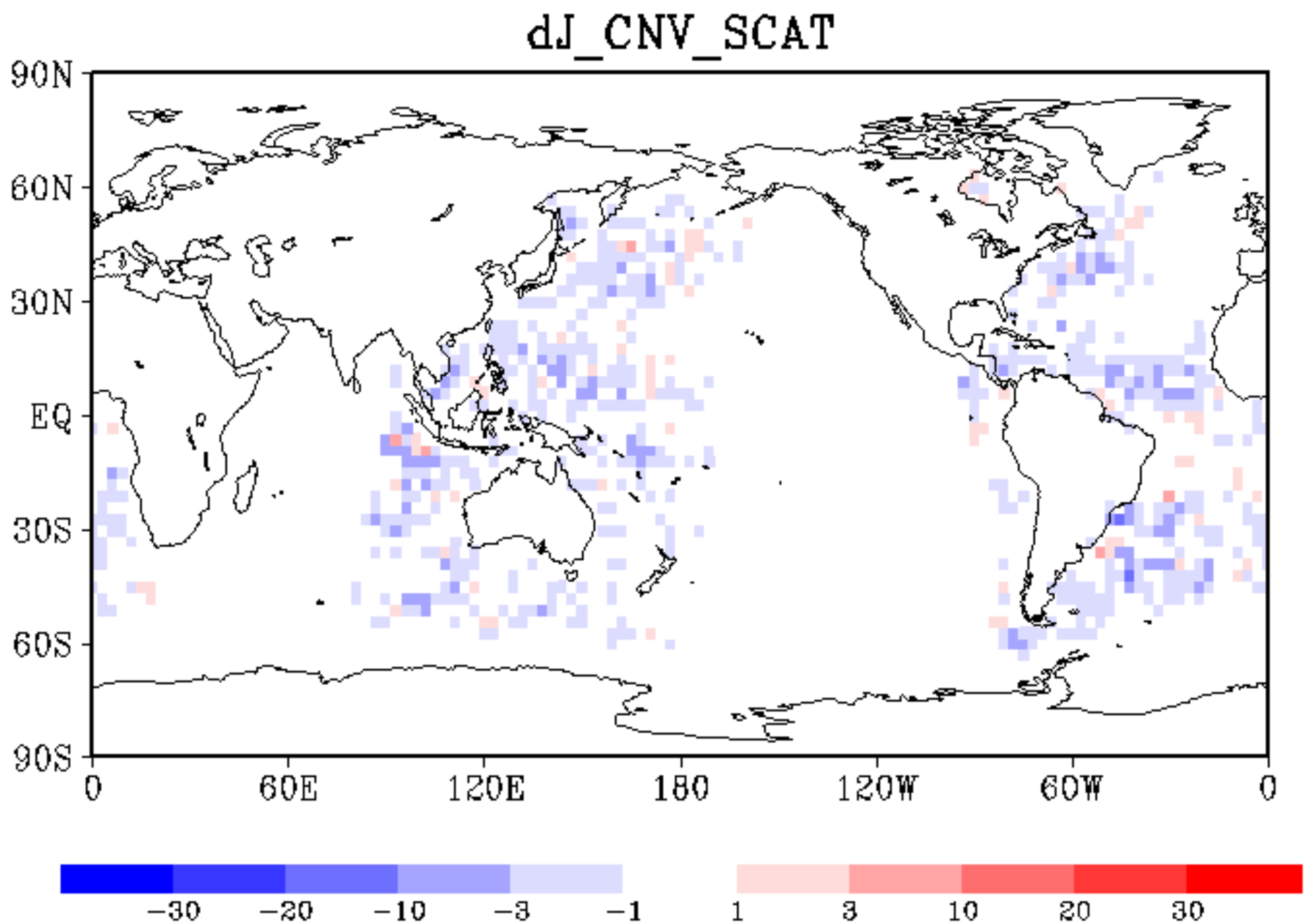
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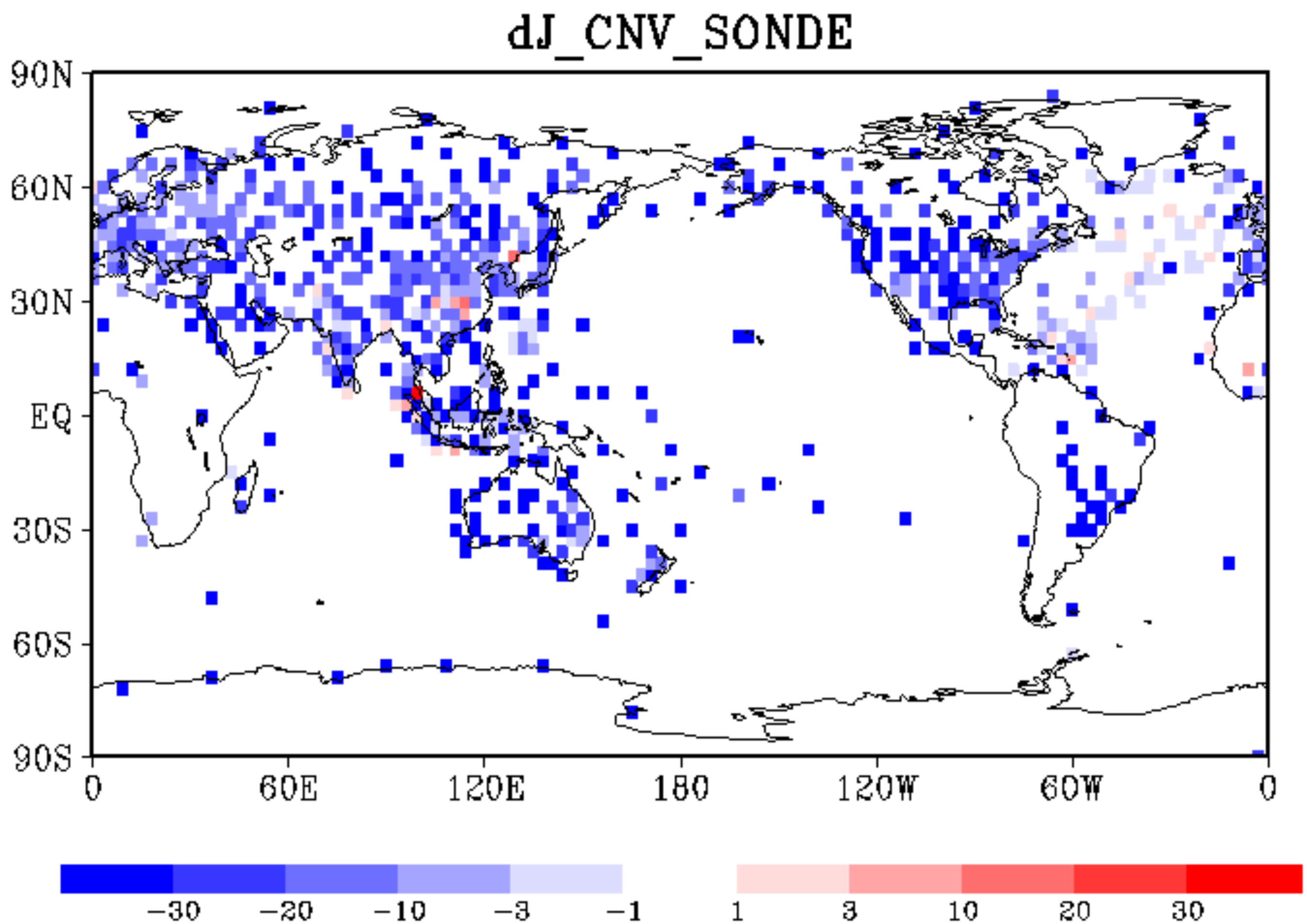
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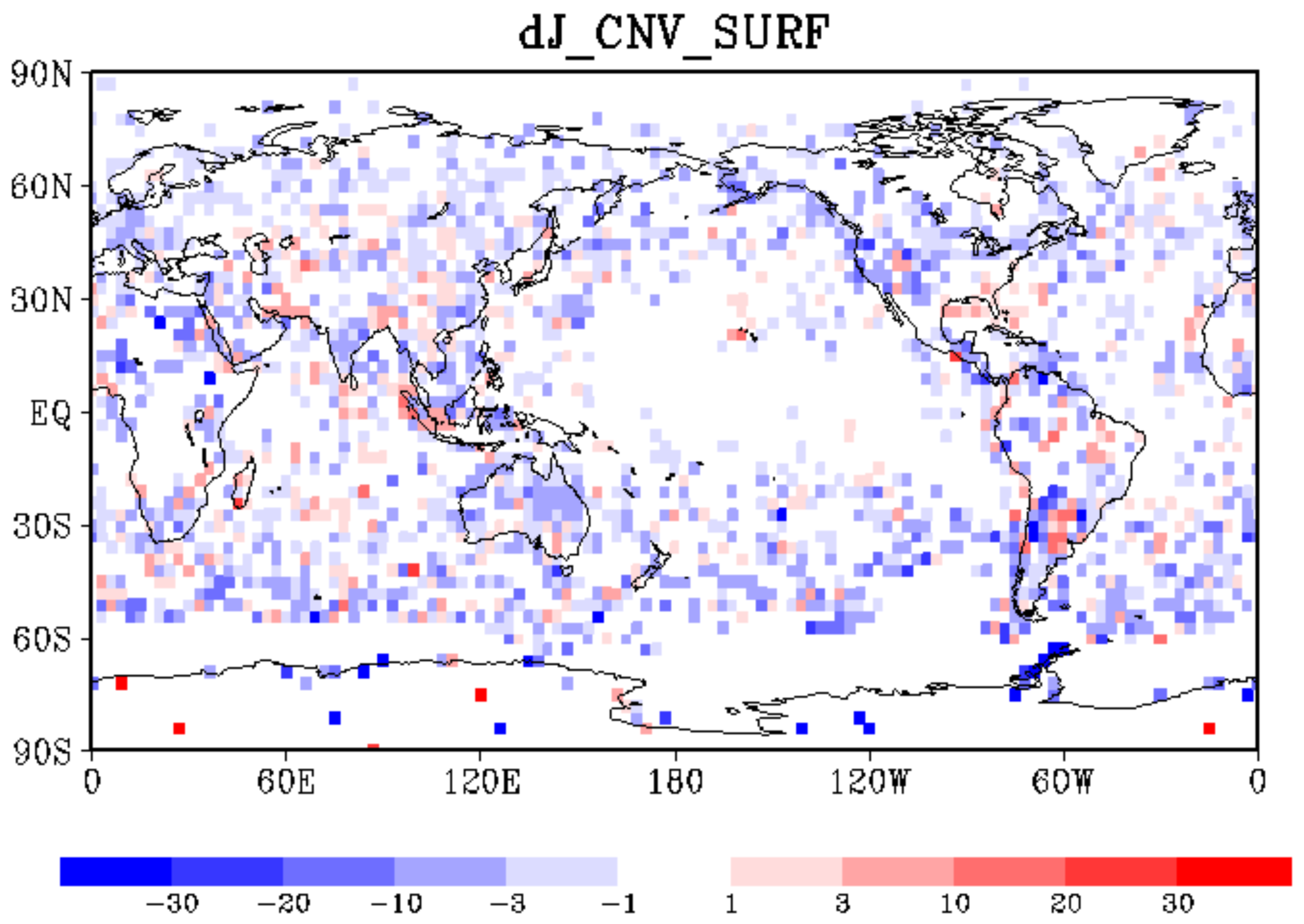

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−10
−3
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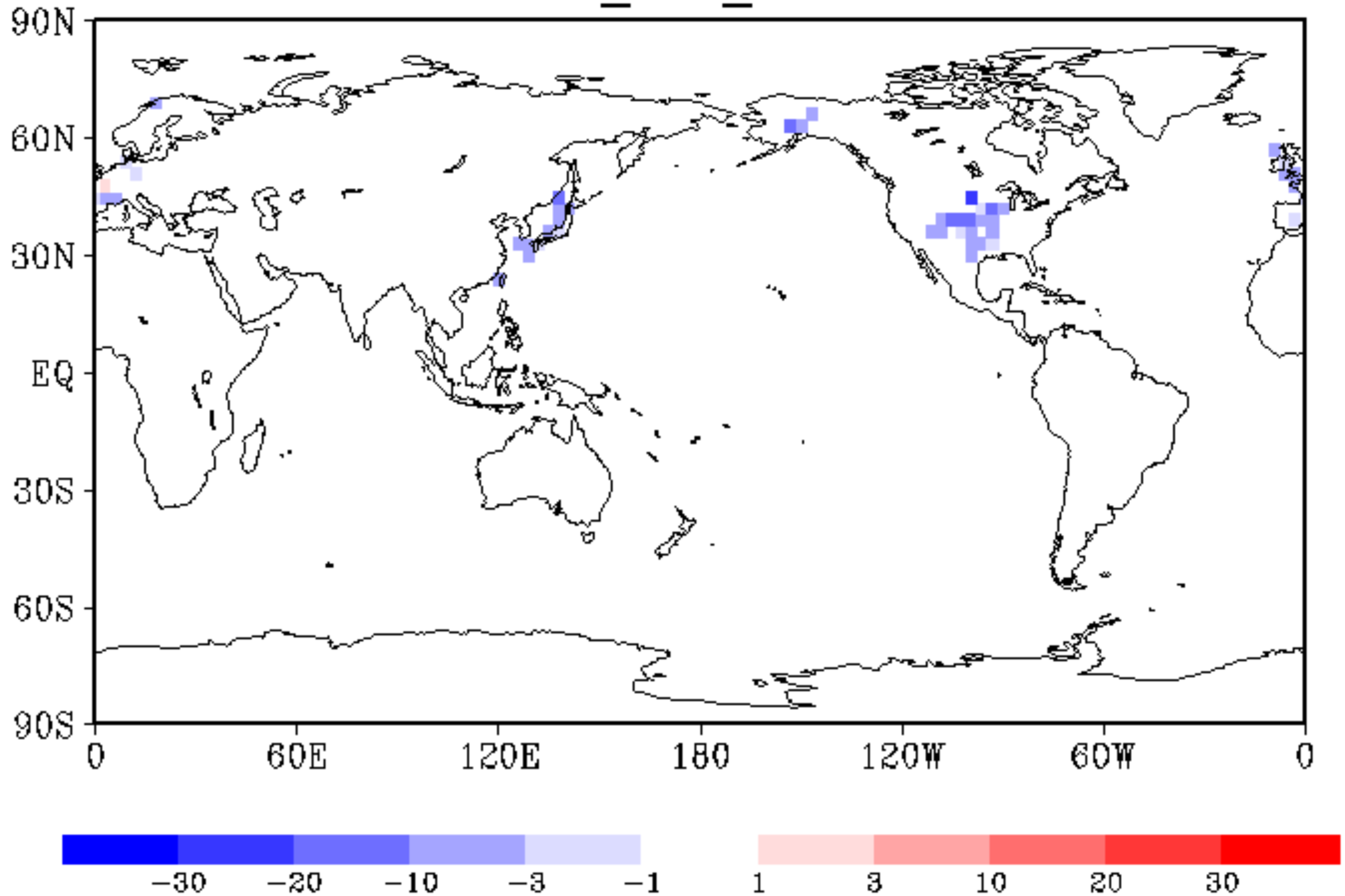
1
3
10
20
30

dJ_CNV_WPR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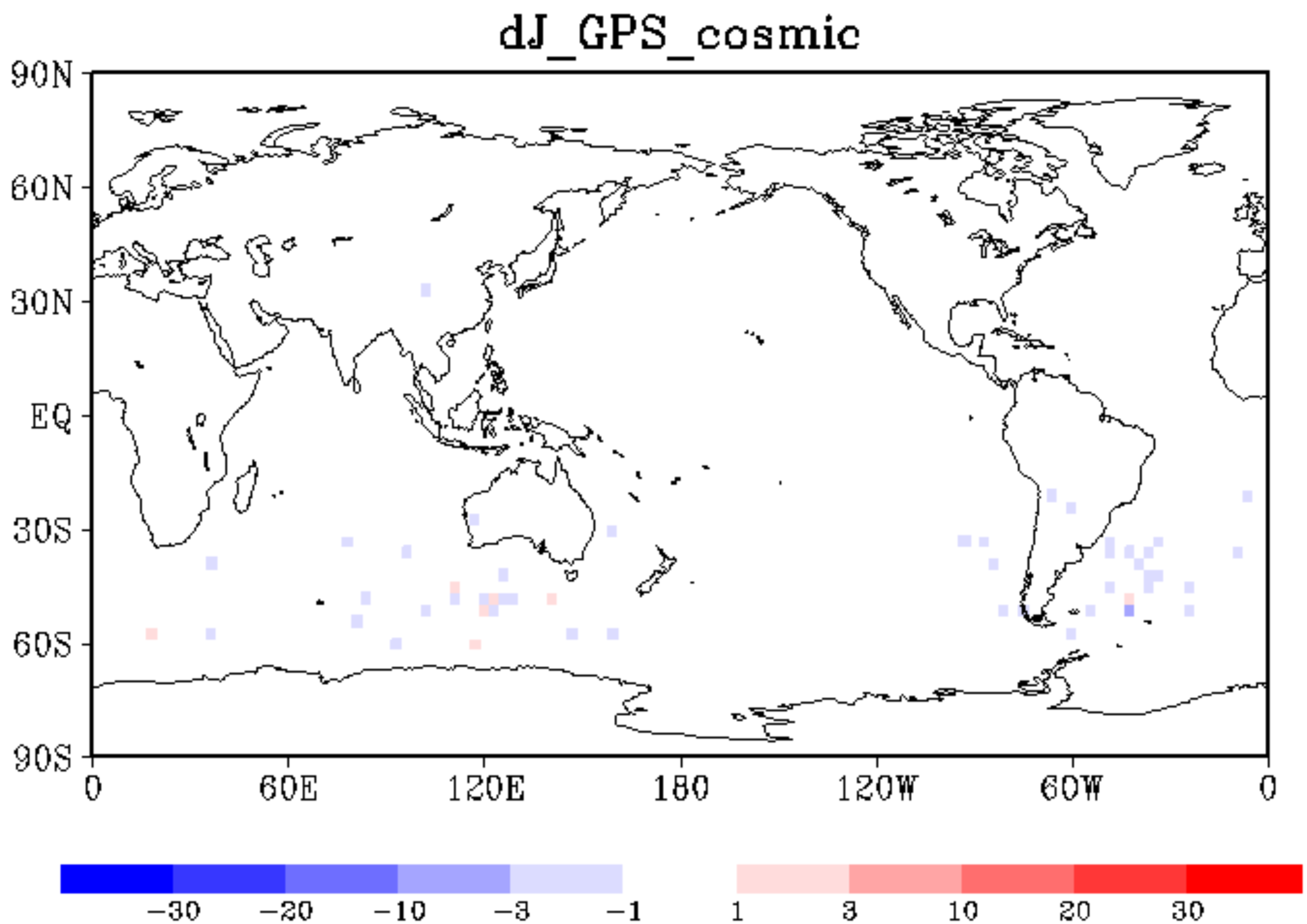
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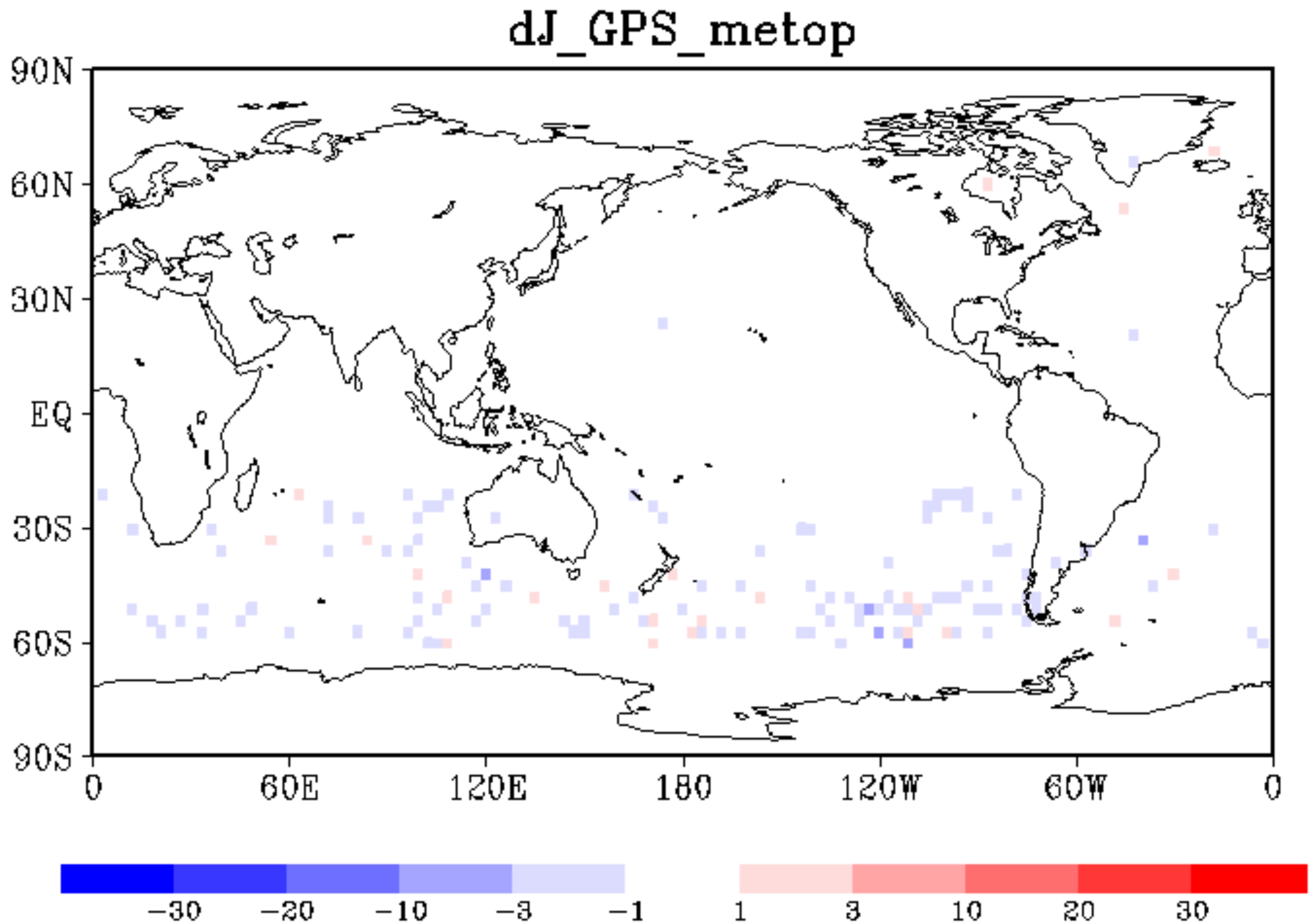
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1
3
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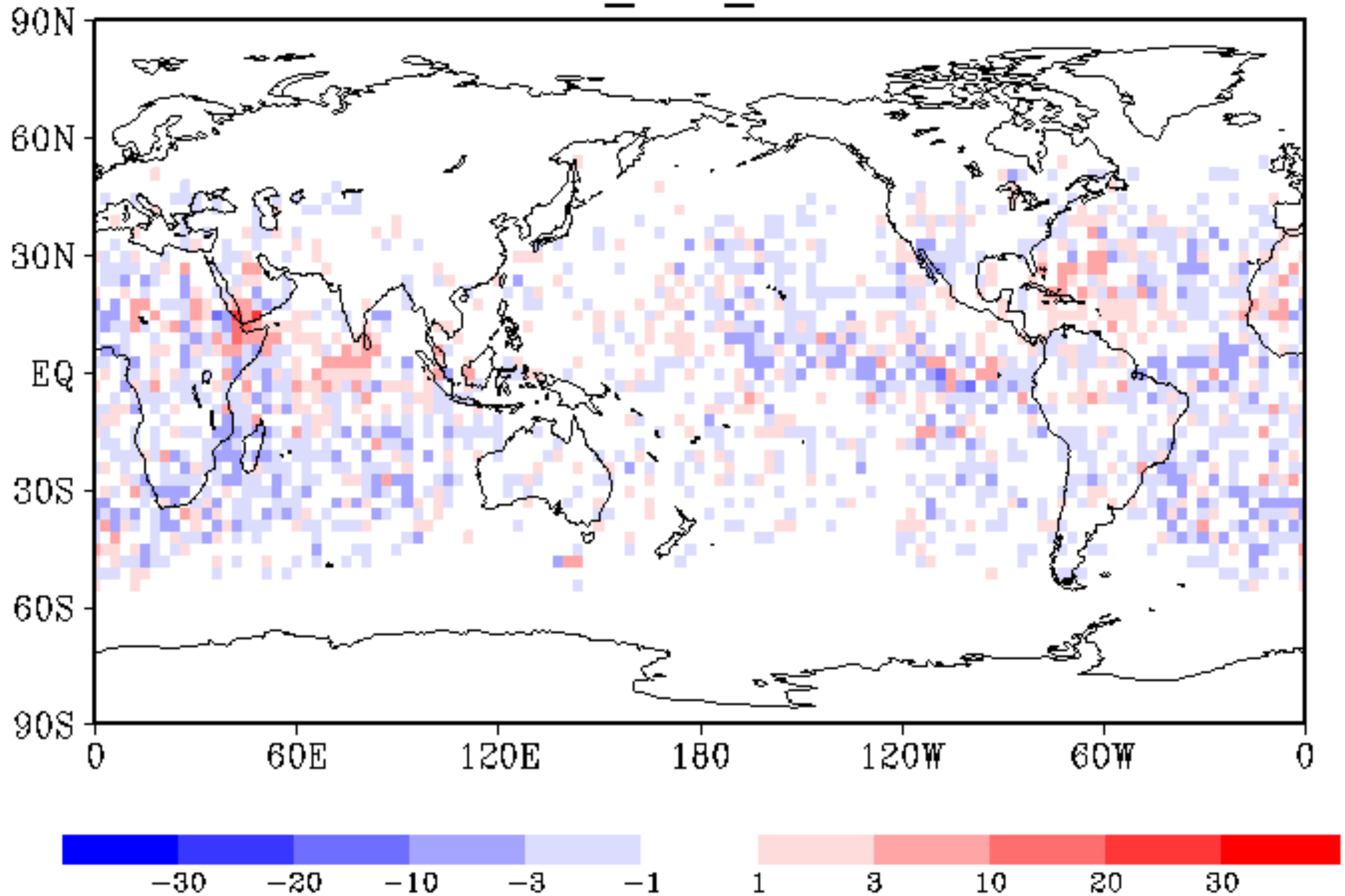
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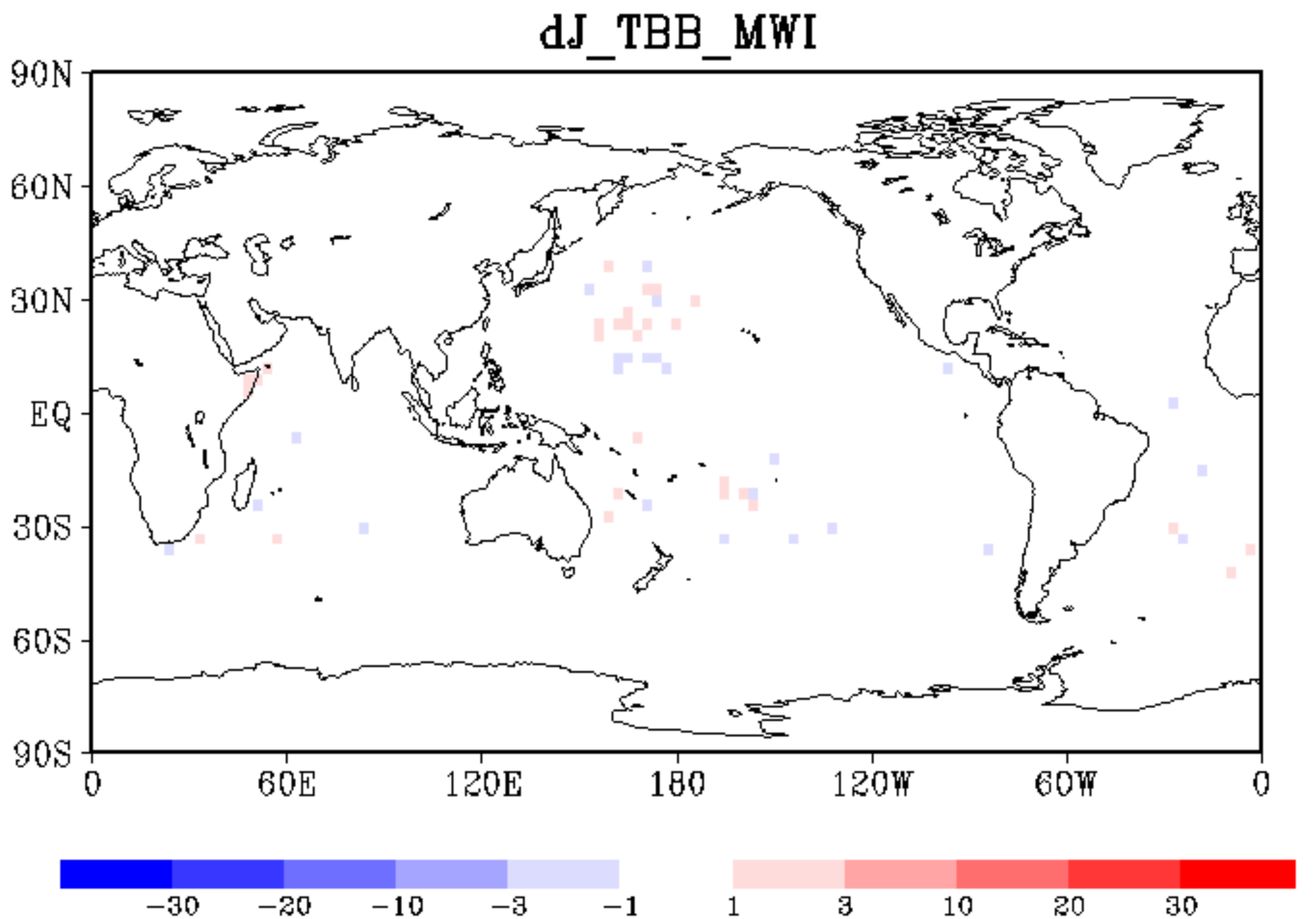
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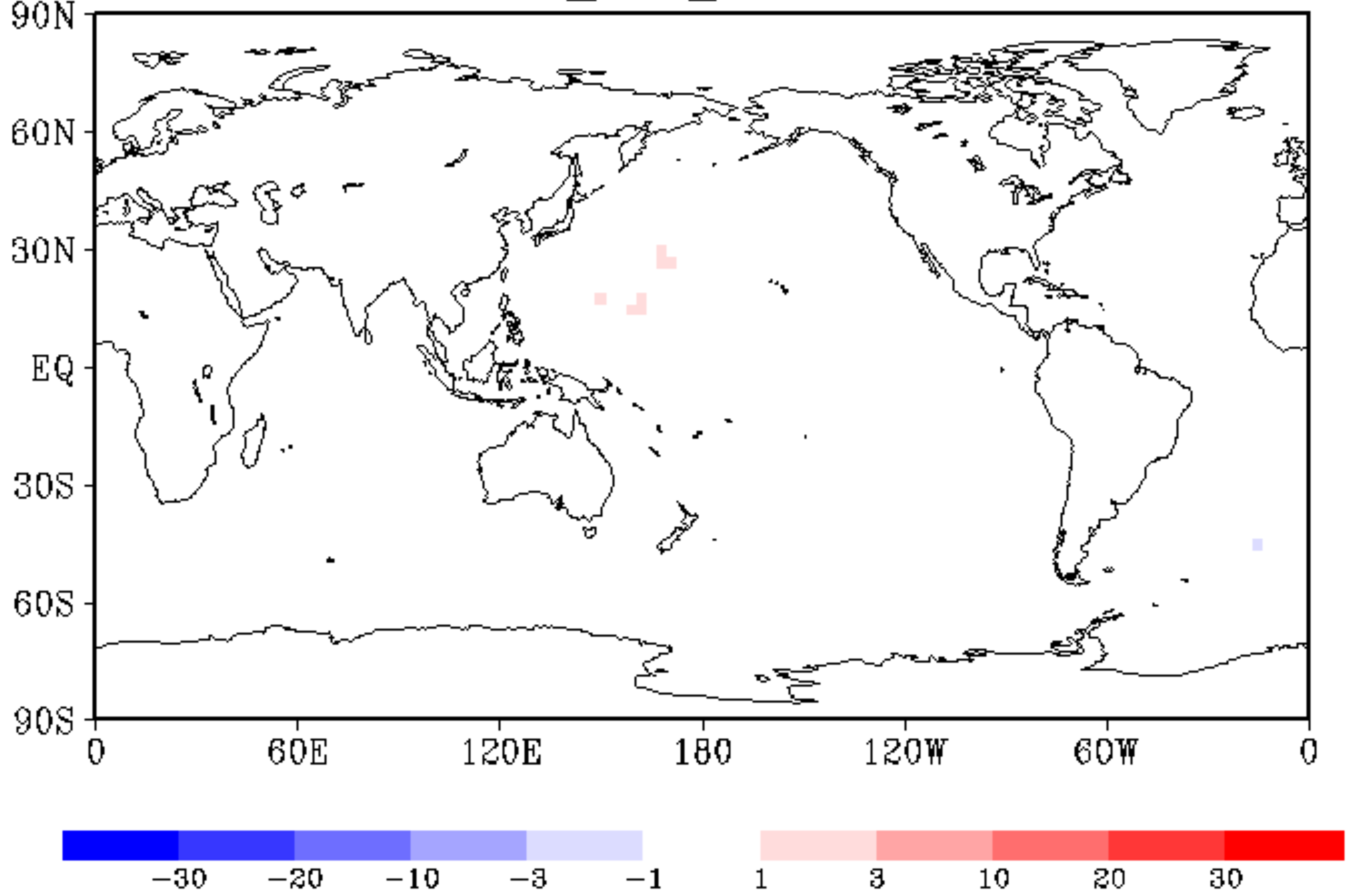
dJ_TBB_MWS-Q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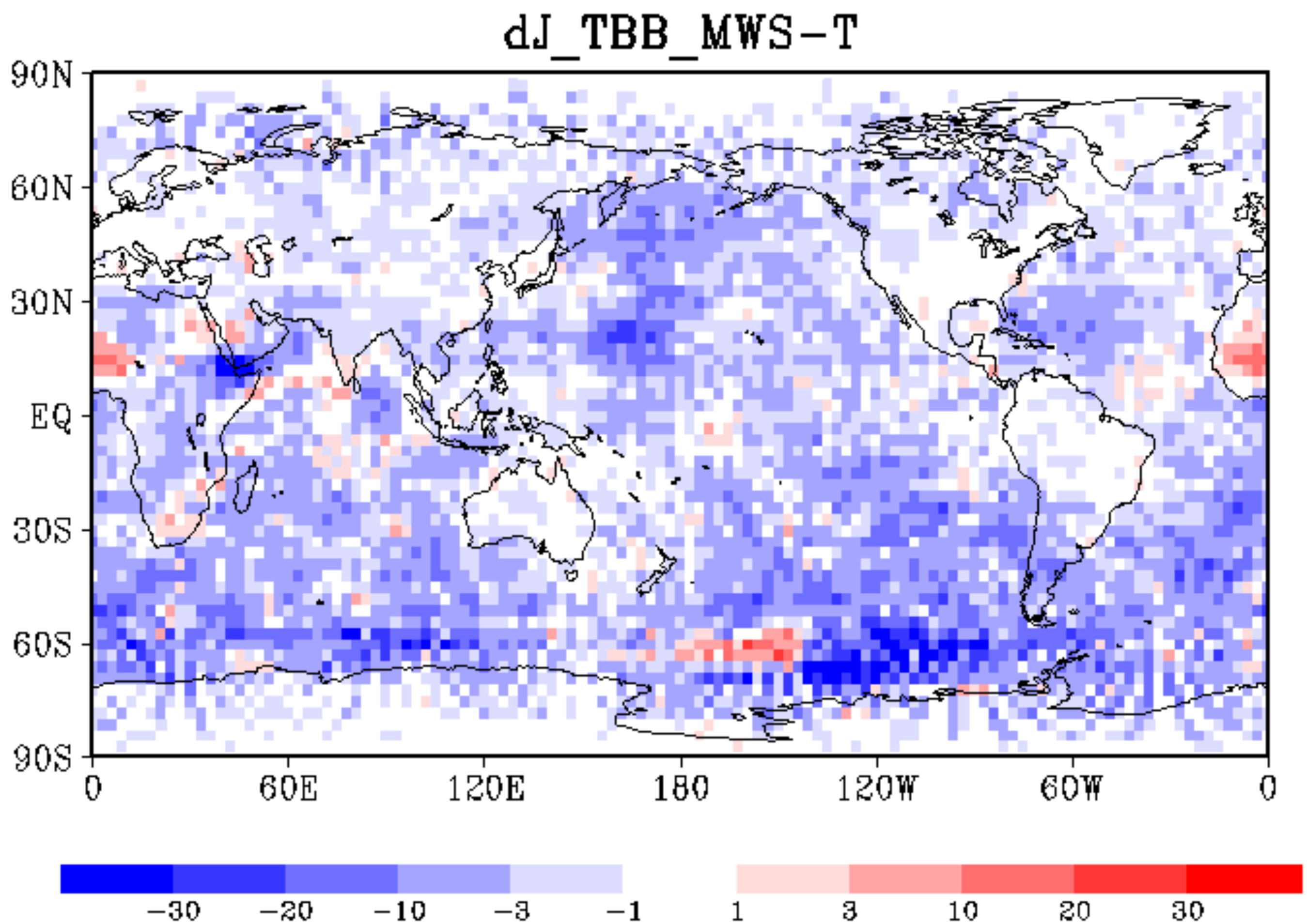
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10
20
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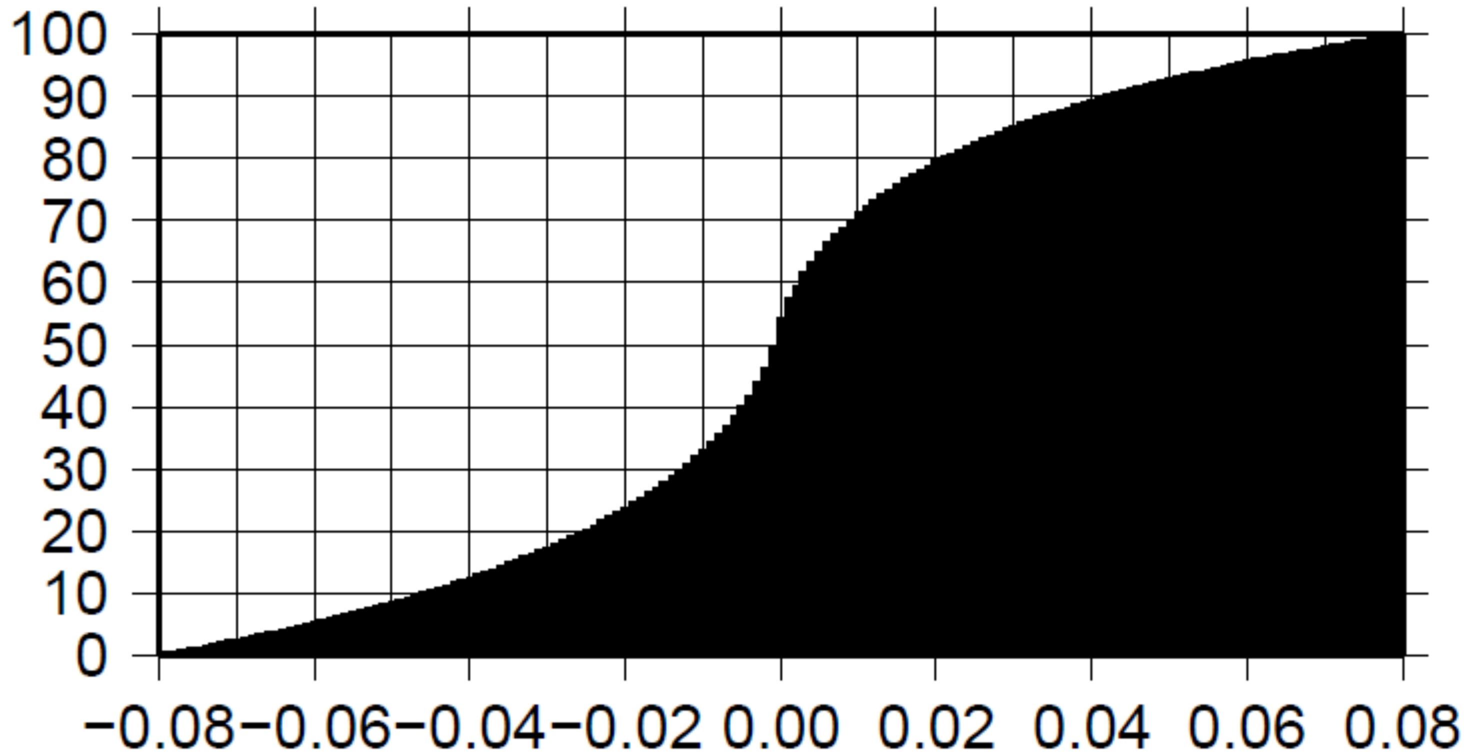
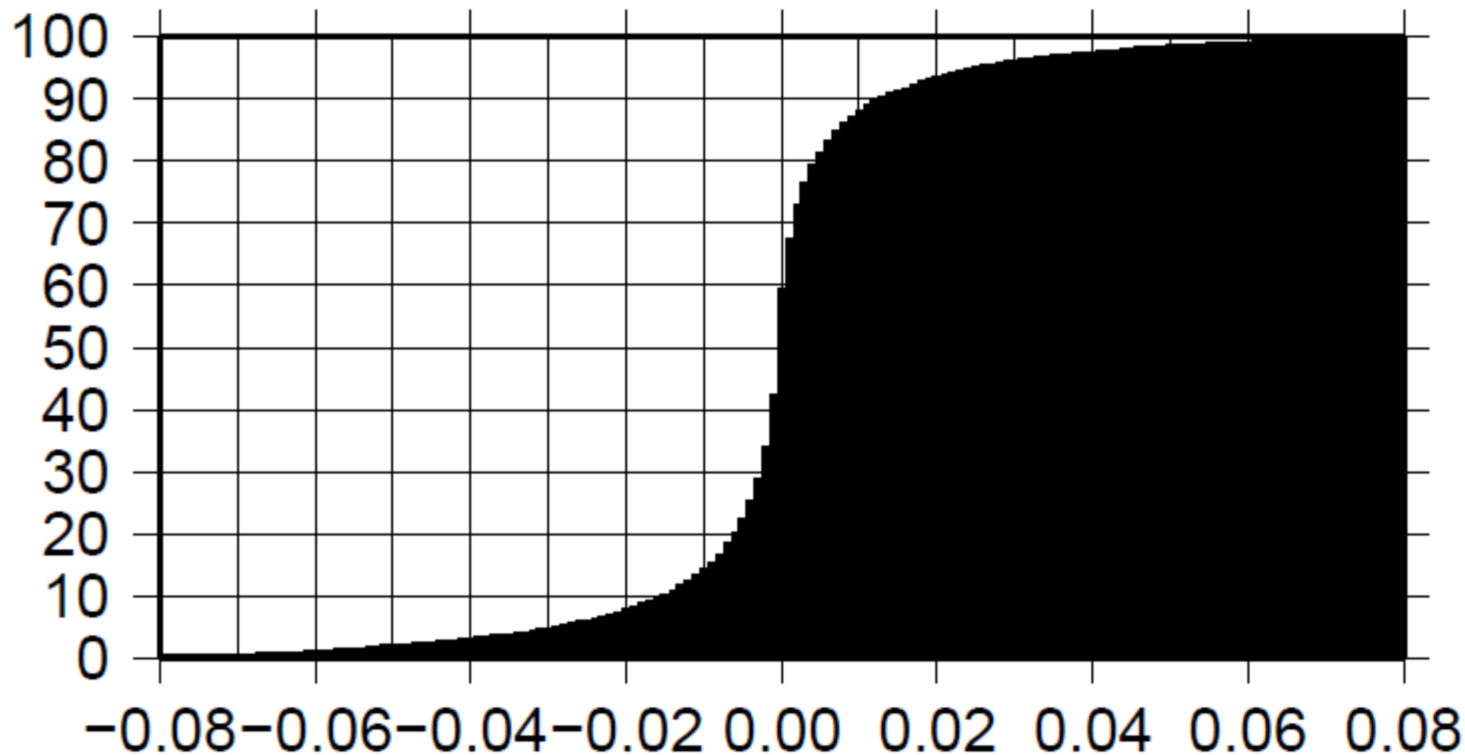
SONDE



COSMIC



Optimization with DIC

- Theoretical relationships;

$$2E\left[J_o\right] = Tr\left[\mathbf{I} - \mathbf{HK}\right] \qquad 2E\left[J_b\right] = Tr\left[\mathbf{KH}\right]$$

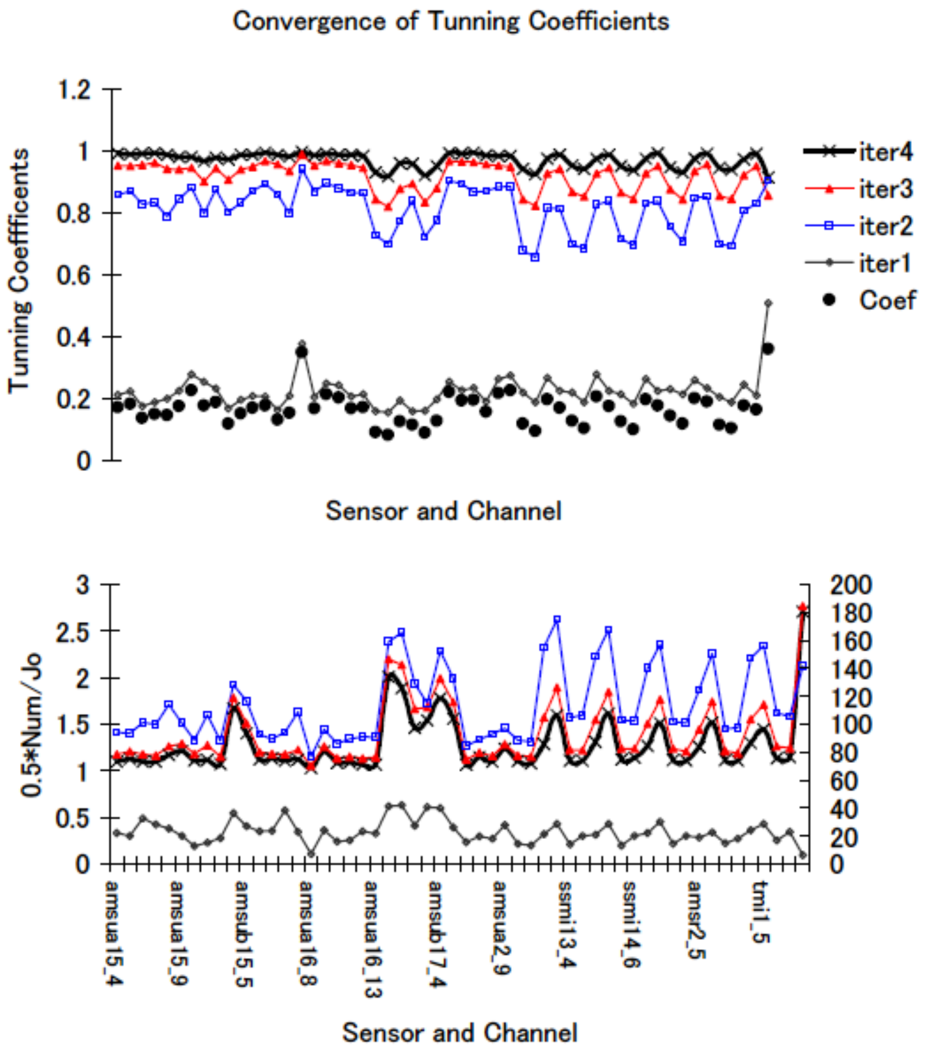
Where, Jo and Jb are observation term, and background term of cost function, respectively

- In real DASs, these relation ships are generally not satisfied because error covariance matrices using in the DASs are different from true ones.
- DIC tunes **B** and **R** using in the DASs to satisfy these relation ships.

DIC experimental results in DAS/ JMA

- About four times iteration of DIC are enough to convergence of tuning coefficients.
- DIC shows optimal observation error settings of satellite radiance are 20 to 30% of current settings.
- After DIC optimization, relation ships between value of cost function and observation data number further meet the theoretical relationship; J=0.5N.

- However, we have also found OSE with these optimized covariance matrices concluded in significant forecast error increase. This is because our forecast model has large dry (wet) biases in mid (low) altitude troposphere, and DIC improve these biased in analysis fields, but model give wrong strong response (rain out watervapors in ITCZ and breal Hadley circulation)

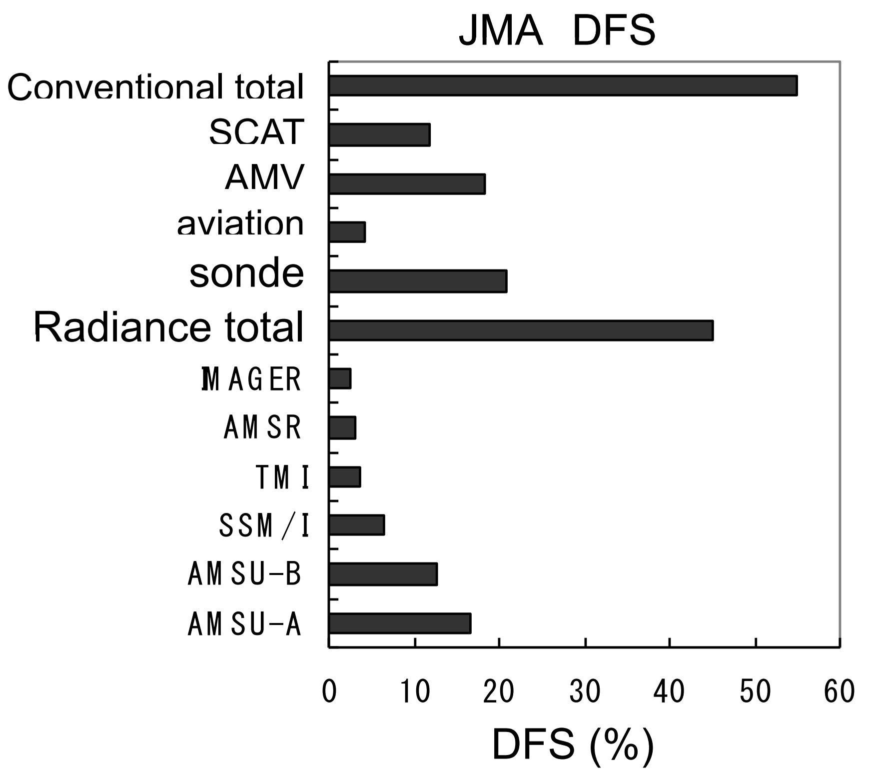


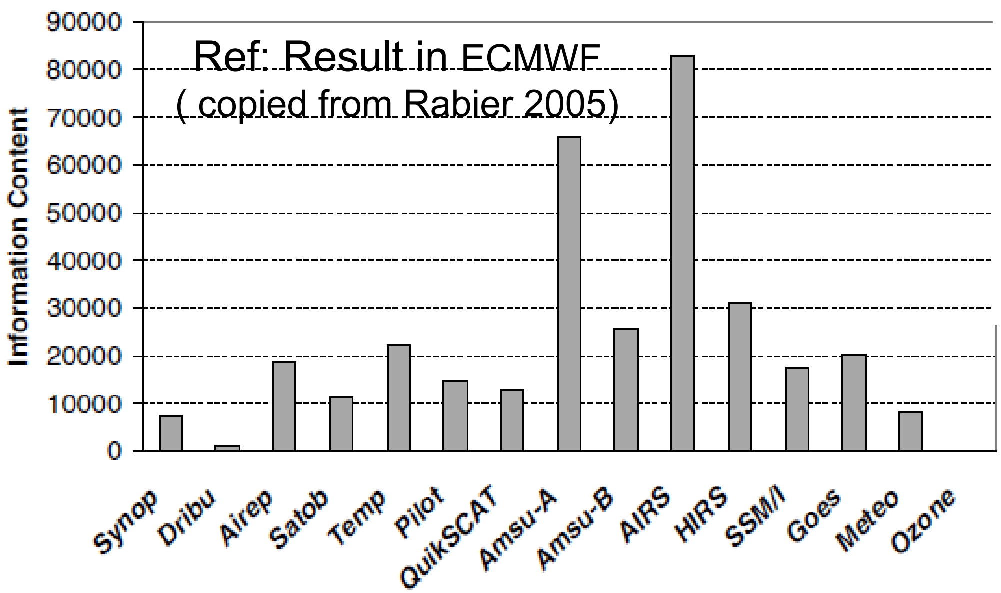
DFS in DAS/JMA

- Definision of DFS

$$\mathrm{DFS} \equiv (\mathbf{B}-\mathbf{A})\mathbf{B}^{-1} = Tr[\mathbf{KH}]$$

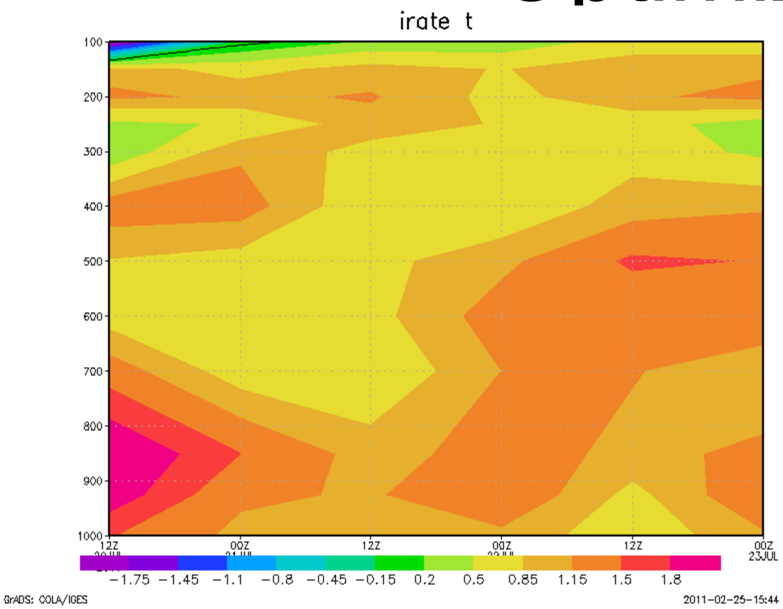
- Radiance total < Conventional total
- Satellite total > others
  - Sattelite includes raiances, scatterometer derived winds, and satellite winds
  - Others=conventional – AMV-SCAT
- Largest contribution from AMSU-A/B.

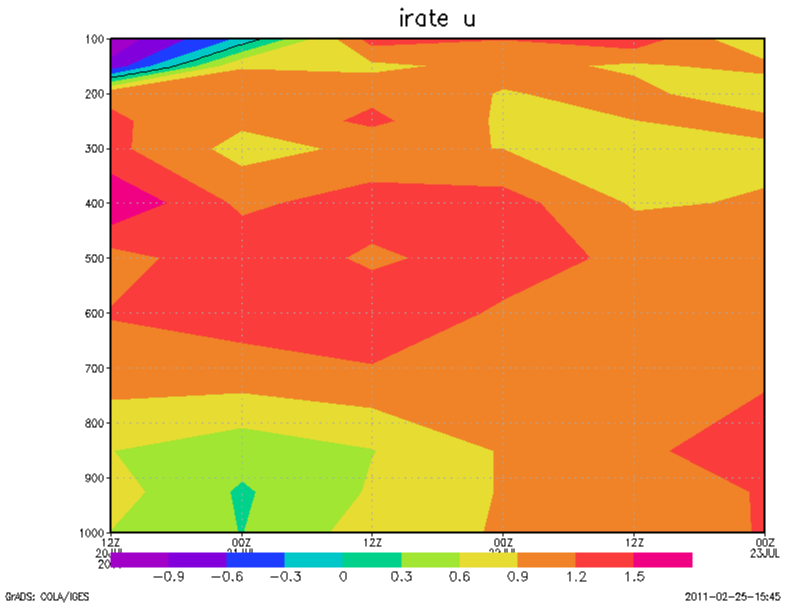


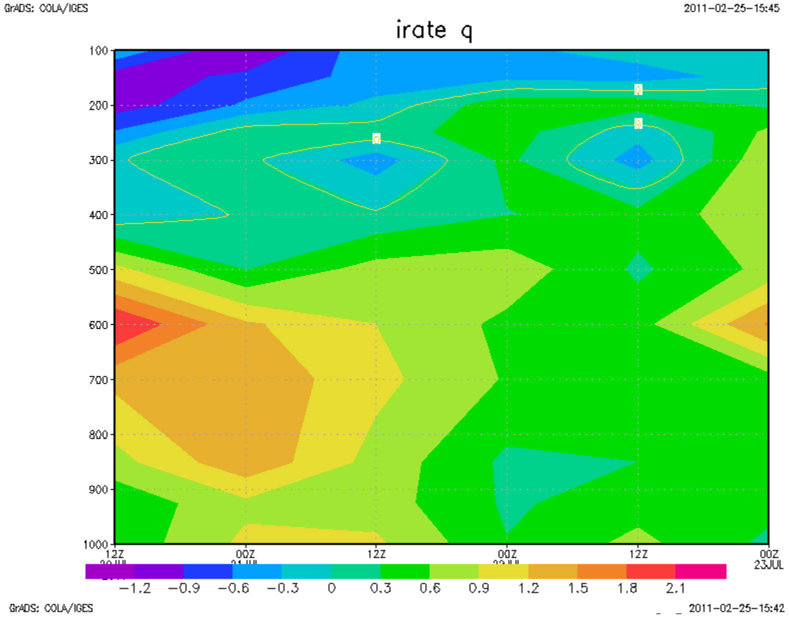


Pilot+Temp<Amsu-A

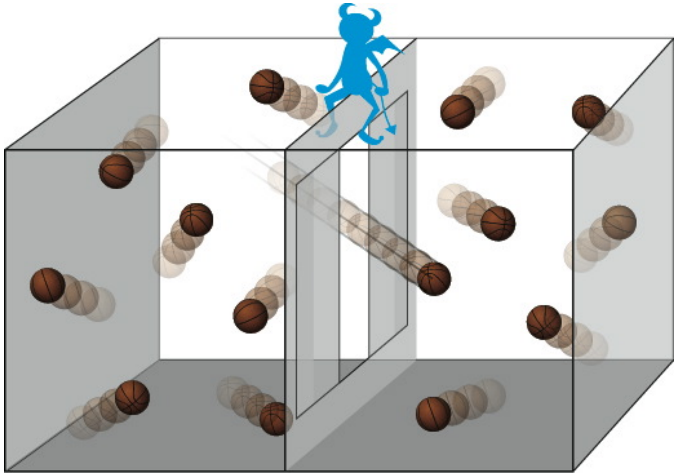
Optimization with D08





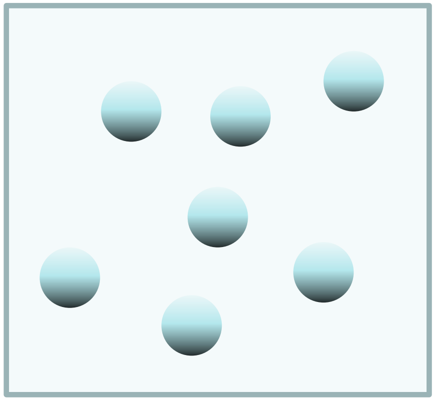


Maxwell demon ?

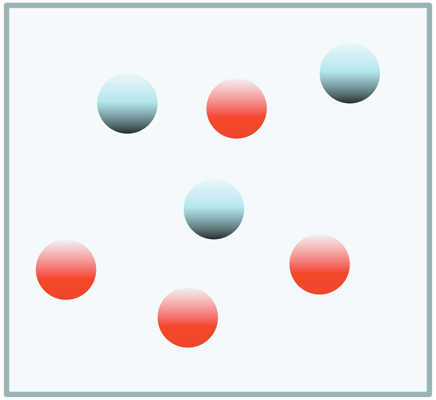


From Bennett CH and Schumacher B (2011)

Thermal equilibrium at temperature T.
We can get usable energy from max entropy state with Maxwell demon.
It knows velocity of each particle, while we know only statistical property, temperature.



Only Statistical property, R and B.



We know property of each observation and can use this information.

Adjoint Method

(Langrad and Baker 2004, Errico 2007)

Sensitivity

Jは予報場の特徴を表す適当なスカラー関数

$$\mathbf{S}_y \equiv \partial J/\partial \mathbf{y} = \mathbf{K}^T(\partial J/\partial \mathbf{x}) = \mathbf{R}^{-T}\mathbf{H}\mathbf{A}^{-T}(\partial J/\partial \mathbf{x}) = \mathbf{R}^{-1}\mathbf{H}\mathbf{A}^{-1}(\partial J/\partial \mathbf{x})$$

$$\mathbf{S}_y \equiv \mathbf{R}^{-1}\mathbf{H}\mathbf{A}^{-1}\mathbf{S}_x \begin{cases} \mathbf{A}^{-1}\mathbf{S}_x \equiv \mathbf{g} \quad \Rightarrow \quad \mathbf{A}\mathbf{g} = \mathbf{S}_x \\ \mathbf{S}_y = \mathbf{R}^{-1}\mathbf{H}\mathbf{g} \end{cases}$$

$$\mathbf{K} = \left(\mathbf{B}^{-1} + \mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{R}^{-1} \equiv \boxed{\mathbf{A}^{-1}}\mathbf{H}^T\mathbf{R}^{-1}$$

$$\delta\mathbf{x} = \mathbf{K}\mathbf{d} = \mathbf{A}^{-1}\mathbf{H}^T\mathbf{R}^{-1}\mathbf{d}$$

$$\mathbf{S}_y \equiv \partial J/\partial \mathbf{y}$$

$$\mathbf{S}_x \equiv \partial J/\partial \mathbf{x}$$

Compare

Analysis equations	*Sensitivity equations*
$\mathbf{A}\delta\mathbf{x} = \mathbf{H}^T\mathbf{R}^{-1}\mathbf{d}$	$\mathbf{A}\mathbf{g} = \mathbf{S}_x$
$\partial J/\partial \mathbf{x} = \mathbf{A}\delta\mathbf{x} - \mathbf{H}^T\mathbf{R}^{-1}\mathbf{d}$	$\partial J/\partial \mathbf{g} = \mathbf{A}\mathbf{g} - \mathbf{S}_x$
$J = 1/2\,\delta\mathbf{x}^T\mathbf{A}\delta\mathbf{x} - \delta\mathbf{x}^T\mathbf{H}^T\mathbf{R}^{-1}\mathbf{d}$	$J = 1/2\,\mathbf{g}^T\mathbf{A}\mathbf{g} - \mathbf{g}^T\mathbf{S}_x$

モデル空間の4DVARの解析とLB2004の感度解析は、ともにAを係数行列とする連立方程式を解く問題である。両者の違いは右辺ベクトルの違いだけである。

Thank you for listening

Comments or Questions ?

Please speak very slowly!