A temperature dependent viscoelastic damage model for creep fracture in ice sheets

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Abstract

We propose a three-dimensional thermo-viscoelastic constitutive damage model law for ice creep, suited for ice in polar regions. The model has been validated by published laboratory experimental data and is implemented in the commercially available finite element code ABAQUS by adopting a strain-based algorithm in a Lagrangian description. The model is used to investigate conditions that enable surface, englacial and basal crevasse formation resulting from different boundary conditions applied to an idealized rectangular slab of ice in contact with the ocean.

Viscoelastic Constitutive Model

- **Additive decomposition:** Assuming small strains [1],

$$\epsilon_{kl} = \epsilon^{e}_{kl} + \epsilon^{d}_{kl} + \epsilon^{v}_{kl}, \tag{1}$$

where the superscripts e, d, and v denote the elastic, delayed elastic and viscous components, respectively.

- **Stress-strain relations:** The strain components are given by,

$$\tilde{\sigma}_{kl} = \frac{E}{3(1-2\nu)}\,\epsilon^{e}_{ii}\delta_{kl} + \frac{E}{(1+\nu)}\left(\epsilon^{e}_{kl} - \tfrac{1}{3}\epsilon^{e}_{ii}\delta_{kl}\right), \tag{2}$$

$$\dot{\epsilon}^{d}_{kl} = A\left(\tfrac{3}{2}K\tilde{\sigma}^{dev}_{kl} - \epsilon^{d}_{kl}\right), \tag{3}$$

$$\dot{\epsilon}^{v}_{kl} = \tfrac{3}{2}\,K_N\left(\tfrac{3}{2}\tilde{\sigma}^{dev}_{mn}\tilde{\sigma}^{dev}_{mn}\right)^{(N-1)/2}\tilde{\sigma}^{dev}_{kl}, \tag{4}$$

where E and ν are the Young's modulus and Poisson's ratio, respectively; K_N and N are the viscous parameters; A and K are delayed elastic material parameters; $\tilde{\boldsymbol{\sigma}}^{dev}$ is the deviatoric part of the effective stress denoted by $\tilde{\boldsymbol{\sigma}}$.

- **Temperature dependence:** The relation for K_N is,

$$K_N(T) = K_N(T_m)\exp\left(\frac{-Q}{R}\left(\frac{1}{T} - \frac{1}{T_m}\right)\right), \tag{5}$$

where Q is the activation energy for creep; R is the universal gas constant or the Boltzmann's constant; T is the temperature and the reference temperature, $T_m = -10^\circ$C.

Continuum Damage Model

- **Effective stress concept:** We define a transformation,

$$\tilde{\sigma}_{ij} = M_{ijkl}\,\sigma_{kl}, \tag{6}$$

$$M_{ijkl} = \tfrac{1}{2}(\omega_{ik}\delta_{jl} + \omega_{jk}\delta_{il}), \;\; \omega_{ik}(\delta_{kj} - D_{kj}) = \delta_{ij}. \tag{7}$$

where $\boldsymbol{M}$ is the damage effect tensor, $\boldsymbol{D}$ is the damage tensor and $\boldsymbol{\sigma}$ denotes the Cauchy stress tensor.

- **Damage rate:** In a Lagrangian framework assuming small strains,

$$\dot{D}_{ij} = \begin{cases} f_{ij}, & \text{if} \quad \max\{\epsilon_{ij}\} \geq \epsilon_{th}, \\ 0, & \text{if} \quad \max\{\epsilon_{ij}\} < \epsilon_{th}. \end{cases} \tag{8}$$

where f_{ij} is the damage evolution function; and ϵ_{th} is a strain threshold for damage initiation.

- **Damage evolution function:** The generalized form reads [2, 3],

$$f_{ij} = B\langle\chi\rangle^{r}\left(\omega_{mn}\xi^{(1)}_{m}\xi^{(1)}_{n}\right)^{k_\sigma}\left[(1-\gamma)\,\delta_{ij} + \gamma\xi^{(1)}_{i}\xi^{(1)}_{j}\right], \tag{9}$$

$$\chi = \alpha\tilde{\sigma}^{(1)} + \beta\sqrt{\frac{3}{2}\tilde{\sigma}^{dev}_{mn}\tilde{\sigma}^{dev}_{mn}} + (1-\alpha-\beta)\tilde{\sigma}_{kk}, \tag{10}$$

where $B, r, k_\sigma, \alpha, \beta, \gamma$ are model parameters; $\tilde{\sigma}^{(1)}$ is the maximum eigenvalue of $\tilde{\boldsymbol{\sigma}}$; $\boldsymbol{\xi}^{(1)}$ is the eigenvector associated with $\tilde{\sigma}^{(1)}$; χ is the Hayhurst's equivalent stress measure and a function of the effective stress tensor, $\tilde{\boldsymbol{\sigma}}$; the Macaulay brackets are defined such that, $\langle\chi\rangle = \chi$ if $\chi \geq 0$ and $\langle\chi\rangle = 0$ if $\chi < 0$.

- **Tension-compression asymmetry:** The different behavior of ice under compression and tension is captured by $k_{\boldsymbol{\sigma}}$ as,

$$k_{\boldsymbol{\sigma}} = \begin{cases} [k_1 + k_2|\sigma_{ii}|], & \text{for} \quad 0 \leq \sigma_{ii} \leq 1\text{ MPa}, \\ -[k_3 + k_4|\sigma_{ii}|], & \text{for} \quad -3\text{ MPa} \leq \sigma_{ii} < 0. \end{cases} \tag{11}$$

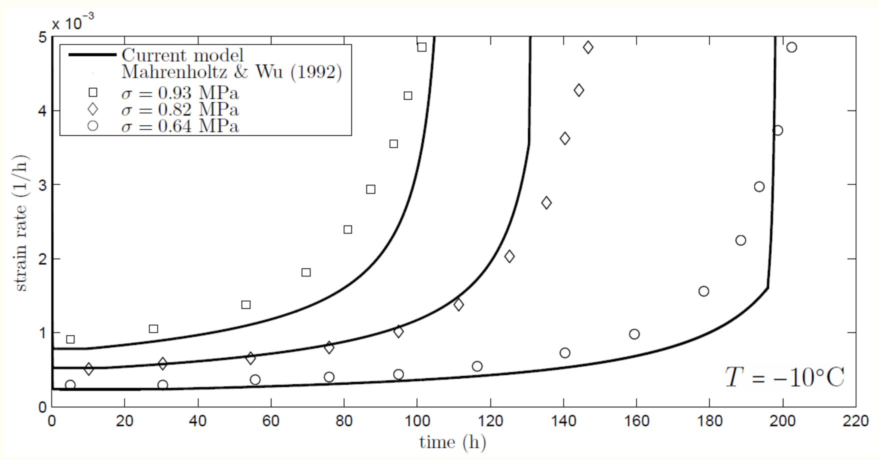
where k_1, k_2, k_3, and k_4 are determined using a linear fit.

- **Temperature dependence:** The relation B is,

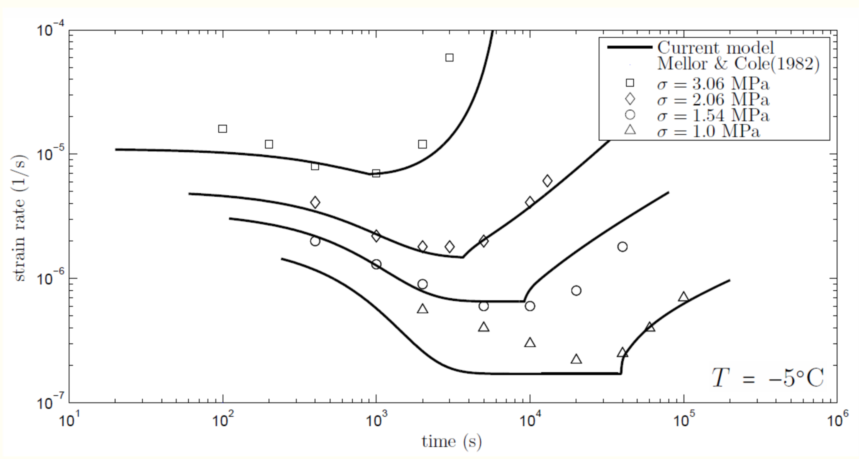
$$B(T) = B(T_m)\exp\left(\frac{-U}{R}\left(\frac{1}{T} - \frac{1}{T_m}\right)\right), \tag{12}$$

where U is the activation energy for damage.

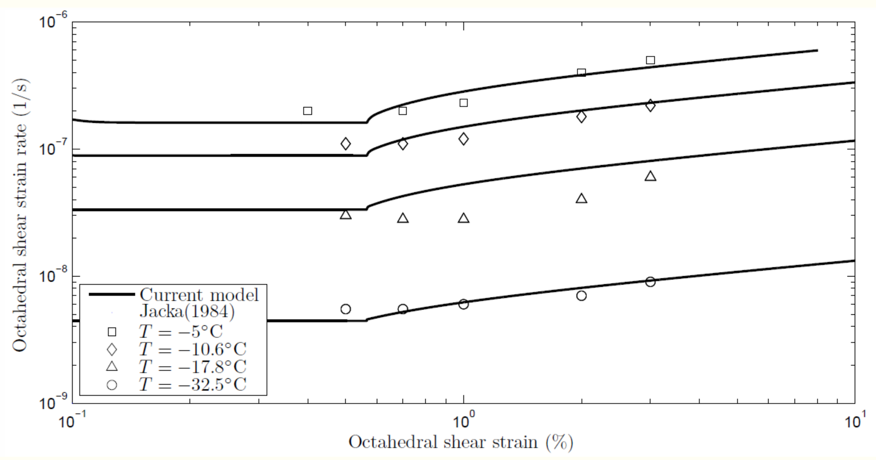
Model Calibration



(a) Ice under uniaxial tension



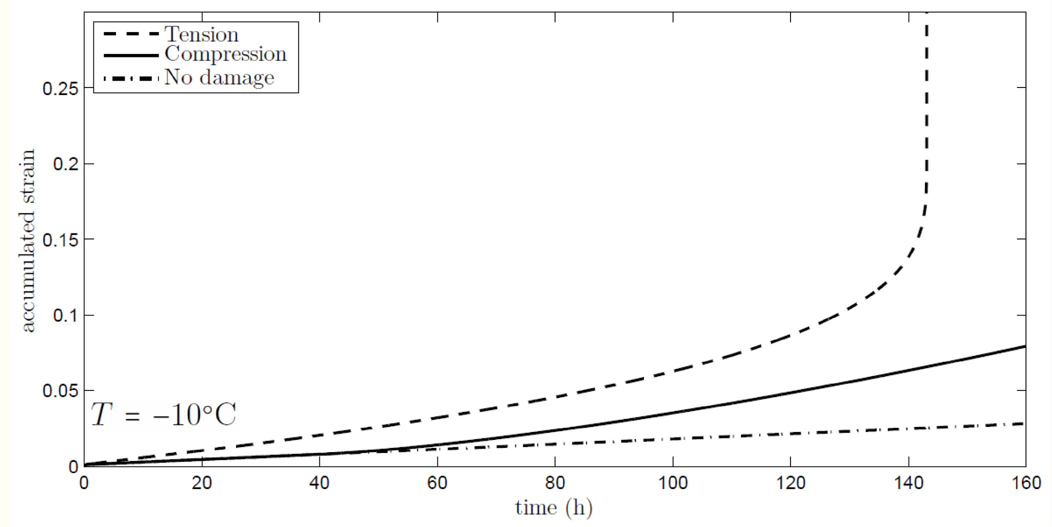
(b) Ice under uniaxial compression



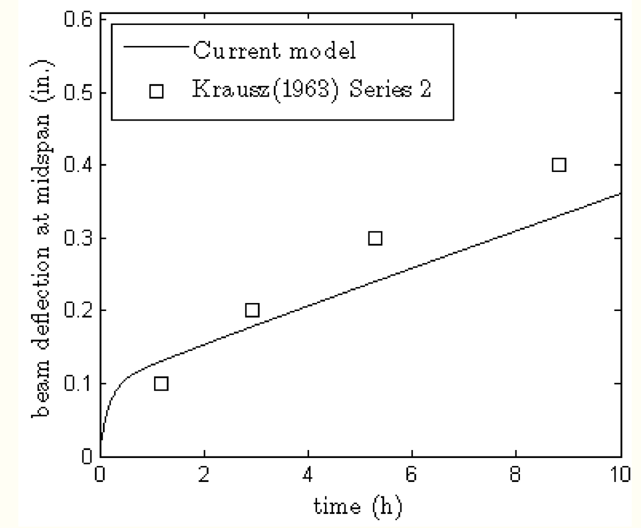
(c) Temperature dependence of K_N and B

The model parameters E, ν, A, K, K_N, N describe the constitutive behavior; $\alpha, \beta, B, r, k_{\boldsymbol{\sigma}}$, and γ describe the orthotropic damage evolution; and Q, U describe the temperature dependence. Herein, these parameters are calibrated from the experimental curves of strain rate vs. time for uniaxial tension, log-log plots of strain rate vs. time for uniaxial compression. The log-log plots of octahedral shear strain rate vs. octahedral shear strain are used to calibrate the temperature dependence.

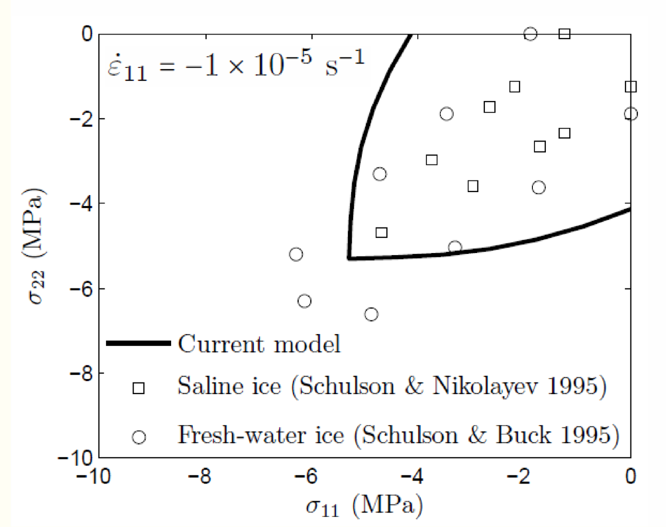
Model Validation



(d) Ice under uniaxial loading $|\sigma| = 0.8$ MPa



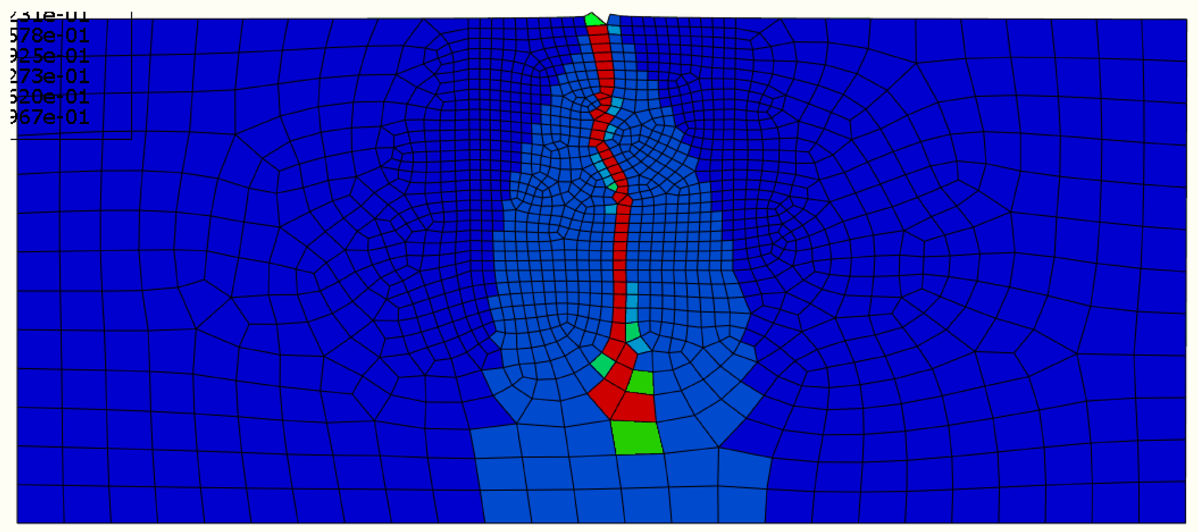
(e) Ice slab under four point bending



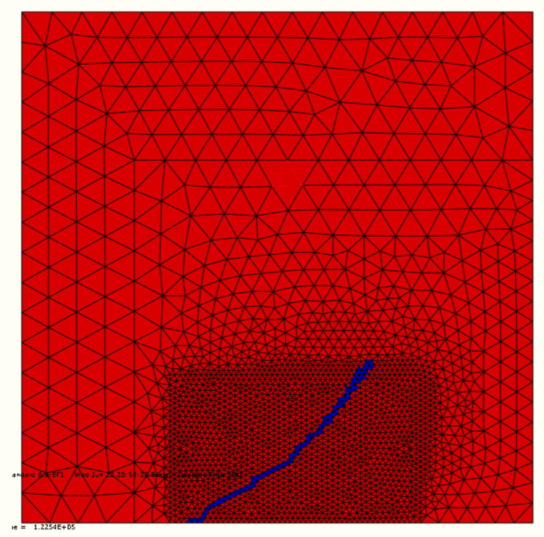
(f) Ice under biaxial compression

From figure (d), it is evident that after 140 hours ice under tension exhibits sudden rupture whereas no such failure is observed under compression, even at a later time. Figure (e) shows the good agreement between model results and experiments for the four point bending test in the initial stages of creep. From figure (f), it is apparent that the model is able to capture the increase in the strength of ice under biaxial compression, quite well.

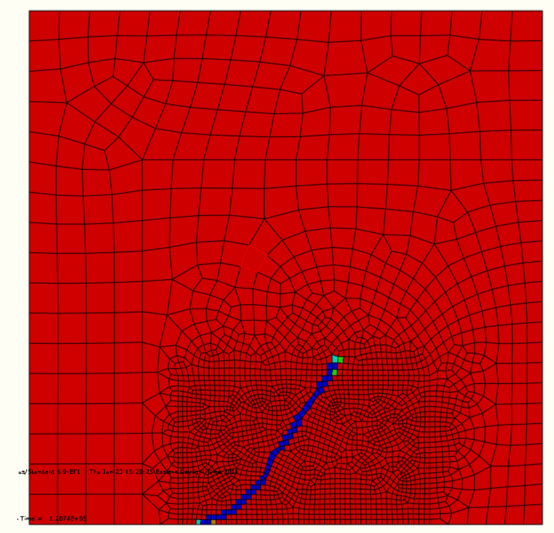
Ongoing Work



(g)Uniaxial tension - pure mode I (quadrilaterals)

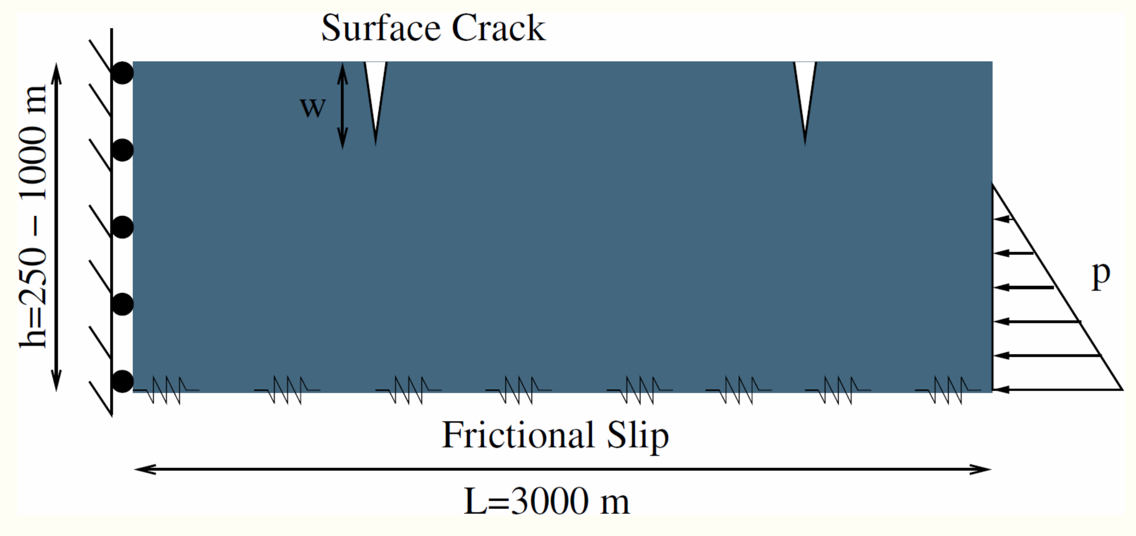


(h) Biaxial tension (triangles)

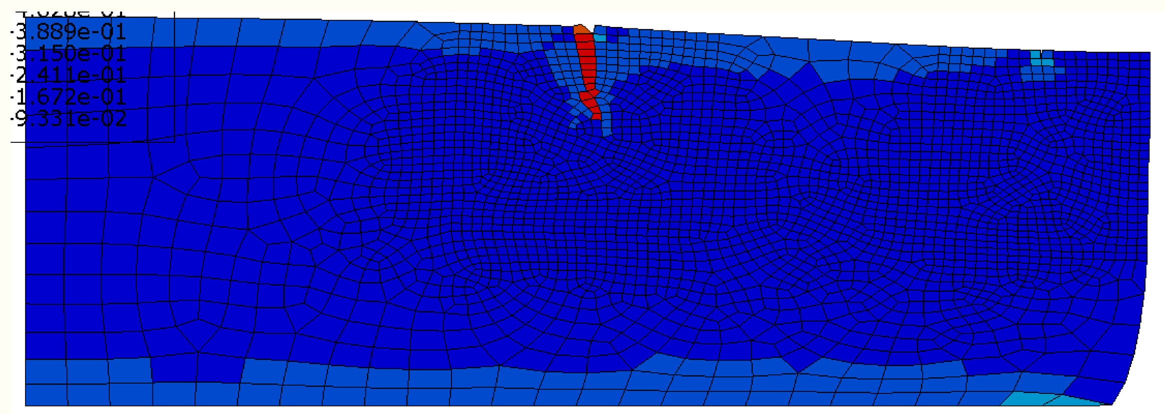


(i) Biaxial tension (quadrilaterals)

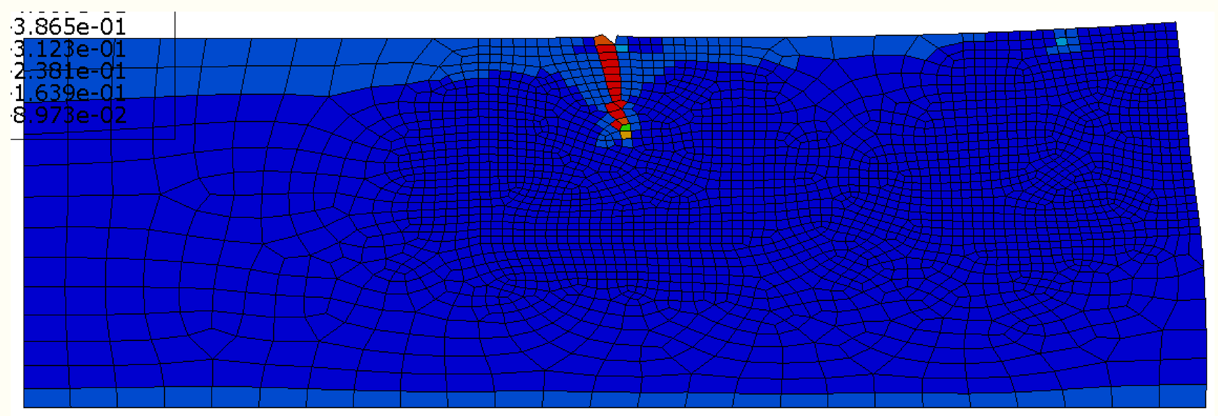
The mesh dependence of numerical results using the current local damage approach is clear from the above figures (g), (h) and (i). In order to ameliorate the numerical scheme and reduce its mesh dependence we are currently pursuing a nonlocal approach for damage evaluation. In the idealized ice slab simulations, three external forces are considered: (1) Gravity as a distributed body force; (2) Water pressure as surface pressure near the ocean-ice boundary; and (3) A stretching horizontal tensile stress for the expanding ice slab. The bottom boundary condition can be varied from free slip to no slip using spring elements at the bottom boundary, as indicated in figure (j).



(j) Crevasse formation in idealized ice slabs (not to scale)



(k) No slip at the bottom



(l) Free slip at the bottom

The simulations results in figures (k) and (l) suggest that the surface crevasses are relatively unaffected by the bottom boundary conditions. The effect of the bottom boundary condition on basal cracks needs to be investigated.

References

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