New climate models: A discontinuous/continuous low order finite element shallow water model on the sphere with grid refinement capability

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Introduction

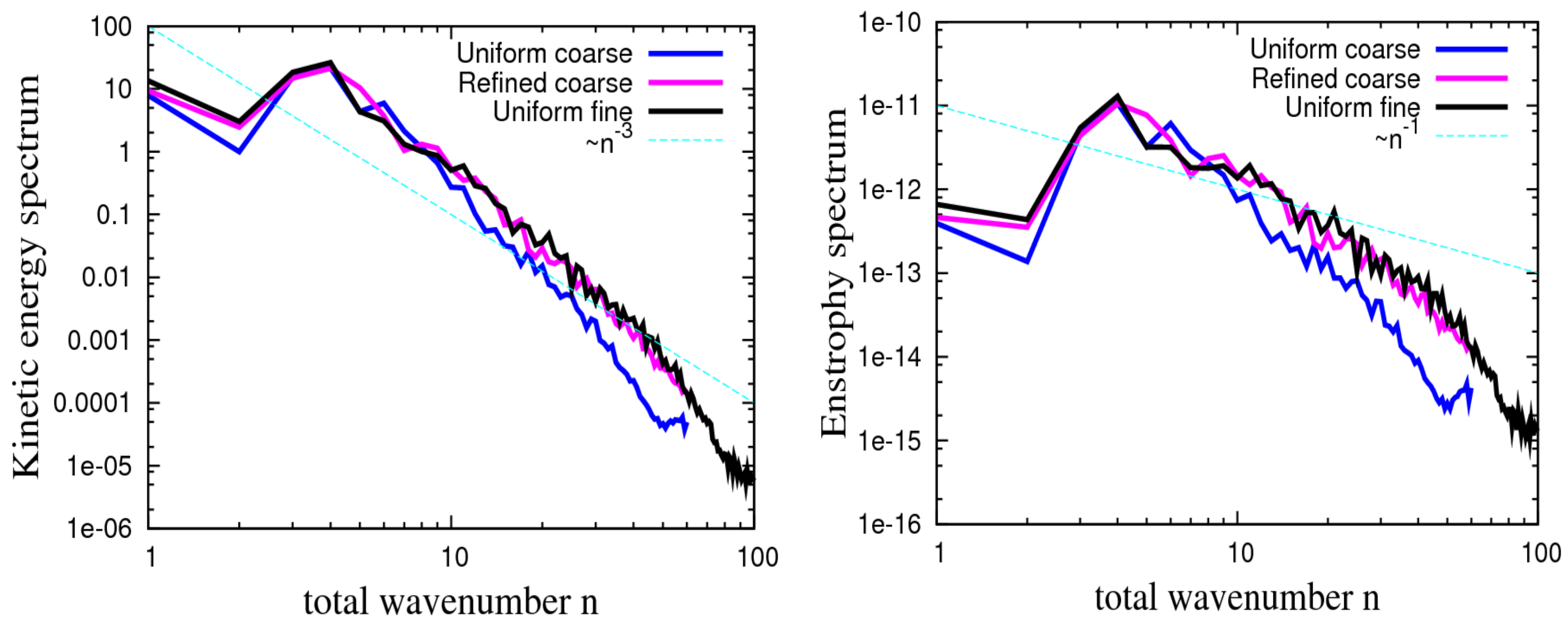
We developed a global shallow water model using a new finite element approach that combines a continuous representation of second order polynomials for the scalar field with a discontinuous representation of first order polynomials for the velocity field on a triangular grid. The specific element is able to hold geostrophic balance and a required stability criteria.[1,2]

The finite element approach was chosen to investigate the potential of local grid refinement in geophysical fluid dynamics. Local grid refinement appears attractive because of its potential to increase efficiency and the ability to resolve local phenomena.

Local refinement: Impact on energy spectra

We simulate a global zonal jet over an isolated mountain[7], behind which Rossby waves form and initialize a turbulent eddy decay. We perform simulations on two uniform icosahedral geodesic grids with an averaged edge length of 240 km and 480 km. A third simulation is performed on a refined grid, where the coarser uniform grid is refined around the mountain by two refinement levels of doubled resolution.

After 100 days the kinetic energy and enstrophy spectra of the refined run are moved towards the higher resolution results.



Local refinement: Boundary separation and dynamical systems theory

Grid refinement allows a detailed representation of boundary currents, and boundary separation. To this end we simulate an Atlantic shaped ocean basin on the sphere with free-slip boundary conditions and plane topography at 1000 m depth. An artificial wind forcing introduces a Munk gyre that is balanced by bottom friction. We use refinement to get a detailed representation of the western boundary current.

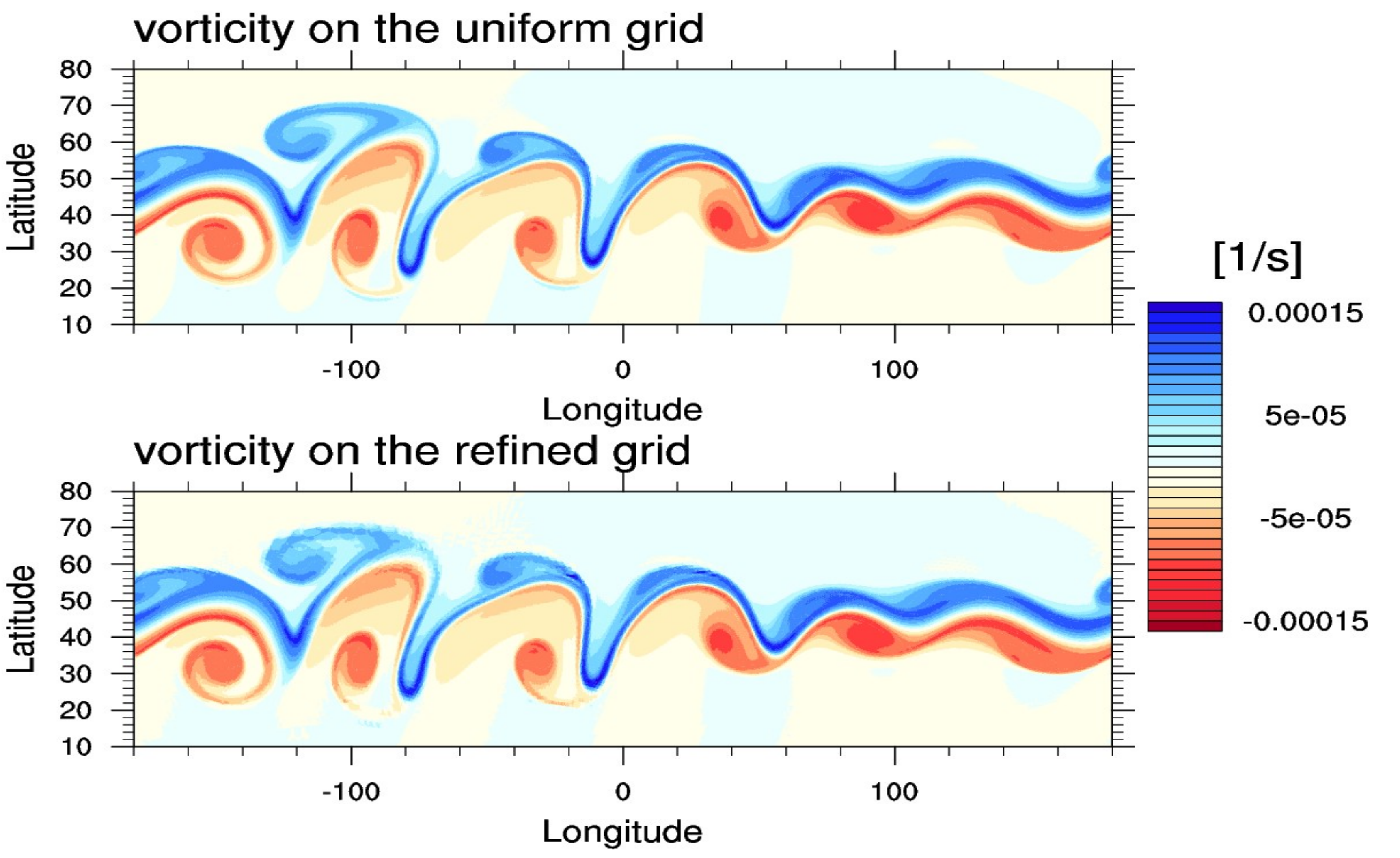
Summary: Model development & grid refinement

The model is carefully tested with the suite of Williamson test cases on uniform grids.[7] The model shows good performance in these classical tests, also with respect to error convergence and energy spectra.[3]

We compare simulations of wave packets, turbulent eddy decays, and geostrophical balanced water hills on uniform grids, and on refined grids with abrupt changes in grid spacing. Tests show that spurious small scale wave patterns are apparent at the transition between coarse and fine grids, but they do not affect larger scale flow features (not shown here)[4]. A deliberated use of grid refinement can improve the model performance and the representation of local phenomena significantly.[4]

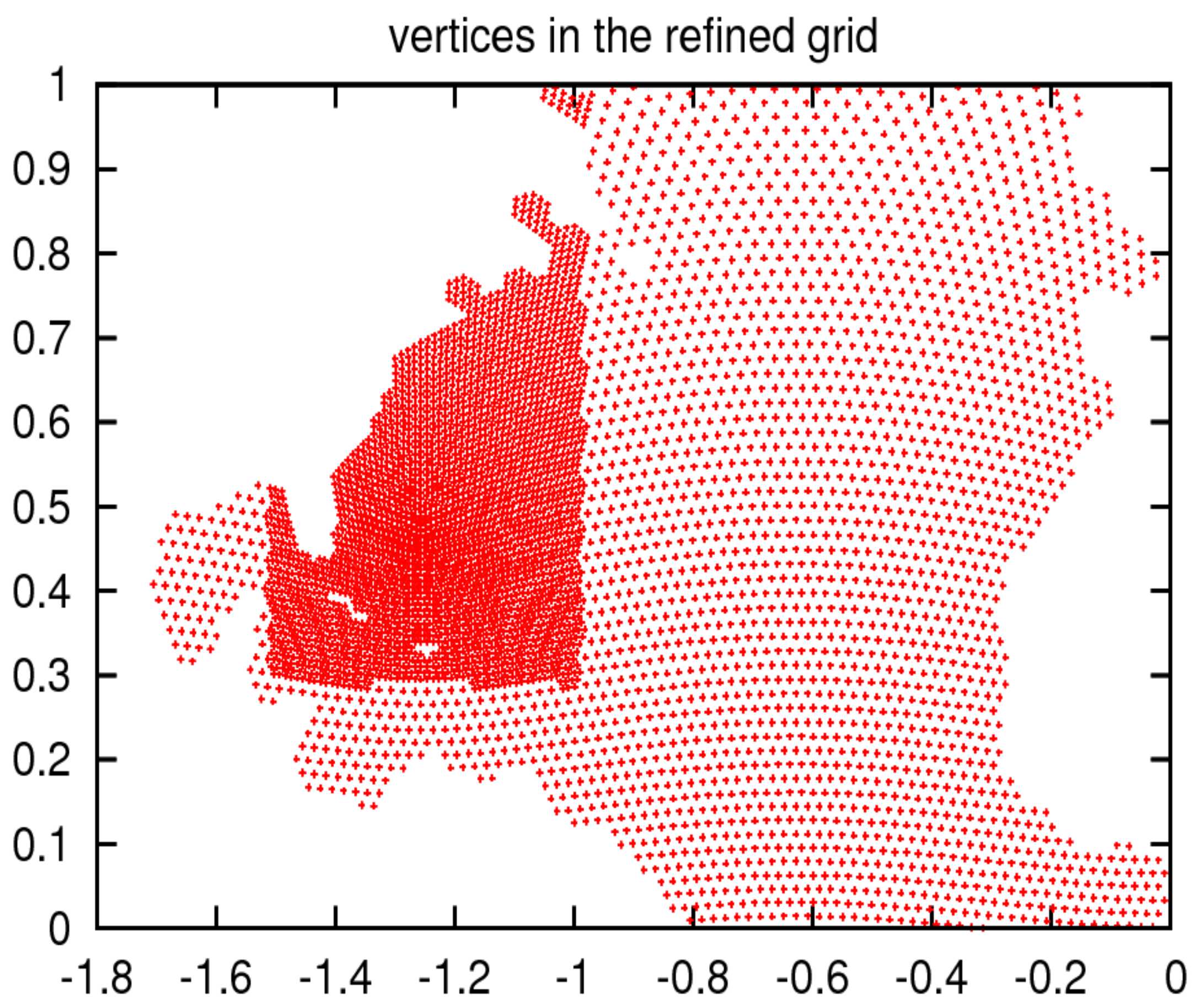
Local refinement: Efficiency:

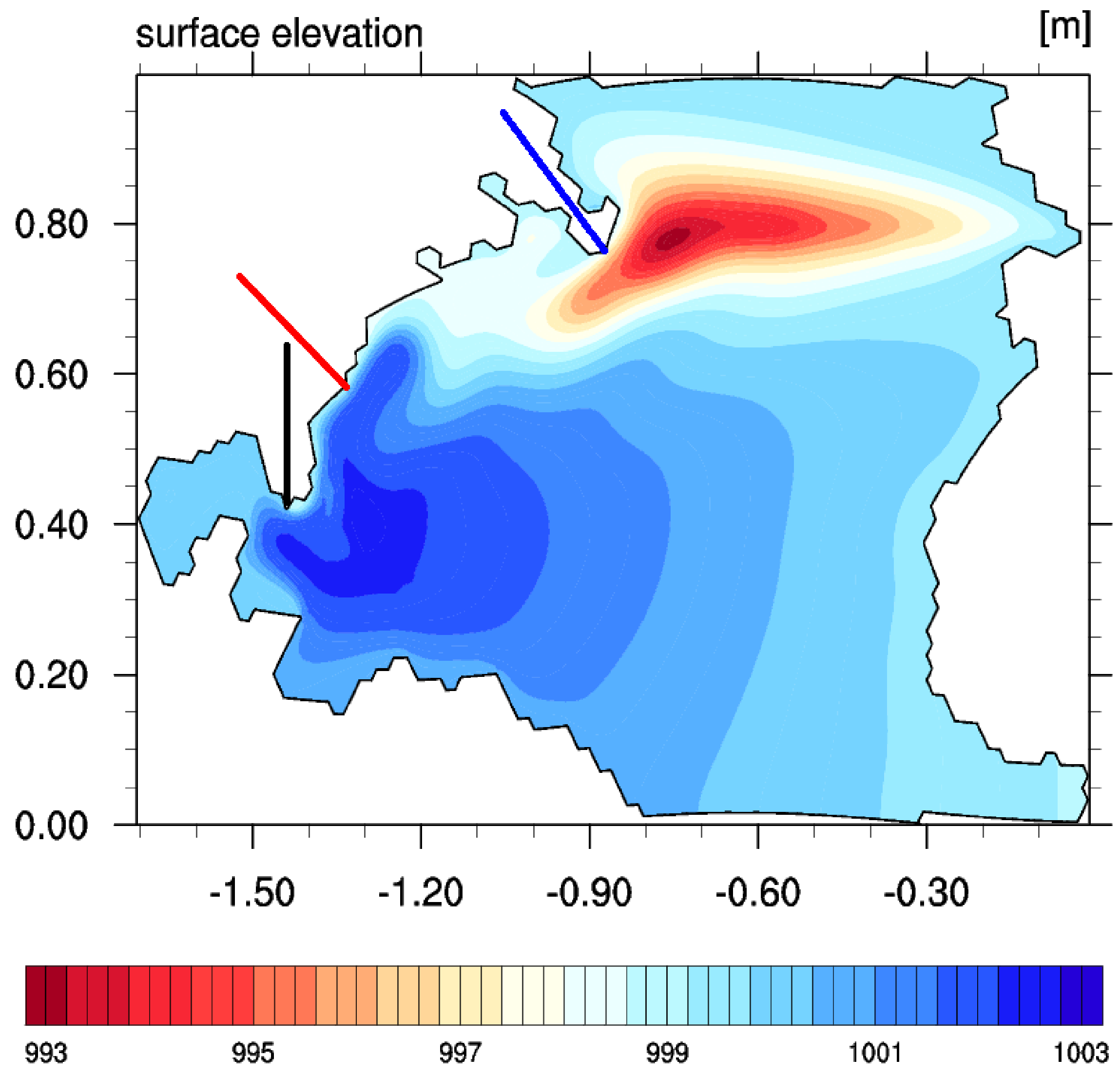
A global barotropically unstable mid latitude jet that is balanced by the height field, is disturbed by a small height perturbation, added to the initial state[5]. After several days of integration the perturbation leads to a turbulent decay of the solution. Simulations are performed on a uniform icosahedral geodesic grid with **1.600.000** degrees of freedom (top) and a refined icosahedral geodesic grid with **370.000** degrees of freedom (bottom). Results show that grid refinement enables to solve this challenging test with only ~23 % of the degrees of freedoms.

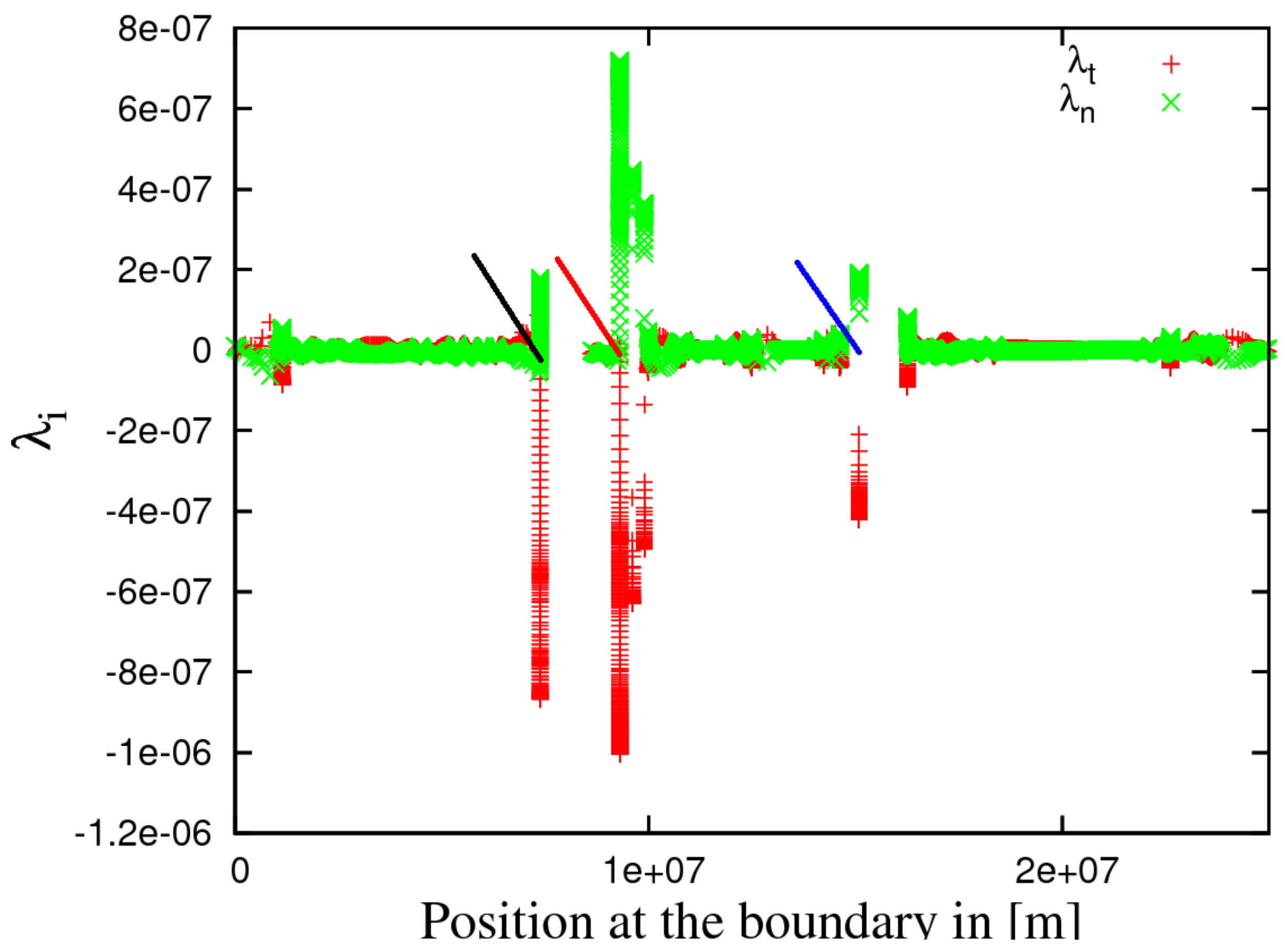


We are able to identify the points of flow separation using an approach rooted in dynamical systems theory. The approach is only dependent on the velocity field along the coastlines. Separation is indicated when two Lyapunov type exponents λ_n and λ_t show a maximum and a minimum.[6] λ_n is given below, integrating along the boundary trajectory $\boldsymbol{x}(s)$. λ_t is calculated analog.

$$\lambda_n(t) = \liminf_{T\to+\infty} \int_{t-T}^{t} T^{-1} < \boldsymbol{n}, (\nabla \boldsymbol{v})\boldsymbol{n} >_{[\boldsymbol{x}(s),s]} ds = 0$$







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