Analysis of relative dynamics of different cycles with the use of phase portraits

Mokhov I.I.

A.M. Obukhov Institute of Atmospheric Physics RAS, Moscow, Russia

mokhov@ifaran.ru

The method of cycles proposed by Mokhov (1993) (see also Mokhov (1995), Mokhov and Eliseev (1998), Mokhov et al., (2000), Mokhov et al., (2004)) can be extended for the analysis of relative dynamics of different cycles (cross-cycles analysis). It is based on the analysis of phase portraits for given time series $X(t)$. In particular, if there is a statistically significant linear regression of $d^2X(t)/dt^2$ on $X(t)$ with a negative regression coefficient $-\omega^2(t)$, then the process can be fitted by a harmonic oscillator:

$$d^2X/dt^2 + \omega^2 X = 0 \qquad (1)$$

or

$$X(t) = A(t)\sin[\omega(t)t + \varphi(t)]\ . \qquad (2)$$

The amplitude of the process $A(t)$, its frequency $\omega(t)$, and period $P(t) = 2\pi/\omega(t)$, as well as the initial phase $\varphi(t)$, are assumed to change sufficiently slowly with time. Equation (1) is considered at an interval of the length $I_0 \geq P$, such that $|I_0 d\Omega/dt| << \Omega$ and $|I_0 dA_c/dt| << A_c$, where $d\Omega/dt$, Ω, dA_c/dt, and A_c are typical values of $d\omega/dt$, ω, dA/dt, and A respectively (Mokhov and Eliseev, 1998; Mokhov et al., 2004).

The variables dX/dt and d^2X/dt^2 can be determined by taking the second-order finite differences of the original time series $X(t)$. The frequency $\omega(t)$ (and the period $P(t)$) are calculated using the least-squares fitting technique at a moving segment of length I_0. Then, using this same moving segment, the amplitude $A(t)$ and phase $\varphi(t)$ can be determined with the use of the least-squares method and (2). To filter out the higher frequency noise, the raw data can be smoothed taking running means at the window I_s.

The method described above is sensitive to the parameter I_0. On the one hand, I_0 must not be less than the characteristic period of variations in the original data set. On the other hand, I_0 acts as a filter preserving this dominant oscillation modulated by variations with time scales longer than I_0. Thus, it is necessary that

$$I > \tau_P\ , \quad \tau_A >> I_0 \geq 2\pi/\Omega >> I_s$$

or at least

$$I - I_s + \delta t > \tau_P , \quad \tau_A > I_0 \geq 2\pi / \Omega > I_s ,$$

where I is the full length of the time series, δt is its time step, τ_P and τ_A are typical times of the changes with a period P and amplitude A respectively (Mokhov and Eliseev, 1998; Mokhov et al., 2004).

This method can be applied to two variables $X_1(t)$ and $X_2(t)$ separately. Then the phase difference $\Delta\varphi(t) = \varphi_2(t) - \varphi_2(t)$ will characterize the relative dynamics of the corresponding cycles of these two variables. Other characteristics of this cross-cycles analysis can be determined from the corresponding linear regressions at a moving segments of length I_0. In this case the coefficient of linear regression and coefficient of correlation of $X_2(t)$ to $X_1(t)$ (or $X_1(t)$ to $X_2(t)$) characterize the cross-amplitude and coherence, respectively. The first estimates have been obtained with this cross-cycles analysis method for relative dynamics of quasi-decadal and quasi-pentadal cyclicity of North Atlantic and Arctic Oscillations, Pacific/North American teleconnection pattern and El-Nino phenomena.

This work has been partly supported by the Russian Foundation for Basic Research and by the RAS program.

References

1. Mokhov, I.I. (1993) Climate changes: Analyses of global cycles. *Ann. Geophys.,* **12** (Suppl. II), C334.
2. Mokhov, I.I. (1995) Diagnostics of climate system structure and its evolution in the annual cycle and interannual variability. Moscow, IAP RAS, 64 pp. (in Russian)
3. Mokhov, I.I., and A.V. Eliseev (1998) Tendencies of change of QBO characteristics for zonal wind and temperature of equatorial lower stratosphere. *Izvestiya, Atmos. Oceanic Phys.,* **34**, 327-336.
4. Mokhov, I.I., A.V. Eliseev, D. Handorf, V.K. Petoukhov, K. Dethloff, A. Weisheimer, and D.V. Khvorostyanov (2000) North Atlantic Oscillation: Diagnosis and simulation of decadal variability and its long-period evolution. *Izvestiya, Atmos. Ocean Phys.,* **36**, 555-565.
5. Mokhov, I.I., D.V. Khvorostyanov, and A.V. Eliseev (2004) Decadal and longer term changes in El Nino – Southern Oscillation characteristics. *Intern. J. Climatol.,* **24,** 401-414.